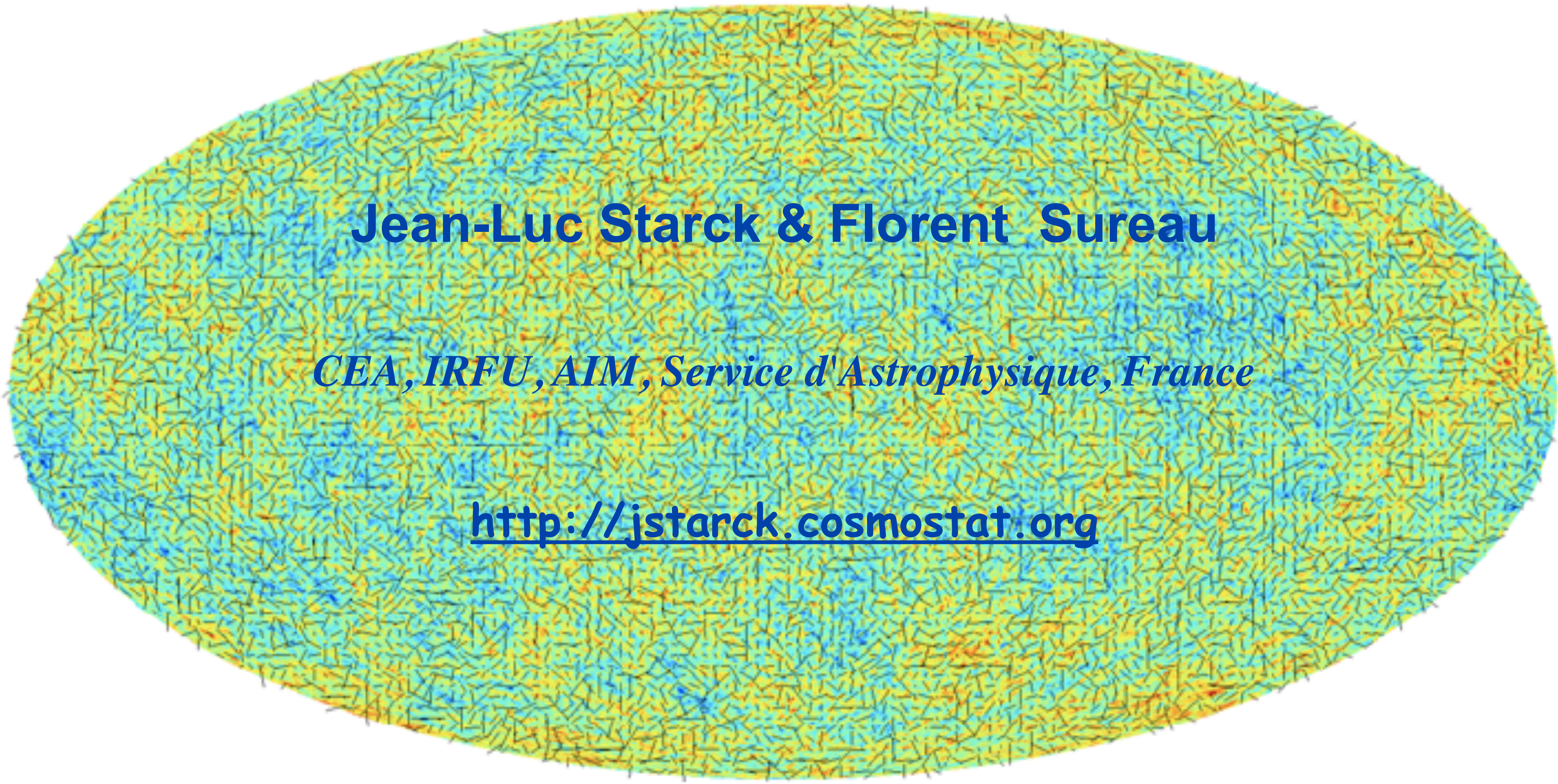
Cosmic Microwave Background Field Estimation



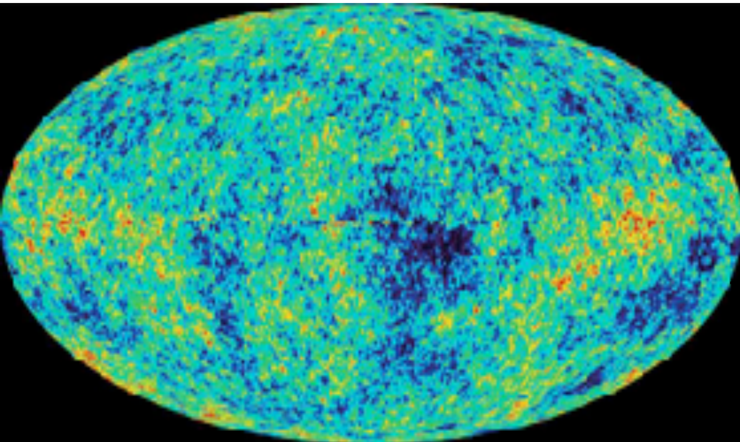
Jean-Luc Starck & Florent Sureau

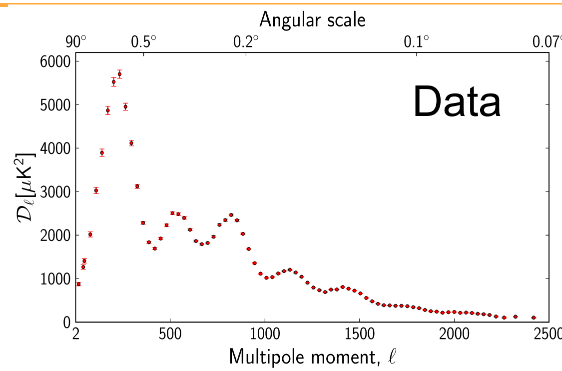
CEA, IRFU, AIM, Service d'Astrophysique, France

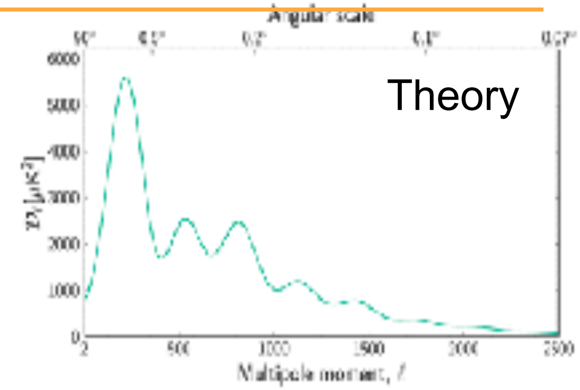
http://jstarck.cosmostat.org

- **Introduction to CMB**
- CMB Temperature Inpainting Story
- CMB Polarized Inpainting

Statistical Properties of the CMB fluctuations



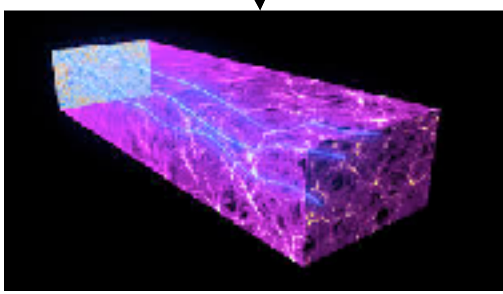




Cosmological Parameters

Search for specific signatures predicted by inflation models

constraints on inflation models (fnl)



Statistical analysis of the weak lensing effect

gravitationnal potential mapping

+

power spectrum

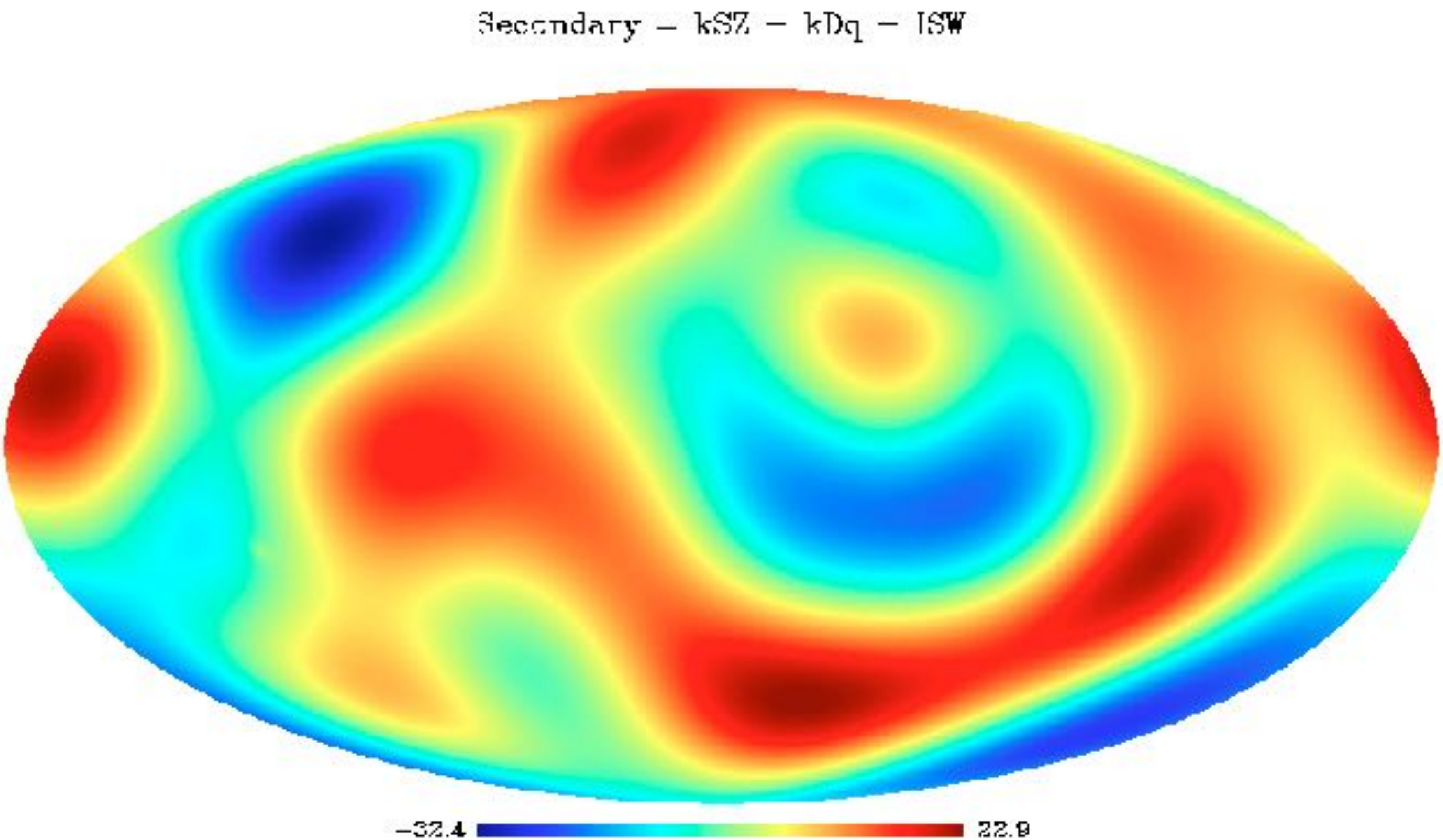
Large scale analysis

Topology of the univers, inflation, ISW, etc

Integrated Sachs-Wolfe Effect (ISW)

Constraint on dark energy

Secondary Temperature Map

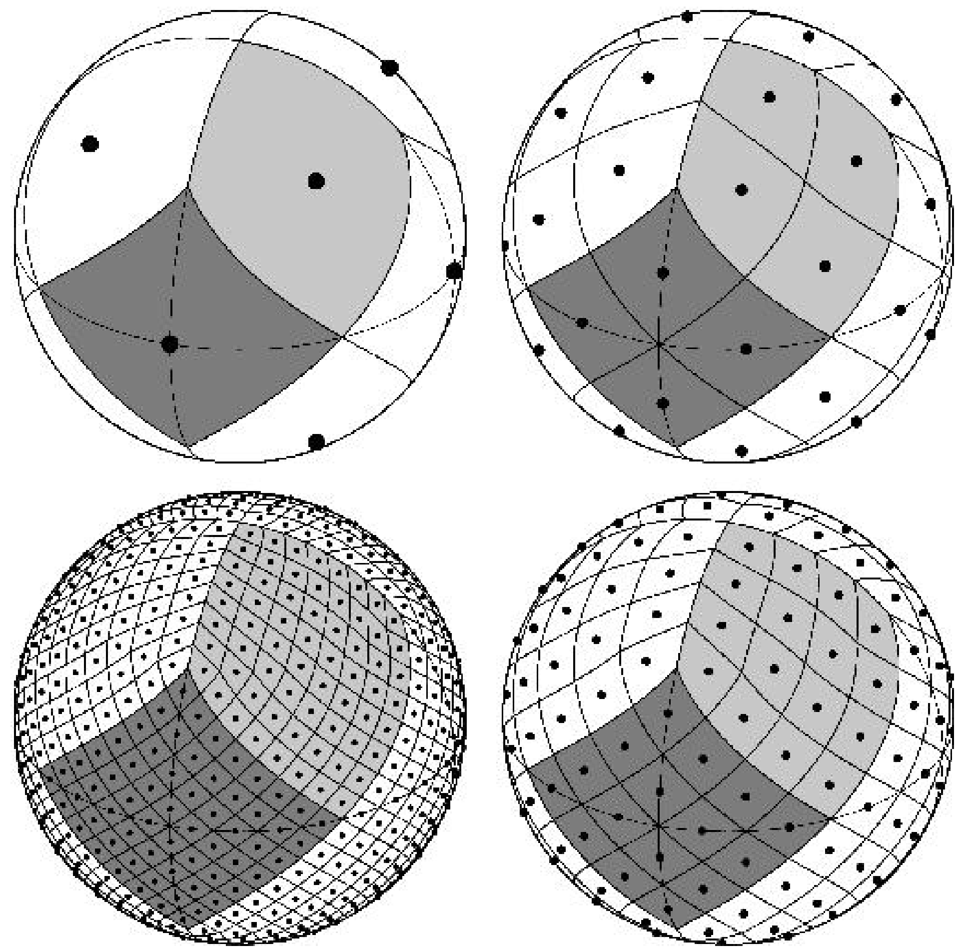


$$T_{CMB}^{obs} = T_{primordial} + kSZ + kDq + ISW$$

Healpix

K.M. Gorski et al., 1999, astro-ph/9812350,
http://www.eso.org/science/healpix

- Pixel = Rhombus
- Same Surfaces
- For a given latitude : regularly spaced
- Number of pixels:
  $12 \times (N_{sides})^2$
- Included in the software:
  - Anafast
  - Synfast



Any function $f(\theta, \vartheta)$ on the sphere S^2 can be decomposed into spherical harmonics:

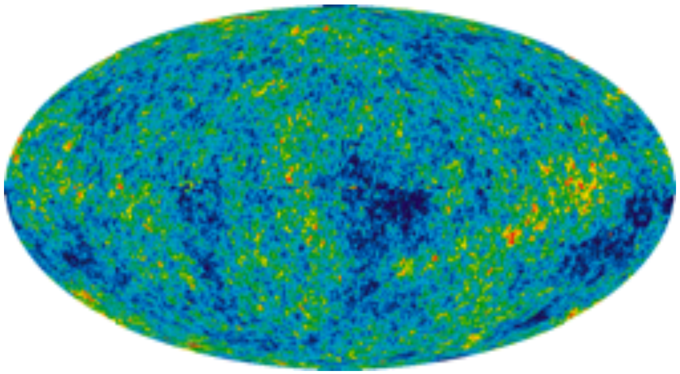
$$f(\theta, \vartheta) = \sum_{l=0}^{+\infty} \sum_{m=-l}^{l} a_{lm} Y_{lm}(\theta, \vartheta),$$

where Y_{lm} are the spherical harmonics defined by:

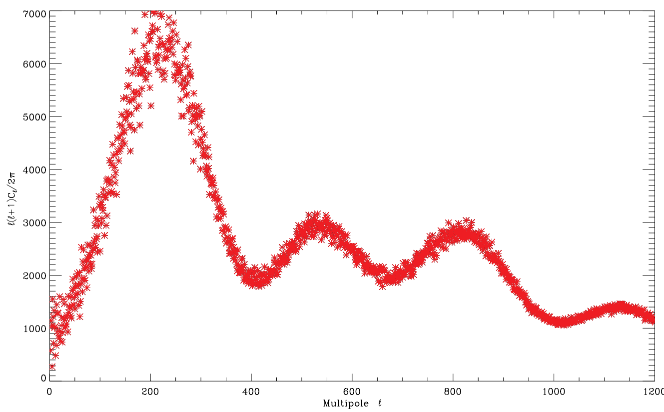
$$Y_{lm}(\theta, \vartheta) = \sqrt{\frac{2l+1}{4\pi} \frac{(l-|m|)!}{(l+|m|)!}} P_{lm}(\cos\vartheta) e^{im\theta}$$

an important property of the Legendre polynomials is that they are orthogonal:

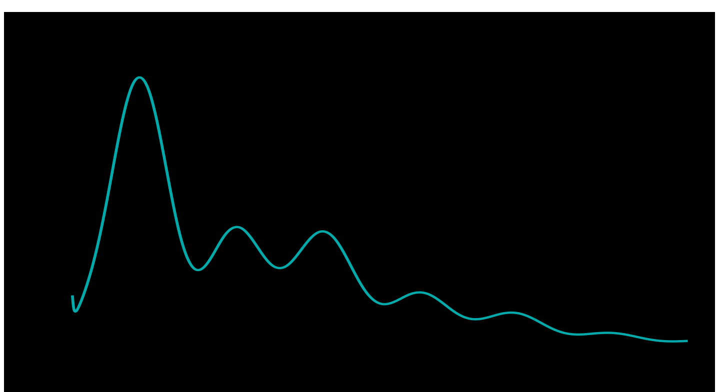
$$\sum_{l \in \mathbb{N}} \sum_{|m| \leqslant l} Y^*_{lm}(\boldsymbol{\omega}') \; Y_{lm}(\boldsymbol{\omega}) = \delta(\boldsymbol{\omega}' - \boldsymbol{\omega}).$$



$$f(\theta,\vartheta)=\sum_{l=0}^{+\infty}\sum_{m=-l}^{l}a_{lm}Y_{lm}(\theta,\vartheta),$$

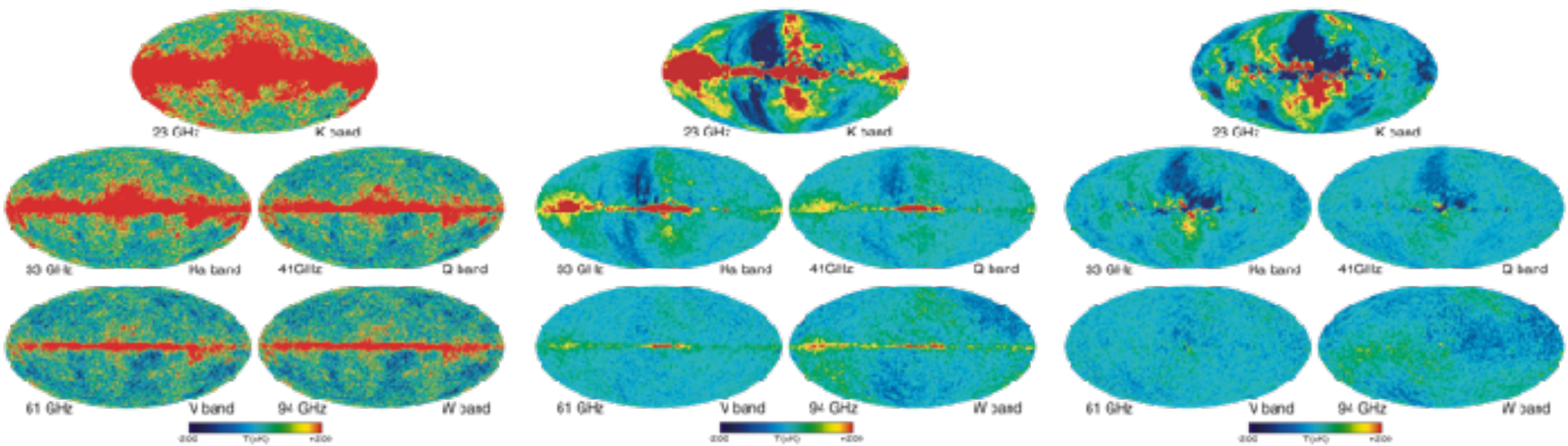


$$C_\ell=\frac{1}{2\ell+1}\sum_{m=-l}^{l}|a_{lm}|^2$$



Polarization Measurements in CMB Missions

- WMAP and Planck (not fully released) measure full sky polarized microwave data
- Described by 3 Stokes parameter: T, Q, U
- Q: difference of amplitude in two orthogonal detectors, U from two orthogonal detectors rotated by 45° from other pair



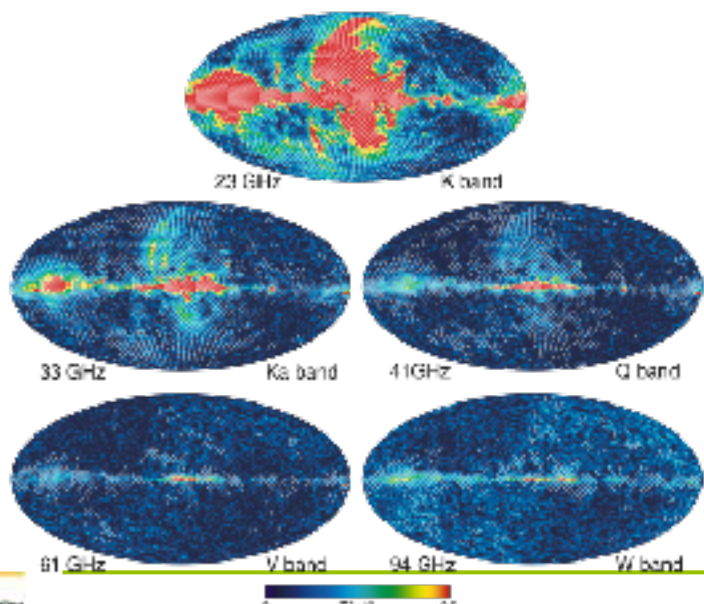
WMAP collaboration

Polarization in Full Sky CMB Missions

- Other representation: $T, P = \sqrt{U^2 + Q^2}, \phi = \mathrm{atan}(U/Q)/2$
- Q,U description ARE NOT invariant to a rotation θ of the coordinate system:

$$_{\pm 2}P' = Q' \pm iU' = \exp(\mp 2i\theta)(Q \pm iU)$$

- $(Q \pm iU)$ are spin-2/spin-(-2) fields
- Can be uniquely decomposed into a "gradient" (E) and "curl" (B) part, independent of the coordinate system through harmonic analysis



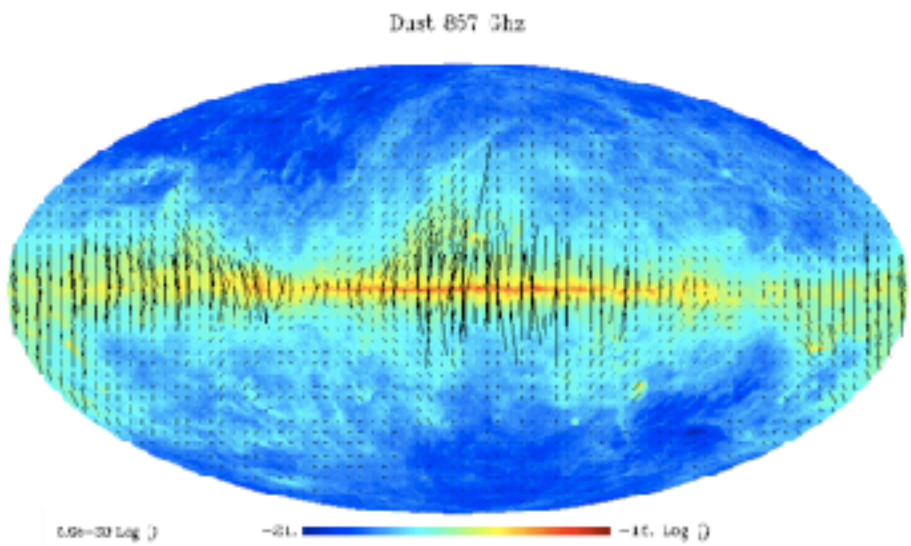
WMAP polarized maps P, ϕ

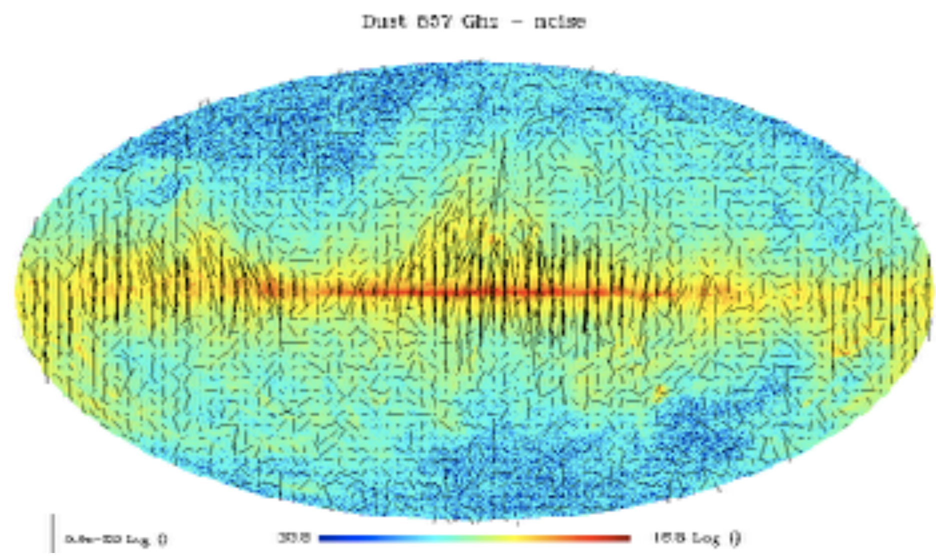
WMAP collaboration

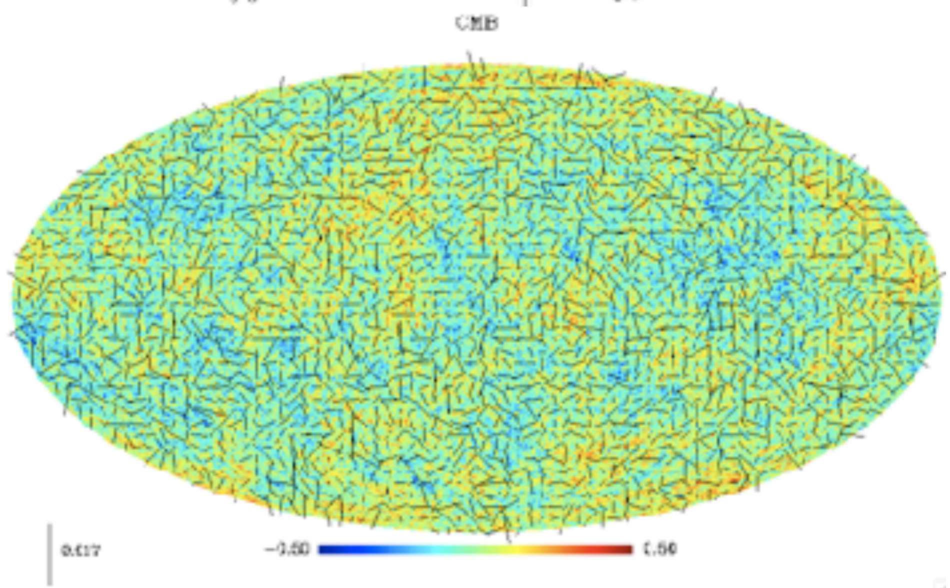
POLARIZED DATA: T, Q, U

Magnitude $P = \sqrt{Q^2 + V^2}$

Orientation $\alpha = \arctan\left(U/Q\right)$



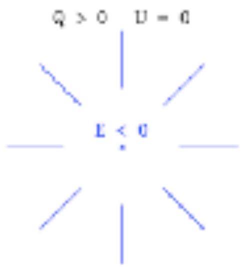




E/B Mode Decomposition

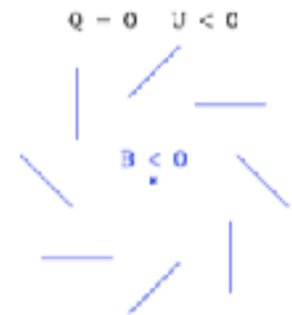
$$E = \sum_{\ell,m} a^E_{\ell m} Y_{\ell m} = \sum_{\ell,m} -\frac{{}_2a_{\ell m} + {}_{-2}a_{\ell m}}{2} Y_{\ell m} \qquad a^E_{lm} = -(a_{2,lm} + a_{-2,lm})/2$$

$$B = \sum_{\ell,m} a^B_{\ell m} Y_{\ell m} = \sum_{\ell,m} i\frac{{}_2a_{\ell m} - {}_{-2}a_{\ell m}}{2} Y_{\ell m} \qquad a^B_{lm} = i(a_{2,lm} - a_{-2,lm})/2$$



$$\mathrm{Q} = - \sum_{l,m} (a^E_{l,m} Z^+_{l,m} + i a^B_{l,m} Z^-_{l,m})$$

$$\mathrm{U} = - \sum_{l,m} (a^B_{l,m} Z^+_{l,m} - i a^E_{l,m} Z^-_{l,m})$$



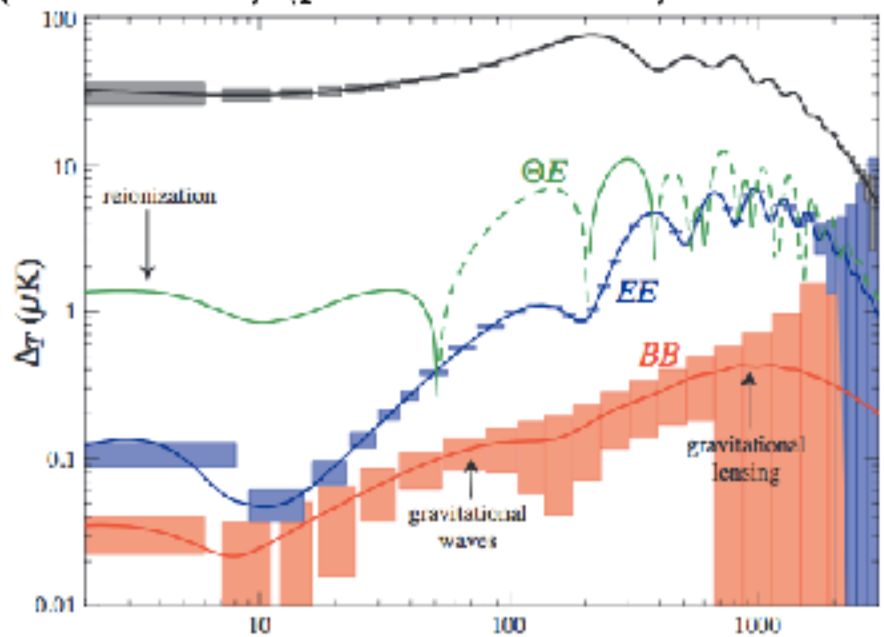
$$Z^+_{l,m} = ({}_2Y_{l,m} + {}_{-2}Y_{l,m})/2$$

$$Z^-_{l,m} = ({}_2Y_{l,m} - {}_{-2}Y_{l,m})/2$$

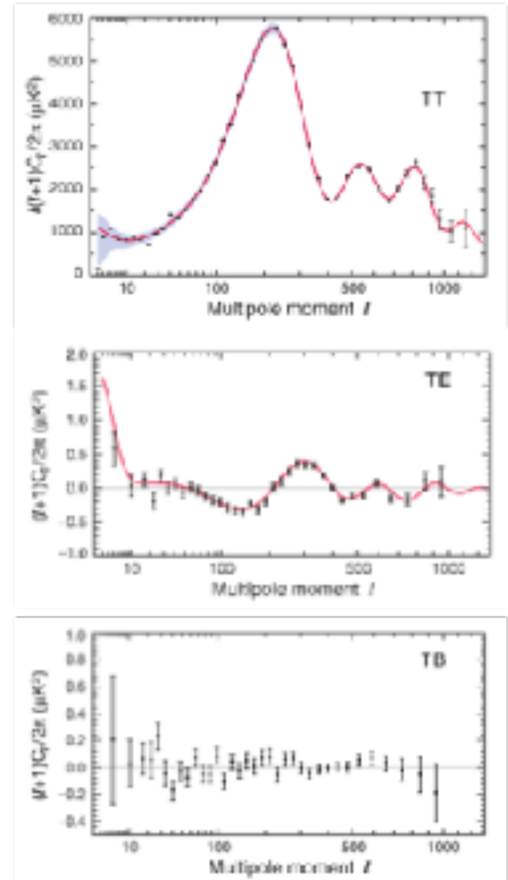
E and B mode are closely related to the curl-free and div-free components of the vector field

CMB Polarization

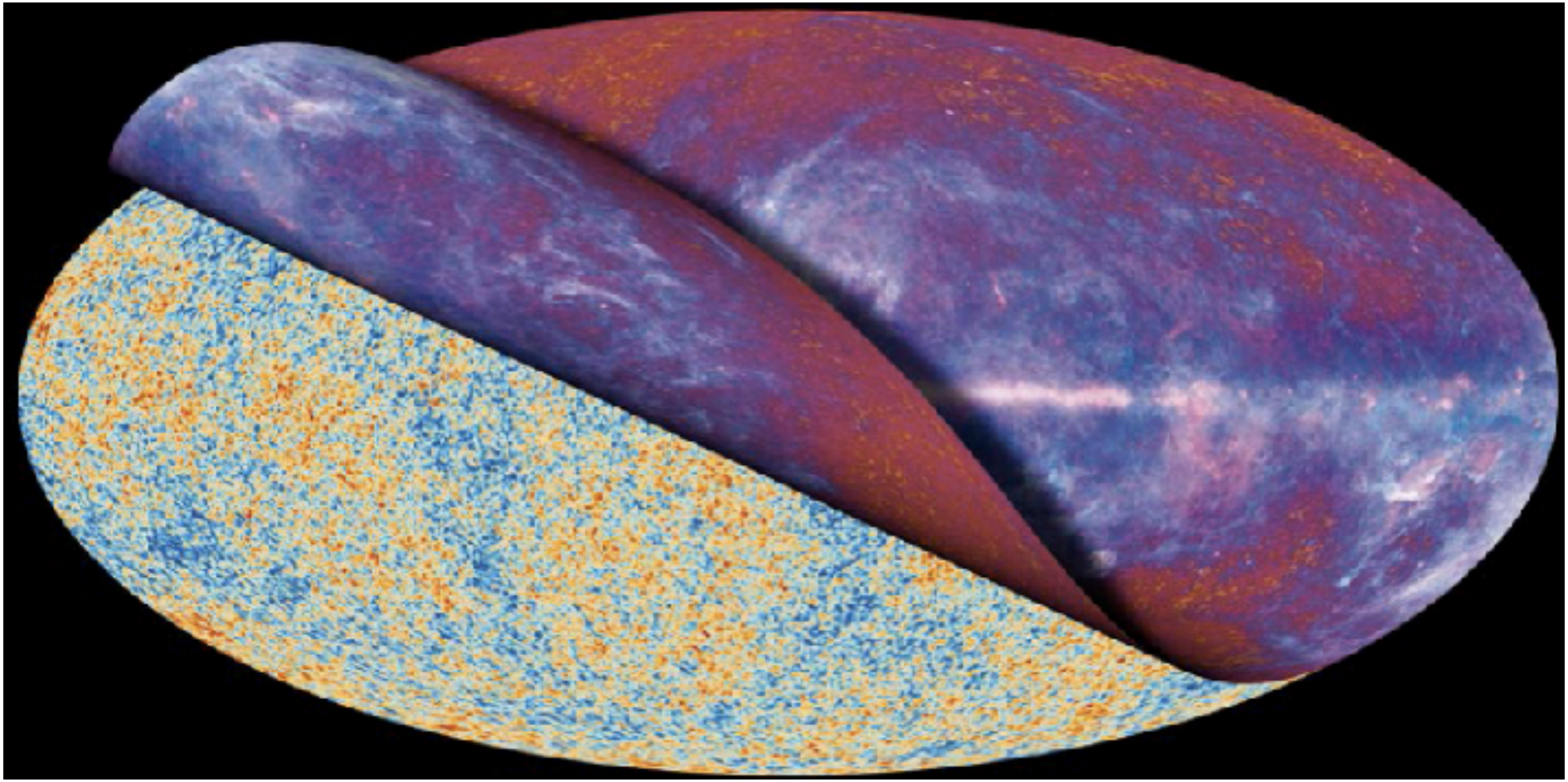
- CMB described by rotation invariant T, E, B
- Standard model: isotropic gaussian fields, characterized by C_ℓ^{TT}, C_ℓ^{EE} , C_ℓ^{BB}, C_ℓ^{TE} (parity: C_ℓ^{TB}=C_ℓ^{EB}=0)
- Density fluctuations at recombination: primordial E mode
- Gravitational waves at recombination: primordial B mode (and E mode) (probe for inflation)



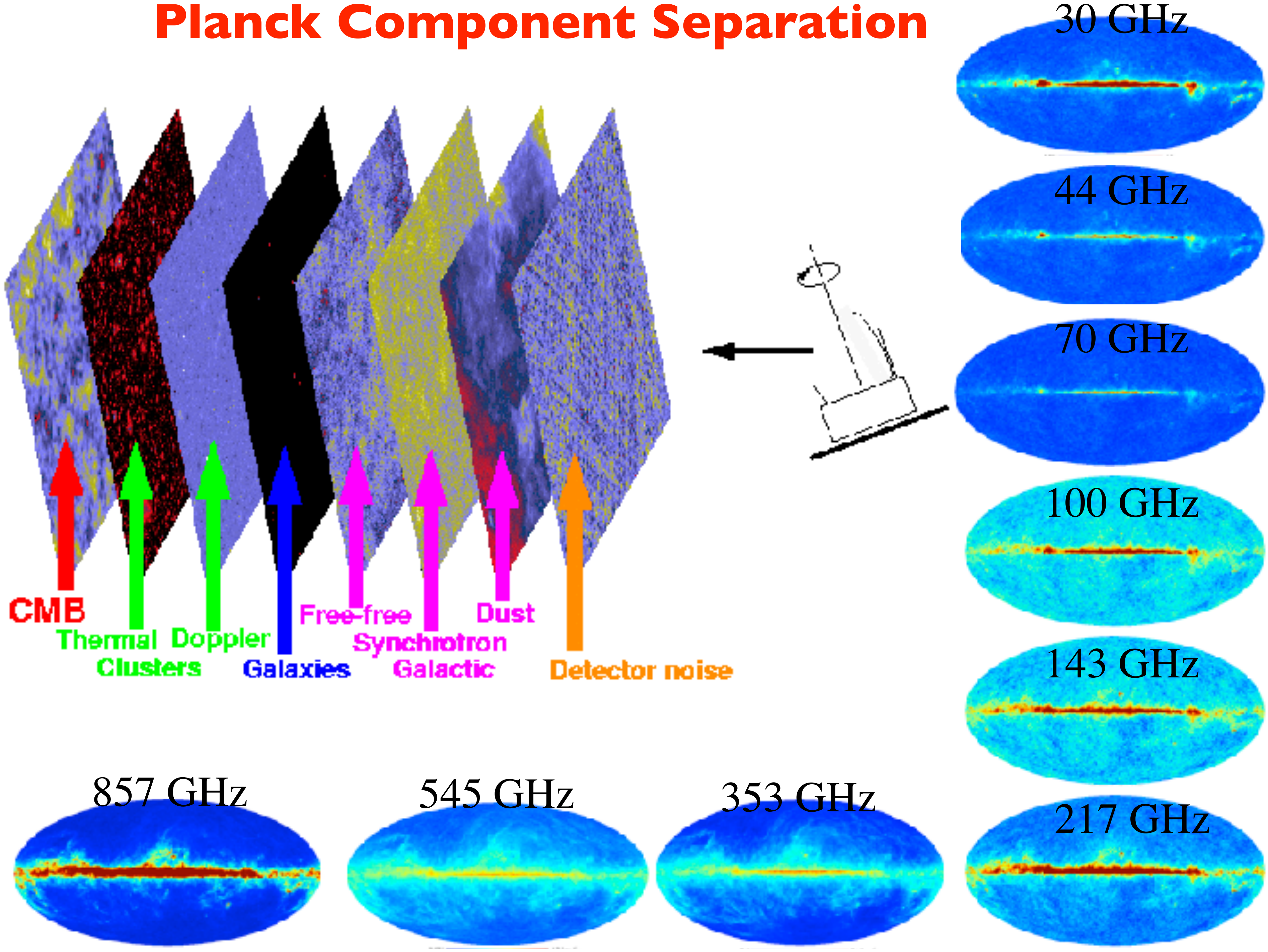
Hu, Dodelson 2002



WMAP collaboration



Planck Component Separation



Blind Source Separation

$$Y = AX + N$$

Sparse Blind Source Separation: the GMCA Method

X and S are estimated alternately and iteratively in two steps :

1) Estimate X assuming A is fixed (iterative thresholding) :

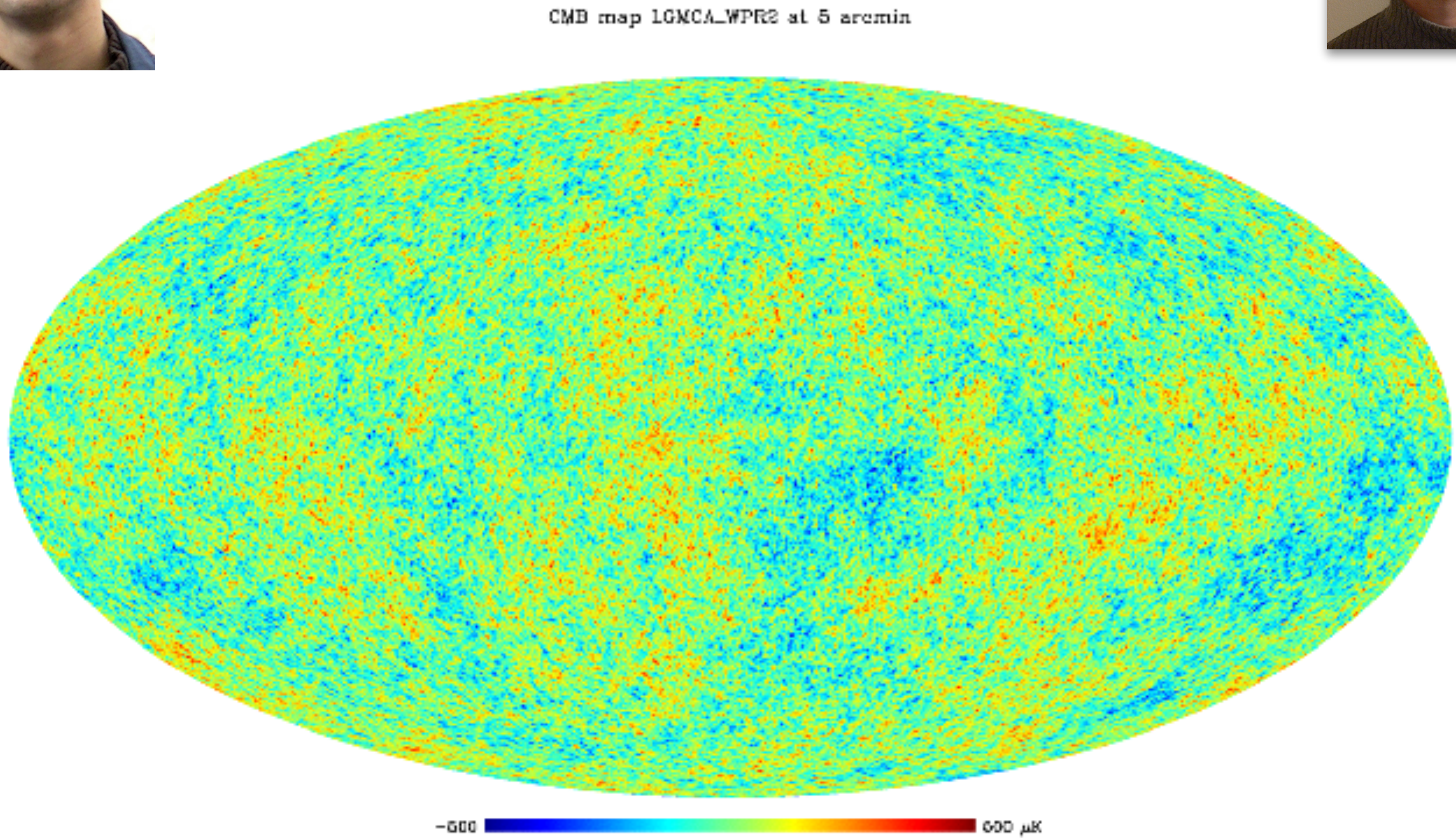
$$\min_X \|Y - AX\|^2_{F,\mathbf{\Sigma}} + \sum_j \lambda_j \|\Phi^t x_j\|_1$$

2) Estimate A assuming X is fixed (a simple least square problem) :

$$\min_A \|Y - AX\|^2_{F,\mathbf{\Sigma}}$$

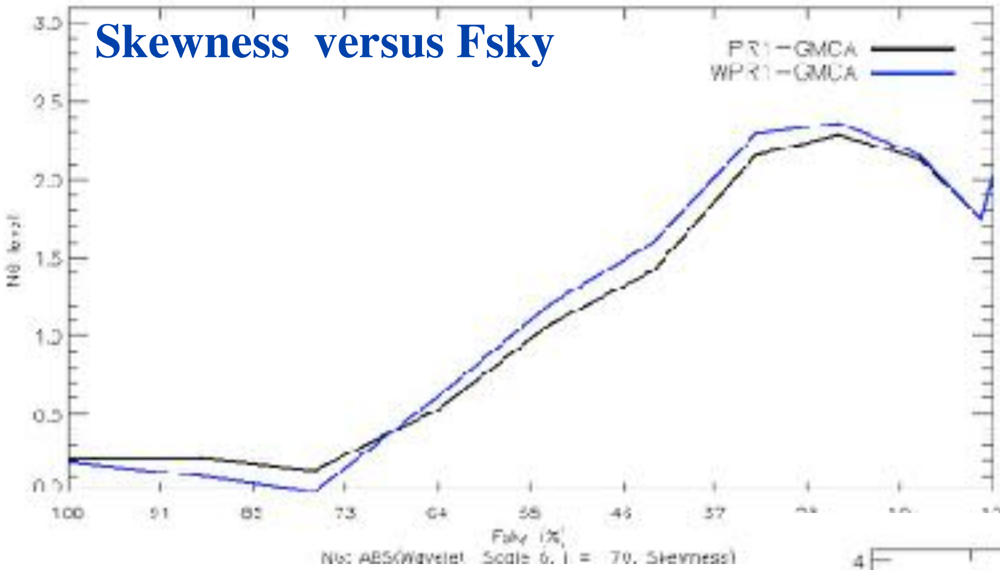
J. Bobin, J.-L. Starck, M.J. Fadili, and Y. Moudden, "Sparsity, Morphological Diversity and Blind Source Separation" , IEEE Transactions on Image Processing , Vol 16, No 11, pp 2662-2674, 2007.
J.Bobin , J. Rapin, J.L. Starck and A. Larue, Sparsity and adaptivity for the blind separation of partially correlated sources, IEEE Transactions on Signal Processing, vol. 63, issue 5, pp. 1199-1213, 2015.

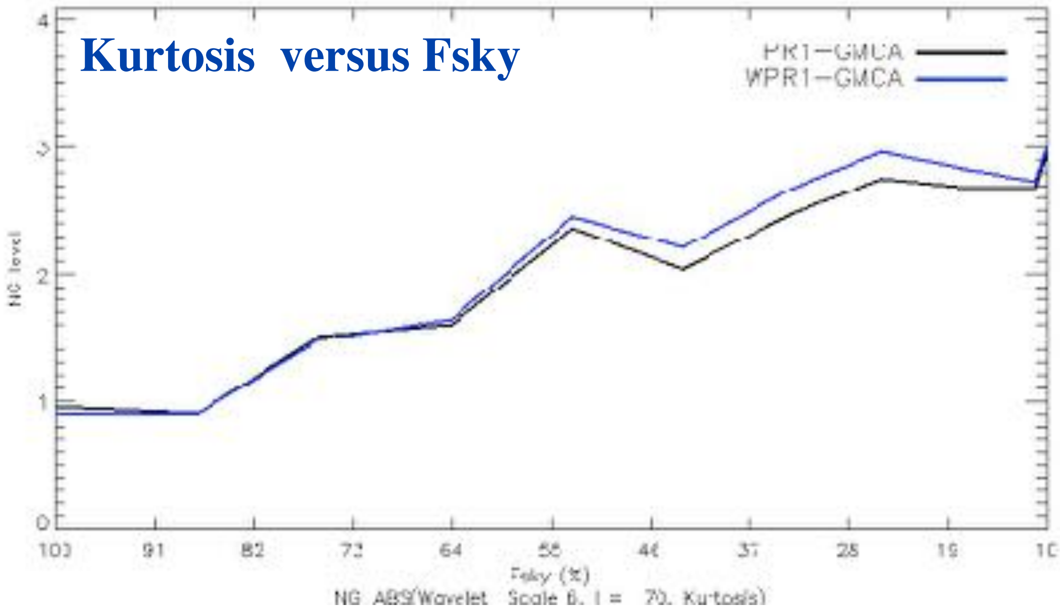
Full Sky Sparse WMAP + Planck-PR2 Map



Bobin J., Sureau F., Starck, CMB reconstruction from the WMAP and Planck PR2 data, A&A, 2016. [arXiv:1511.08690]

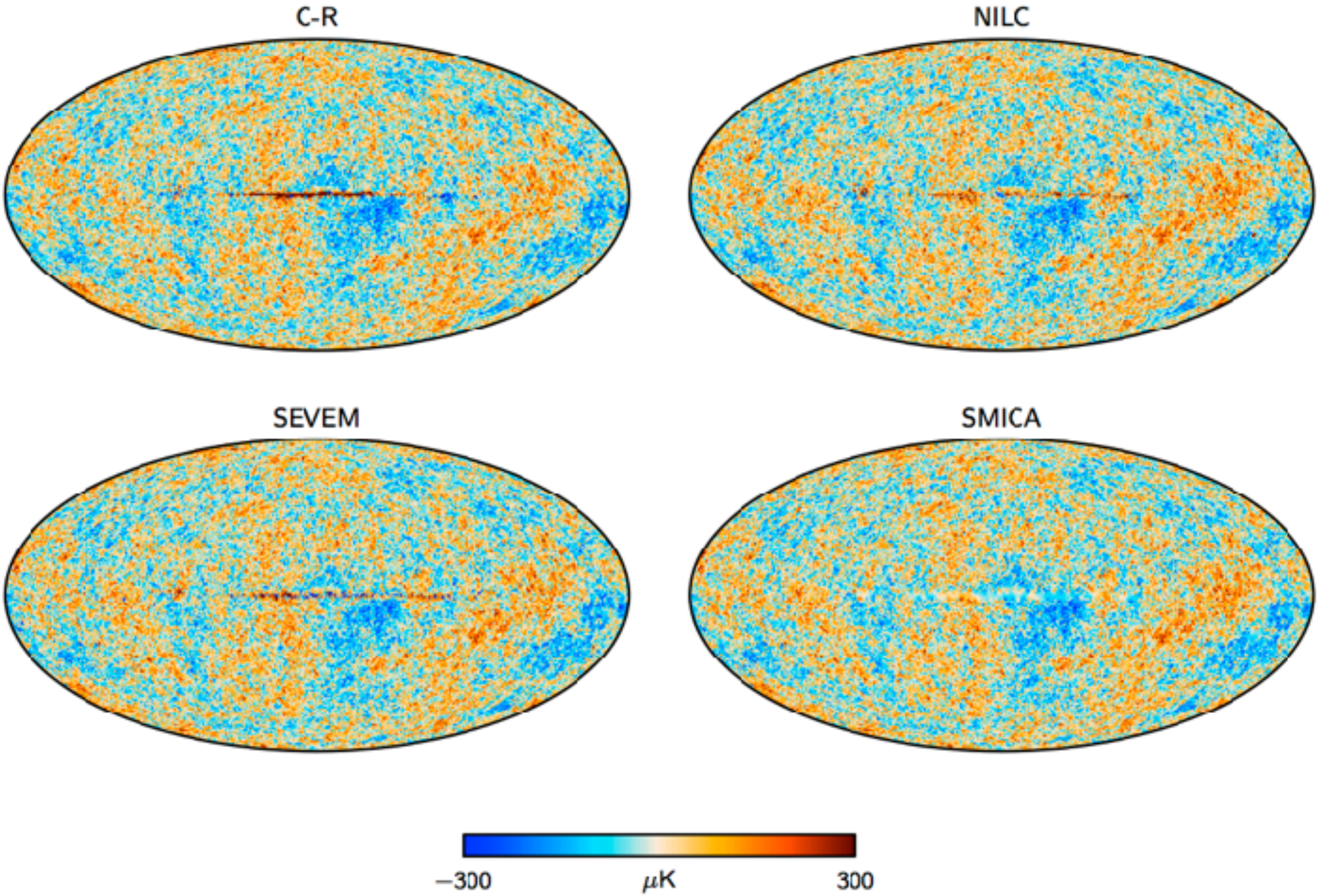
Normalized Skewness & Kurtosis





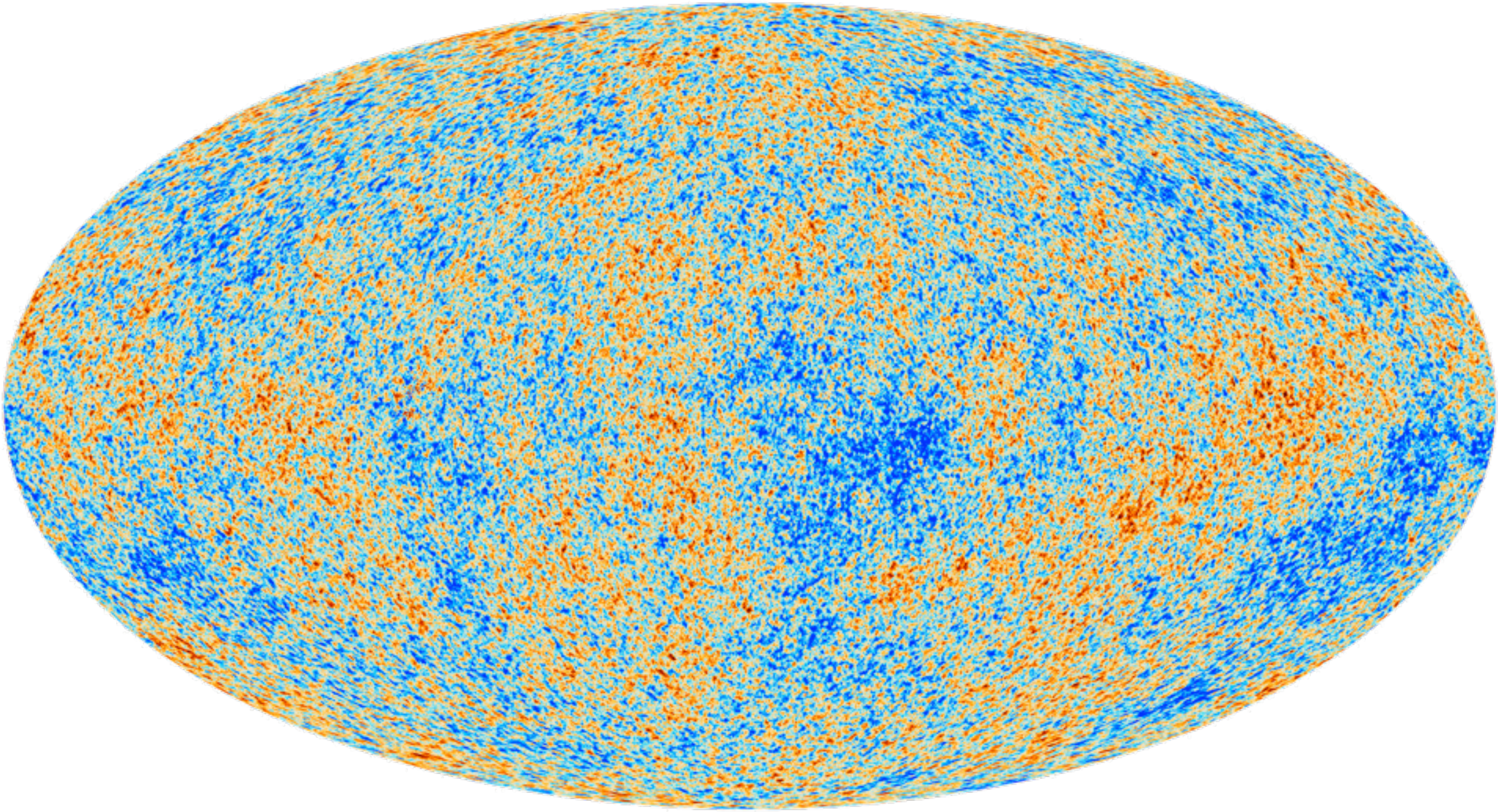
Commander-Ruler, Sevem, NILC, Smica

Planck Collaboration: *Planck* 2013 results. XII. Component separation



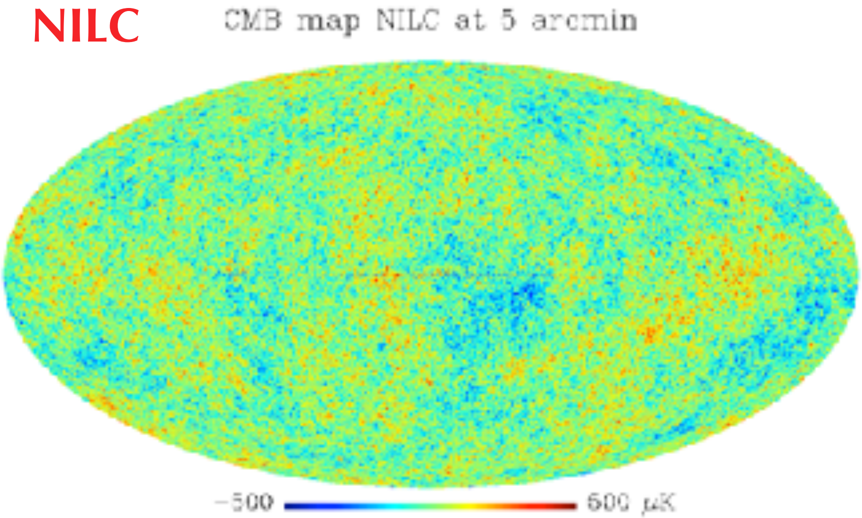
INPAINTING for the Planck-PR1 Public Press Release

Constraint Realization **Inpainting**

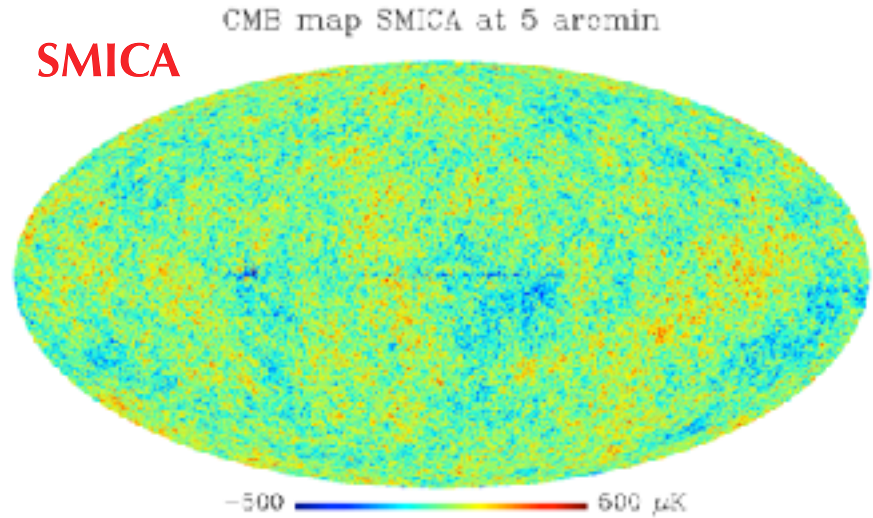


PLANCK PR2 CMB MAPS

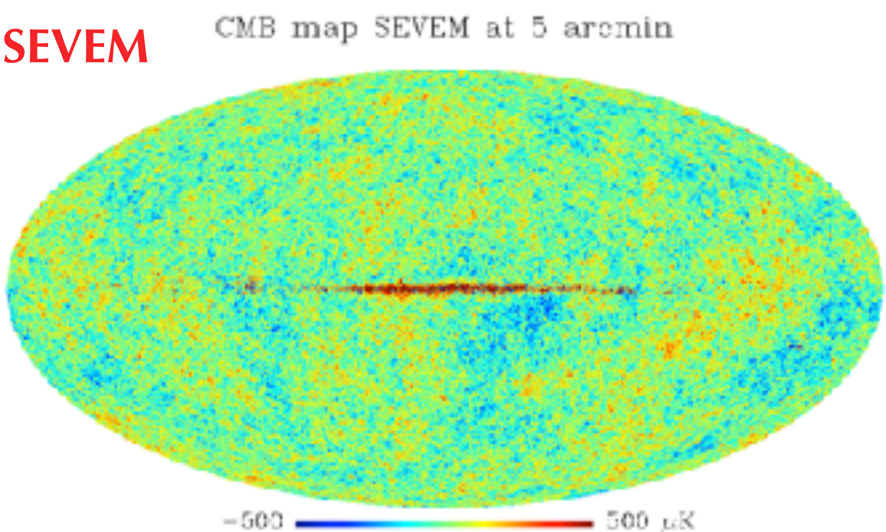
NILC



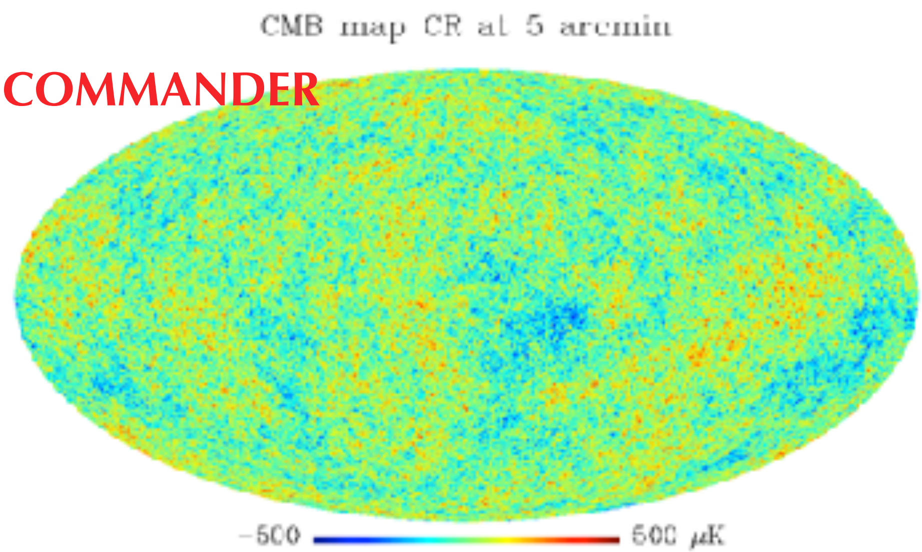
SMICA



SEVEM

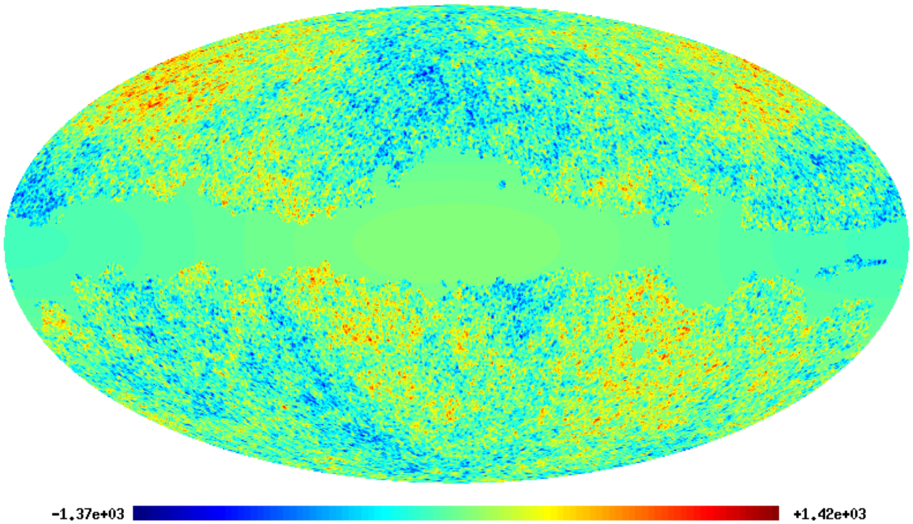


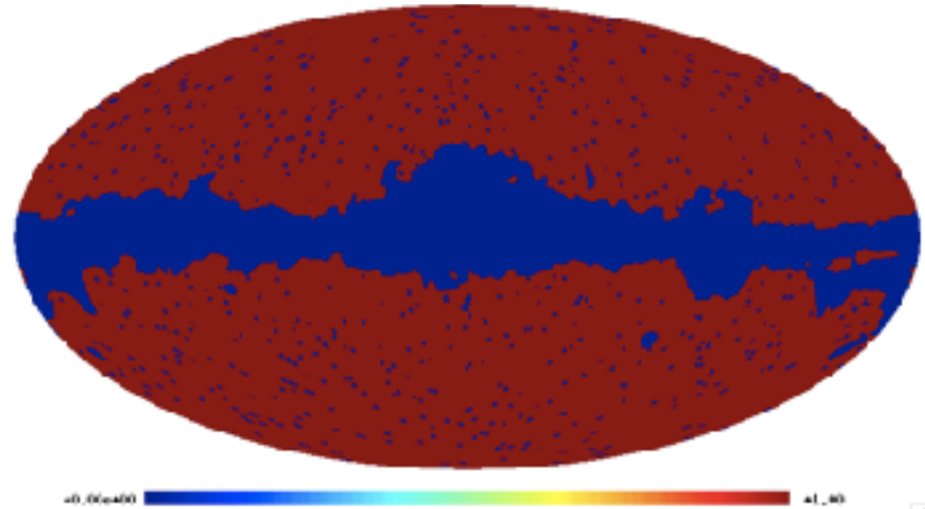
COMMANDER



- Introduction to CMB
- **CMB Temperature Inpainting Story**
- CMB Polarized Inpainting

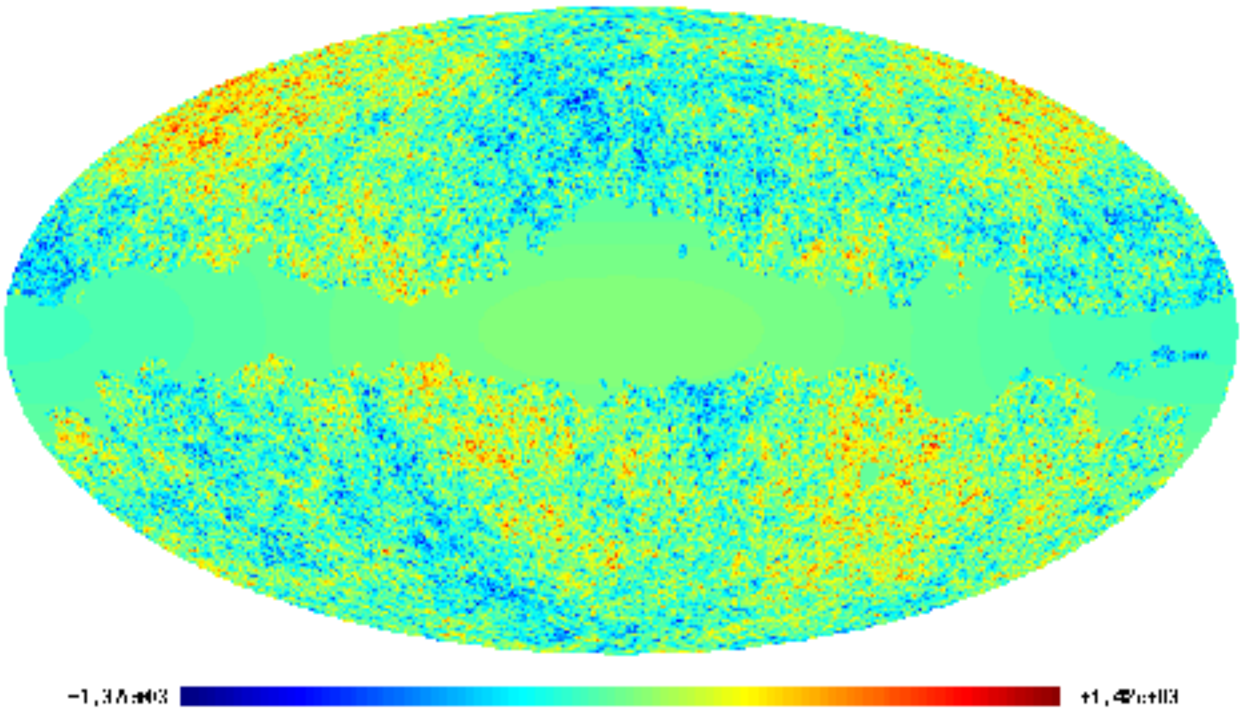
The Curse of Masking





•Point sources problem
•Gaussianity/isotropy test, especially in the spherical harmonic domain (bispectrum analysis, lensing estimator, large scale studies, etc).
•Any analysis where the mask is a problem.

WHY INPAINTING IS USEFUL FOR THE CMB ?

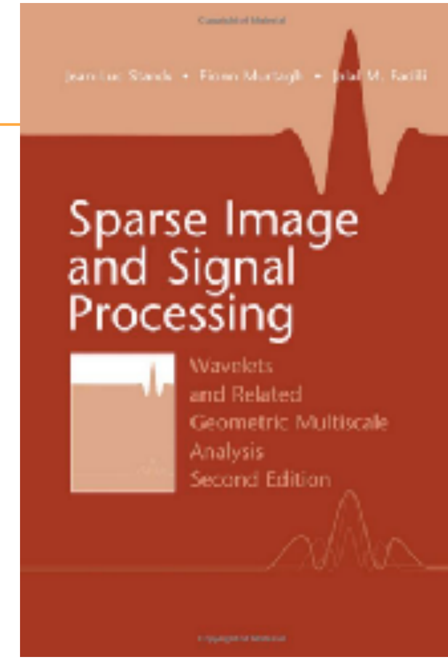


- Gaussianity test.
- Any analysis where the mask is a problem.

P. Abrial et al, *“**Inpainting on the Sphere**”, Astronomical Data Analysis Conference IV, September 18-20, Marseille, 2006*.

P. Abrial, Y. Moudden, J.L. Starck, et al, , "***Morphological Component Analysis and Inpainting on the Sphere: Application in Physics and Astrophysics***", Journal of Fourier Analysis and Applications (JFAA), 2007.

Inverse Problems: Sparse Recovery

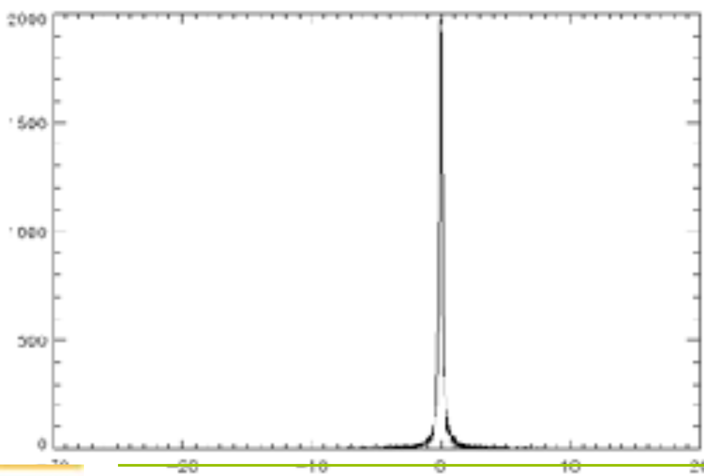


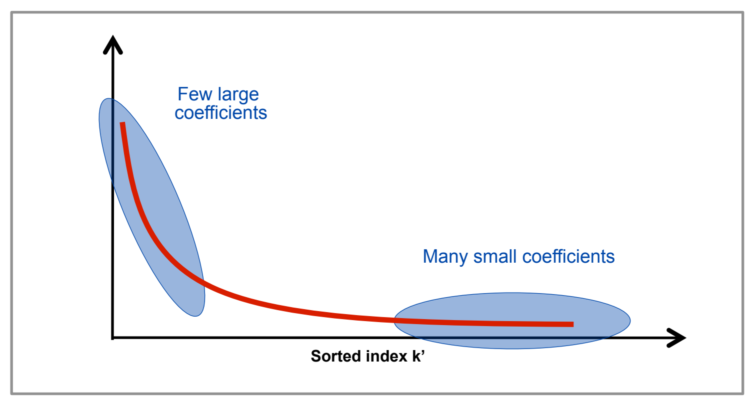
$$Y = HX + N$$

$$\min_{X} ||Y - HX||^2 + \mathcal{C}(\mathcal{X})$$

Sparse model: $X = \Phi\alpha$

$$\boxed{\min_{\alpha} ||Y - H\Phi\alpha||^2 + \lambda||\alpha||_p^p}$$





Interpolation of Missing Data

Sparse Inpainting

- *M. Elad, J.-L. Starck, D.L. Donoho, P. Querre, "Simultaneous Cartoon and Texture Image Inpainting using Morphological Component Analysis (MCA)", ACHA, Vol. 19, pp. 340-358, 2005.*

$$\mathbf{\Theta}_\Lambda = \mathbf{Id}_\Lambda \qquad \min_{\alpha} \|\alpha\|_{\ell_0} \text{ s.t. } y = Mx$$

Where M is the mask: M(i,j) = 0 ==> missing data
M(i,j) = 1 ==> good data

$$x^{(n+1)} = \mathcal{S}_{\Phi,\lambda^{(n)}} \left\{ x^{(n)} + M\left(y - x^{(n)}\right) \right\}$$

Iterative Thresholding with a decreasing threshold.

80%

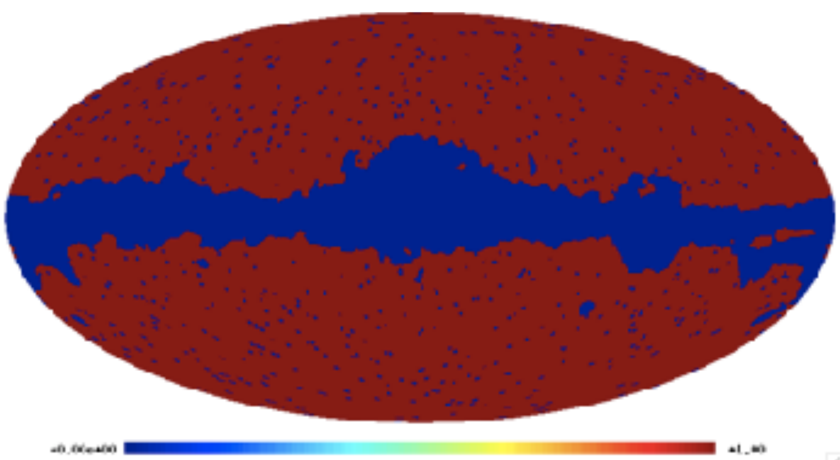
Interpolation of Missing Data: Sparse Inpainting

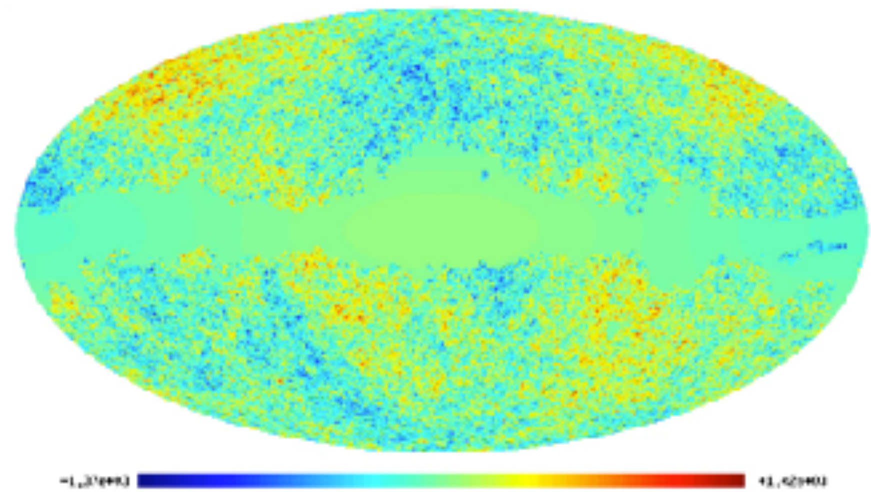
▪Abrial, Moudden, Starck, et al, , "***Morphological Component Analysis and Inpainting on the Sphere: Application in Physics and Astrophysics***", Journal of Fourier Analysis and Applications (JFAA), 13, 6, pp 729-748, 2007.

$$\min_{\alpha} \|\alpha\|_p \quad \text{subject to} \quad Y = M\Phi\alpha$$

Where M is the mask: M(i,j) = 0 ==> missing data
M(i,j) = 1 ==> good data

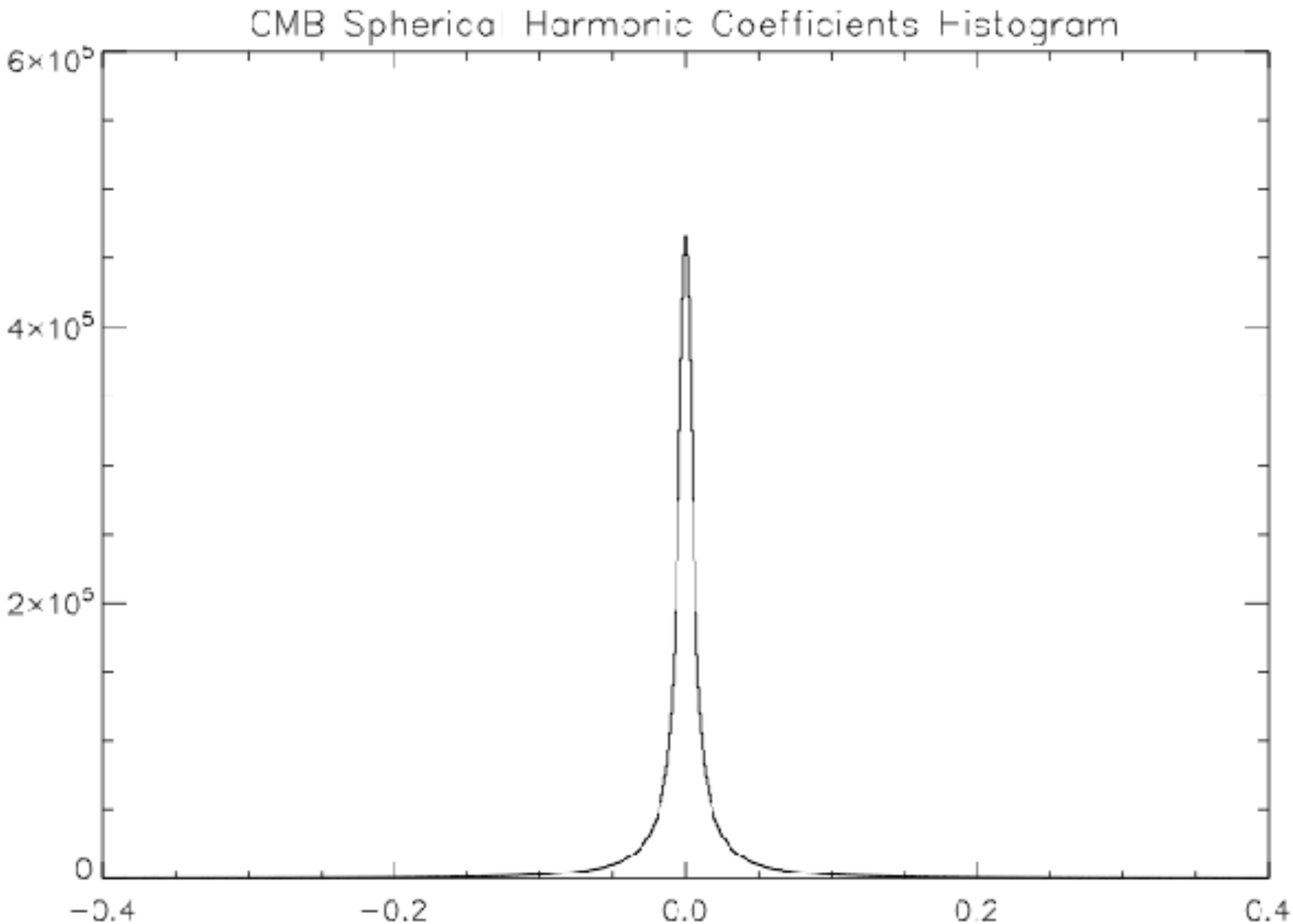
$$x^{(n+1)} = \mathcal{S}_{\Phi,\lambda^{(n)}} \left\{ x^{(n)} + M\left(y - x^{(n)}\right) \right\}$$





Sparse-Inpainting preserves the ISW and the weak lensing signal.

- L. Perotto, J. Bobin, S. Plaszczynski, J.-L. Starck, and A. Lavabre, "Reconstruction of the CMB lensing for Planck", *Astronomy and Astrophysics*, 2010.
- F.-X. Dupe, A. Rassat, J.-L. Starck, M. J. Fadili , "An Optimal Approach for Measuring the Integrated Sachs-Wolfe Effect", arXiv:1010.2192, *Astronomy and Astrophysics*, 534, A51+, 2011.
- S. Plaszczynski, A. Lavabre, L. Perotto, J-L Starck, "An hybrid approach to CMB lensing reconstruction on all-sky intensity maps", arxiv.org/abs/1201.5779, *Astronomy and Astrophysics*, 544, A27, 2012.


CMB Spherical Harmonic Coefficients Histogram
6×10^5
4×10^5
2×10^5
0
−0.4
−0.2
0.0
0.2
0.4

Iterative Harmonic Expansion

Inoue and Cabella and {Komatsu, Harmonic inpainting of the cosmic microwave background sky: Formulation and error estimate, Phys. Rev. D, 77, 2008.
Nishizawa and Inoue, « Reconstruction of missing data using iterative harmonic expansion », MNRAS, 462, 2016.

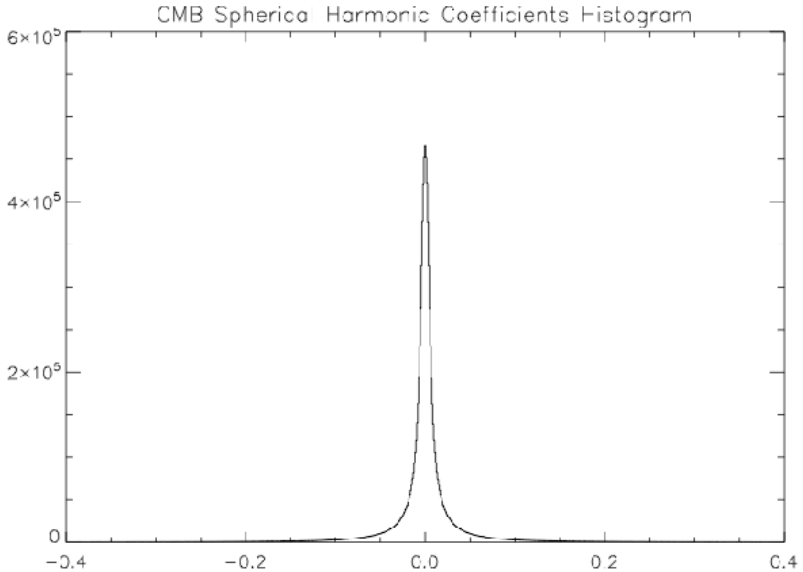
$$X^{(l+1)} = Md + (1-M)\mathcal{P}_{l+1}(X^{(l)})$$

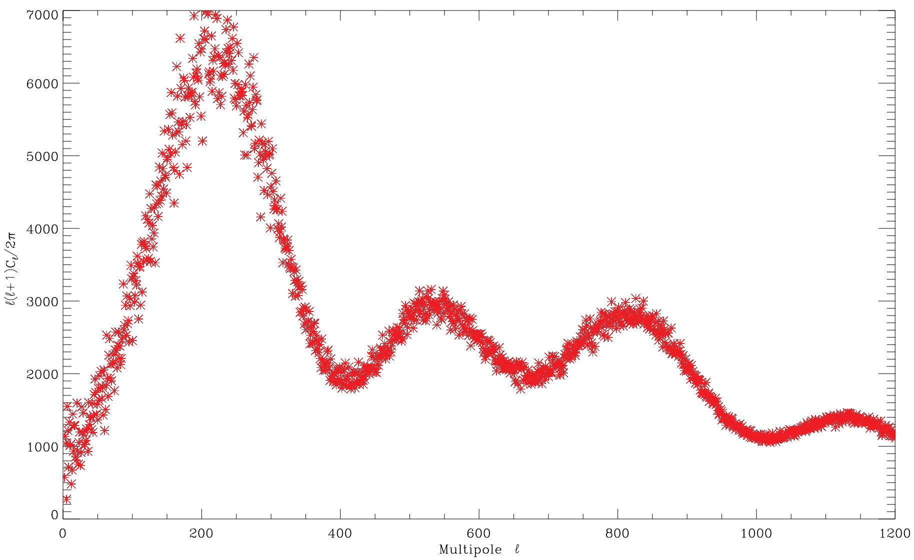
$$\text{where } \mathcal{P}_{l_{max}}(X) = \sum_{l=0}^{l_{max}} \sum_{m=-l}^{l} a_{lm} Y_{lm}$$

IHE method is equivalent to brute–force inversion in the limit that the number of iterations approaches infinity.

Iterative Harmonic Expansion

This method is somewhere connected to the Iterative Hard Thresholding Inpainting





Wiener INPAINTING

Feeney et al, « Avoiding bias in reconstructing the largest observable scales from partial-sky data », Phys. Rev. D, 2011.

$$x = Wd \qquad\qquad \mathbf{W} = [\mathbf{Y}^t\mathbf{C}^{-1}\mathbf{Y}]^{-1}\mathbf{Y}^t\mathbf{C}^{-1}$$

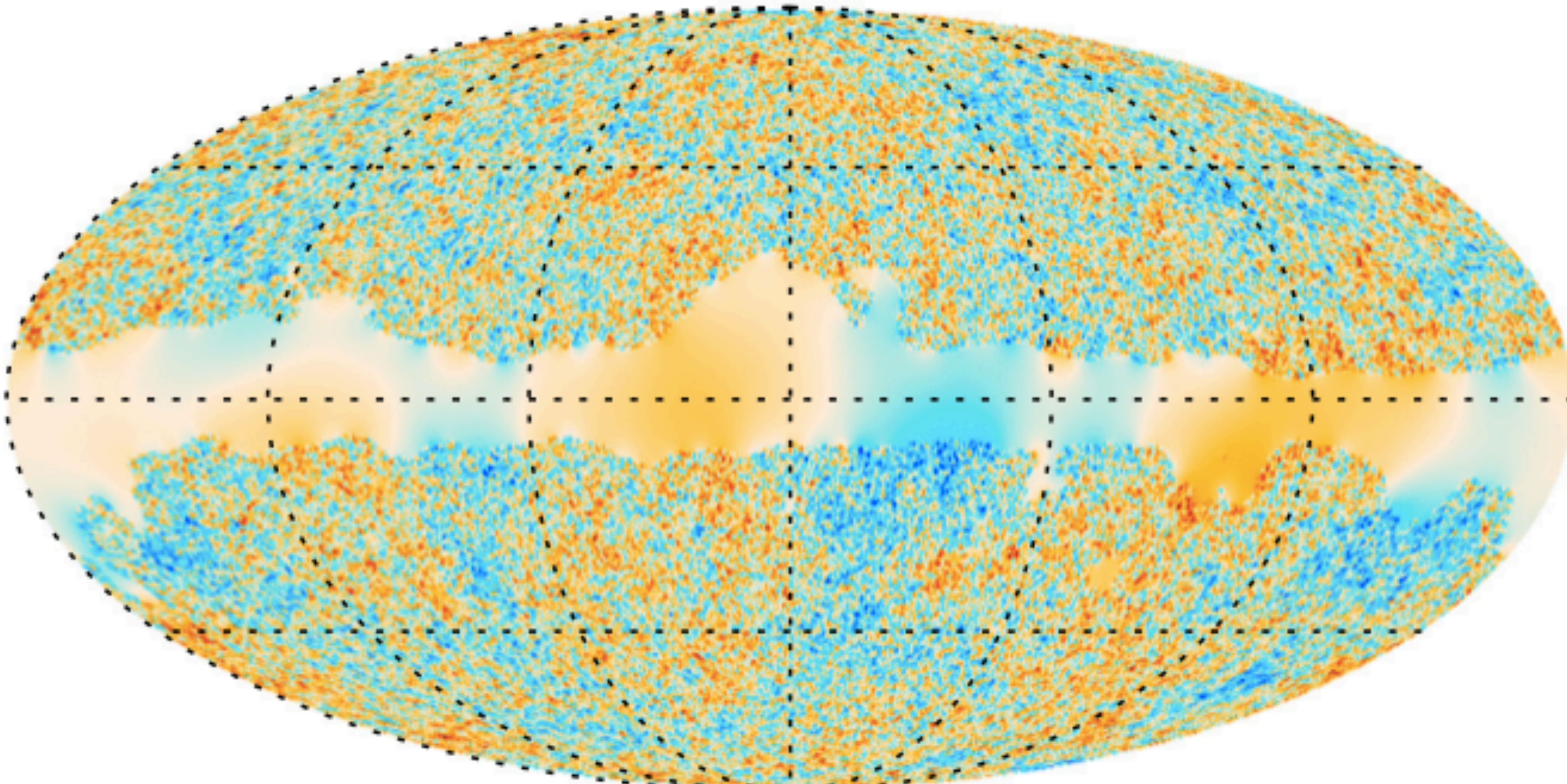
Y are the spherical harmonics calculated at each unmasked pixel
C is the pixel-space noise covariance matrix

$$C_{ij} = R_{ij} + \sum_{\ell=\ell_{\mathbf{max,\,rec}}+1}^{\ell_{\mathbf{max}}} \frac{2\ell+1}{4\pi}\bar{C}_\ell P_\ell(\hat{\mathbf{r}}_i \cdot \hat{\mathbf{r}}_j)$$

where $\mathbf{R}$ is uncorrelated, low-amplitude regularizing noise added to prevent $\mathbf{C}$ from becoming singular, $\bar{C}_\ell$ is the smoothed theory CMB angular power spectrum, and P_ℓ are the Legendre polynomials at unmasked pixels i, j.

Wiener INPAINTING

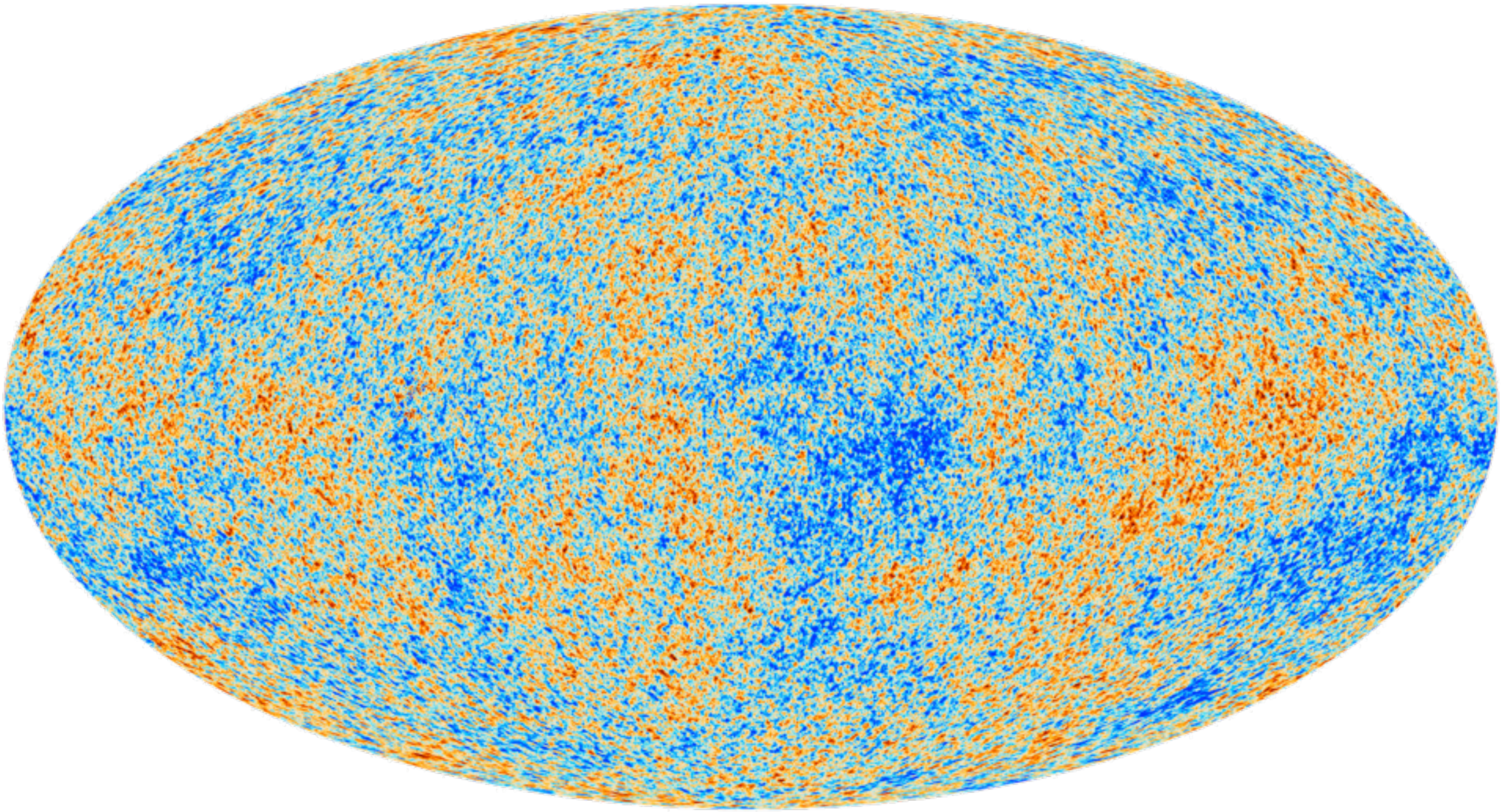
Mask 70% of the sky



Constraint Gaussians Realizations

Bucher et Louis, « Filling in cosmic microwave background map missing data using constrained Gaussian realizations", MNRAS, 2012

Mask 70% of the sky



Inpainting using Proximal Theory

J.-L. Starck, A. Rassat, and M.J. Fadili, "Low-l CMB Analysis and Inpainting", **Astronomy and Astrophysics** , 550, A15, 2013.

$$X = \Phi a \qquad \Phi = \text{Spherical Harmonics}$$

$$\boxed{\min_a \mathcal{C}(a) \quad \text{s.t} \quad D = M\Phi a}$$

l_1 norm $\quad \mathcal{C}(a) = \sum_{l,m} |a_{l,m}| \qquad |x| = \sqrt{\mathrm{Re}(x)^2 + \mathrm{Im}(x)^2}$

l_2 norm $\quad \mathcal{C}(a) = \sum_{l,m} \parallel a_{l,m} \parallel^2_{C_l^{-1}} \qquad \parallel a_{l,m} \parallel^2_{C_l^{-1}} = \sum_{l,m} \frac{|a_{l,m}|^2}{C_l}$

Isotropy $\quad \mathcal{C}(a) = ||a_{l,m}| - \mu_l| \leq \epsilon, \ \forall \ell, m$

Douglas-Rachford Algorithm l_1 norm

P.L. Combettes and J.-C. Pesquet, A Douglas-Rachford splitting approach to nonsmooth convex variational signal recovery, IEEE Journal of Selected Topics in Signal Procesing (2007).

$$\boldsymbol{a}^{n+\frac{1}{2}} = \mathrm{Proj}_{\{\boldsymbol{a}:\boldsymbol{z}=\mathbf{MS}^*\boldsymbol{a}\}}\left(\boldsymbol{a}^n\right),$$

$$\boldsymbol{a}^{n+1} = \boldsymbol{a}^n + \alpha_n\left(\mathrm{prox}_{\beta\|\cdot\|_1}\left(2\boldsymbol{a}^{n+\frac{1}{2}} - \boldsymbol{a}^n\right) - \boldsymbol{a}^{n+\frac{1}{2}}\right)$$

$\mathrm{prox}_{\beta\|\cdot\|_1}$ is the proximity operator of the l_1-norm which can be easily shown to be soft-thresholding

$$\mathrm{prox}_{\beta\|\cdot\|_1}(a) = \Delta^{\mathrm{S}}_{\beta}(a)$$

$$\Delta^{\mathrm{S}}_{\beta}(a) = \mathrm{sign}(a)\max(0, |a| - \beta)$$

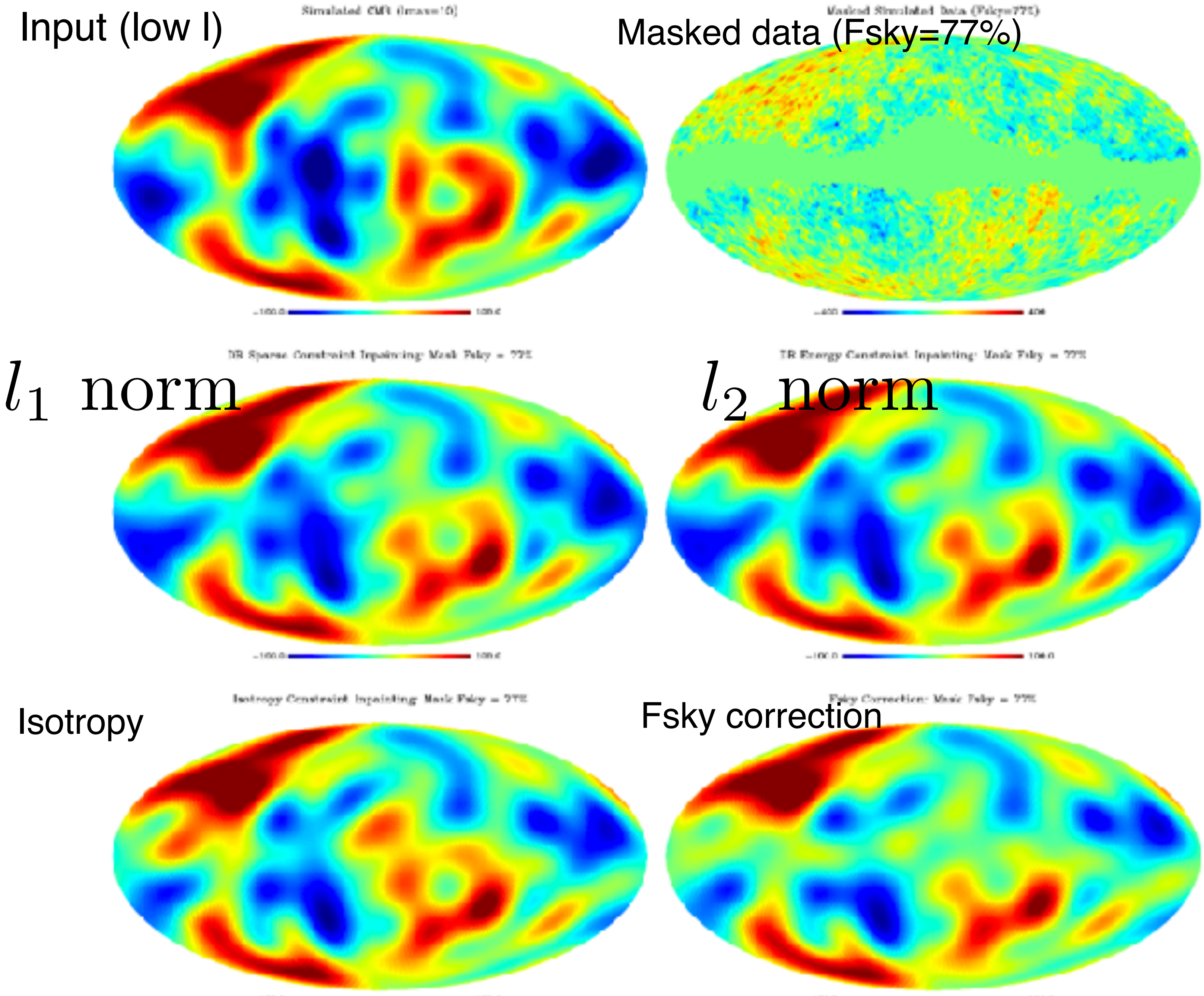
Douglas-Rachford Algorithm l_2 norm

P.L. Combettes and J.-C. Pesquet, A Douglas-Rachford splitting approach to nonsmooth convex variational signal recovery, IEEE Journal of Selected Topics in Signal Procesing (2007).

$$\boldsymbol{x}^{n+\frac{1}{2}} = \mathrm{Proj}_{\{\boldsymbol{x}:\boldsymbol{z}=\mathbf{M}\boldsymbol{x}\}}\left(\boldsymbol{x}^{n}\right),$$

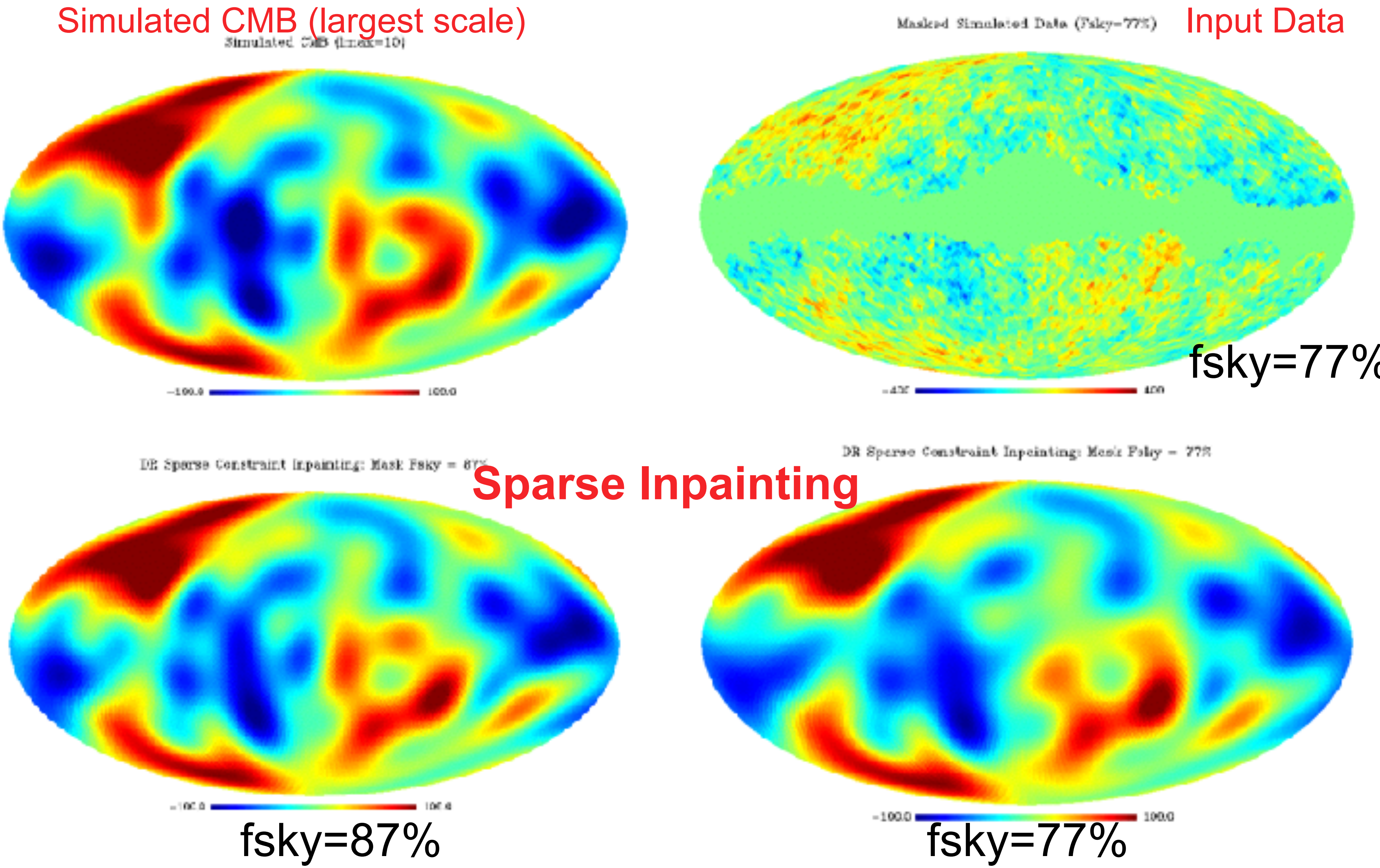
$$\boldsymbol{x}^{n+1} = \boldsymbol{x}^{n} + \alpha_n \Bigg(\mathrm{prox}_{\beta\|\mathbf{S}(\cdot)\|^2_{C-1}}\left(2\boldsymbol{x}^{n+\frac{1}{2}} - \boldsymbol{x}^{n}\right) - \boldsymbol{x}^{n+\frac{1}{2}} \Bigg),$$

$$\mathrm{prox}_{\beta\|\mathbf{S}(\cdot)\|^2_{C-1}}(\boldsymbol{x}) = \mathbf{S}^{*}\left((\mathbf{S}\boldsymbol{x}) \otimes \left(\frac{C}{\beta + C} \right) \right)$$


Input (low l)
Masked data (Fsky=77%)
l_1 norm
l_2 norm
Isotropy
Fsky correction

2013.

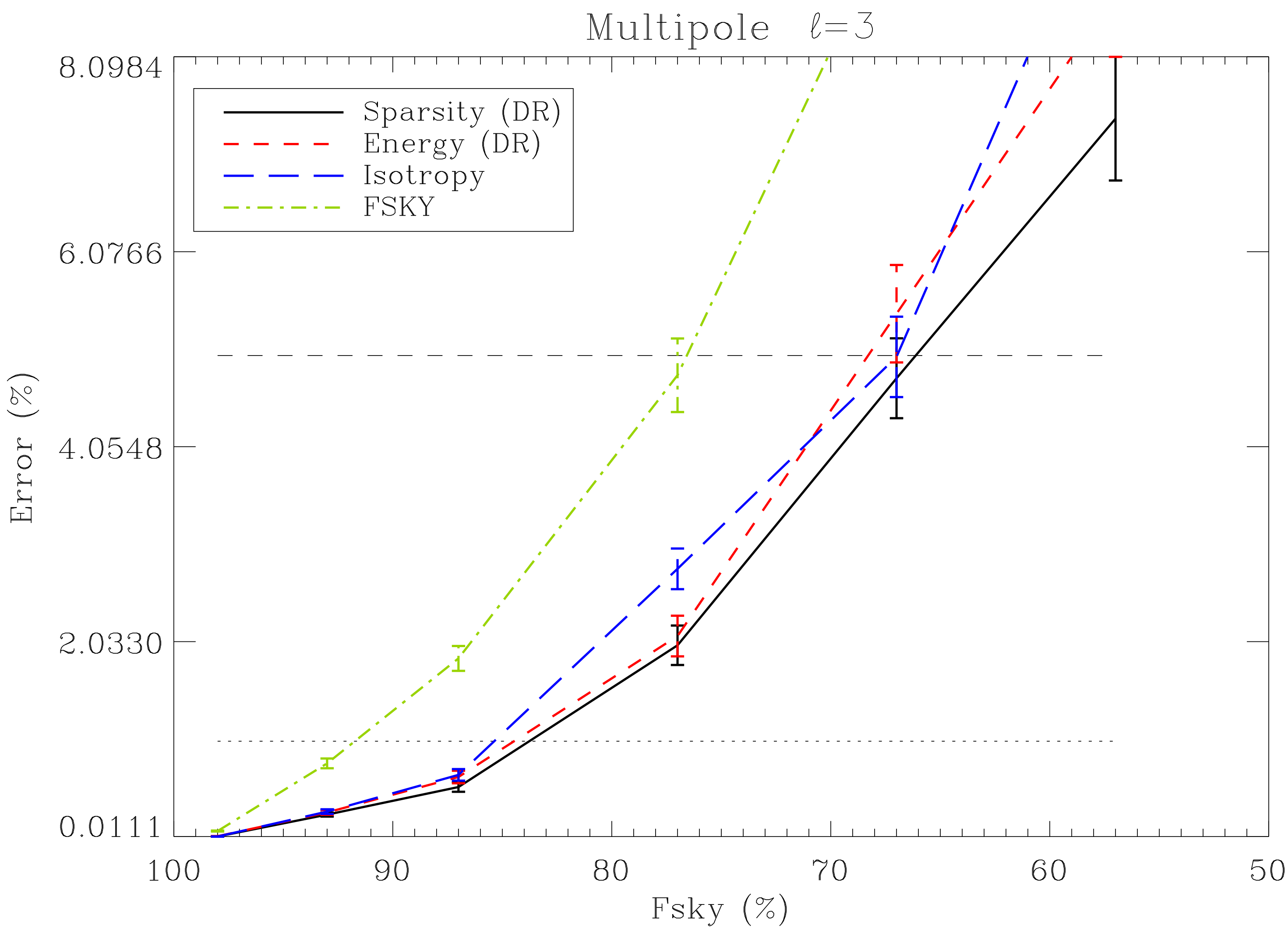
Large CMB Scale Analysis: Douglas-Rachford Sparse Inpainting



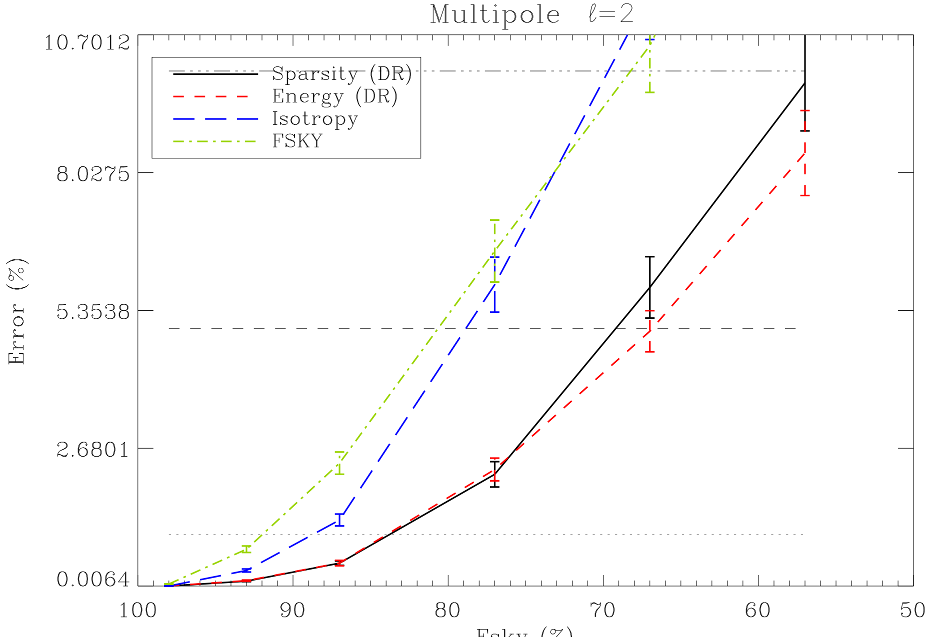
J.-L. Starck, A. Rassat, and M.J. Fadili, "Low-l CMB Analysis and Inpainting", **Astronomy and Astrophysics**, 550, A15, 2013.

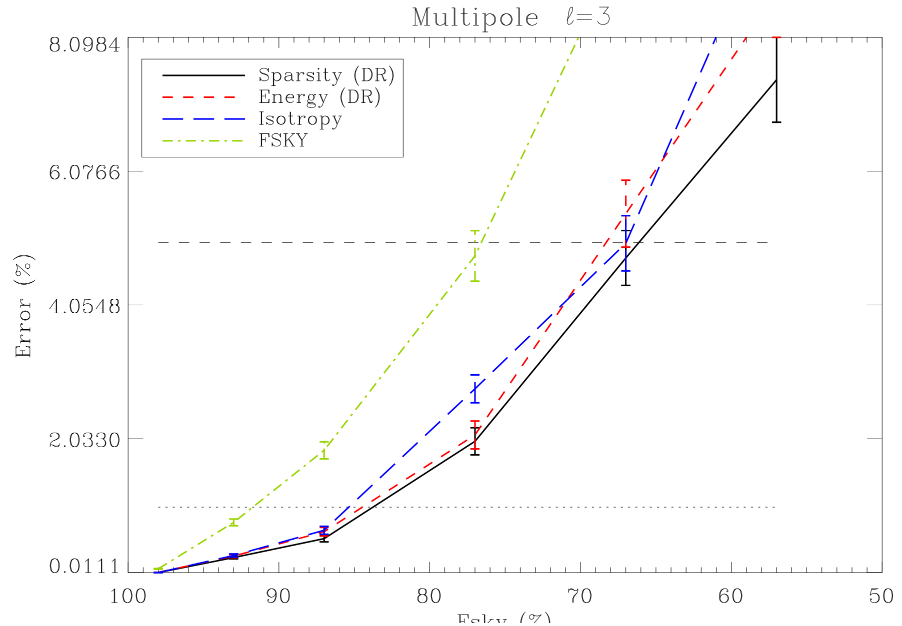
Inpainting (100 CMB Maps, nside=32, lmax=64)

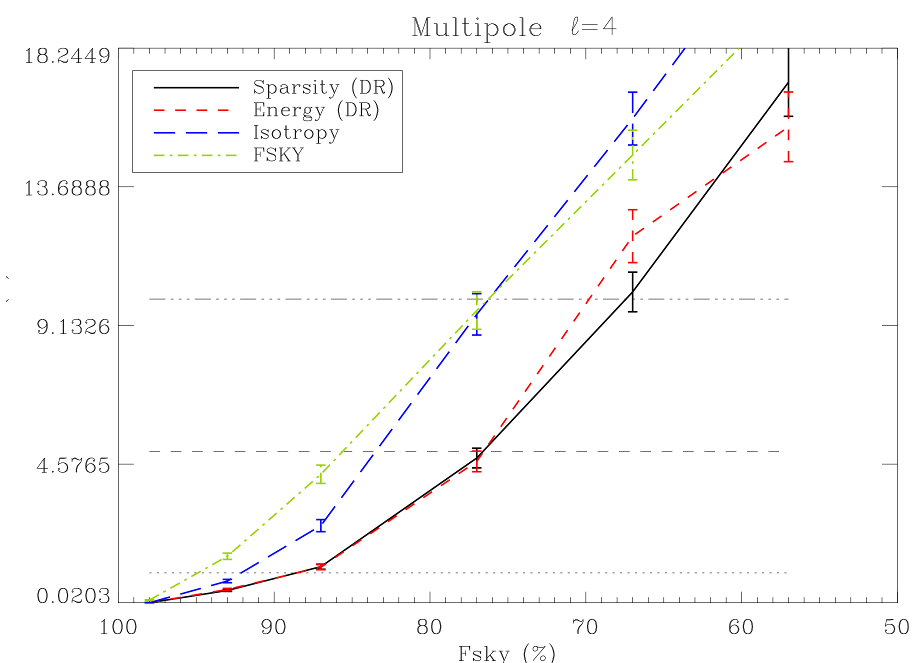
$$e^{(i)}[\ell] = \left\langle \frac{\left| \boldsymbol{a}^{\mathrm{True}}_{\ell,m} - \boldsymbol{a}^{(i)}_{\ell,m} \right|^2}{C_\ell} \right\rangle_m \qquad E[\ell] = 100 \times \left\langle e^{(i)}[\ell] \right\rangle_i \ (\%)$$

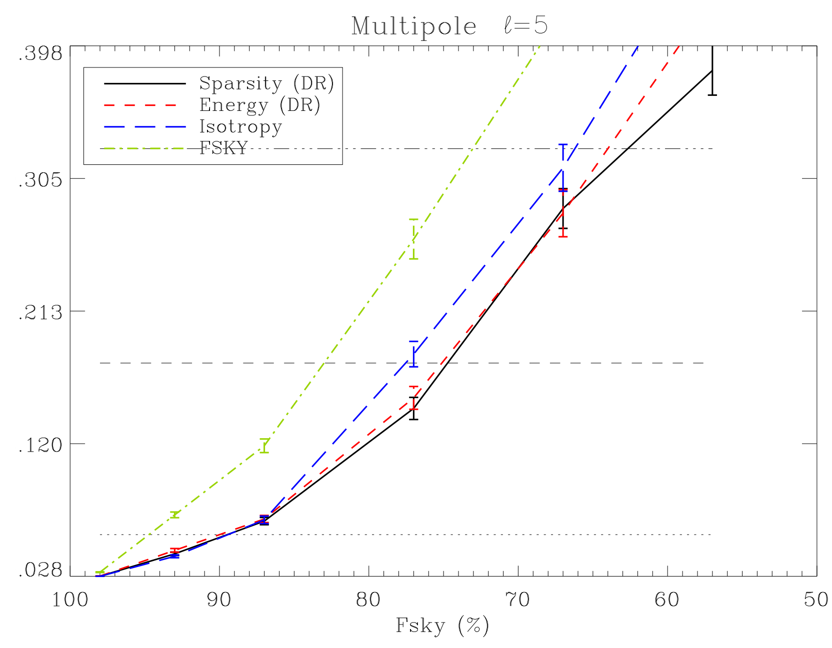


Inpainting


Multipole ℓ=2
Sparsity (DR)
Energy (DR)
Isotropy
FSKY
Error (%)
10.7012
8.0275
5.3538
2.6801
0.0064
100
90
80
70
60
50


Multipole ℓ=3
Sparsity (DR)
Energy (DR)
Isotropy
FSKY
Error (%)
8.0984
6.0766
4.0548
2.0330
0.0111
100
90
80
70
60
50


Multipole ℓ=4
Sparsity (DR)
Energy (DR)
Isotropy
FSKY
18.2449
13.6888
9.1326
4.5765
0.0203
100
90
80
70
60
50
Fsky (%)


Multipole ℓ=5
Sparsity (DR)
Energy (DR)
Isotropy
FSKY
.398
.305
.213
.120
.028
100
90
80
70
60
50
Fsky (%)

Cosmic Microwave Background Field Estimation

- Introduction to CMB
- CMB Temperature Inpainting Story
- **CMB Polarized Inpainting**

Multichannel Inpainting with Correlations

$$\text{find} \quad \operatorname{argmin}_{\mathbf{x}\in\mathbb{R}^{3n}} \mathcal{R}(\mathbf{x}) \;\; s.t.\; \mathbf{My} = \mathbf{Mx}, \quad \mathcal{R}(\mathbf{x}) = \{\beta\|\mathbf{Sx}\|^2_{2,\Sigma^{-1}}\}$$

$\mathbf{y} \in \mathbb{R}^{3n}$: TQU/TEB data, $\mathbf{M} \in \mathbb{R}^{3n\times 3n}$: TQU/TEB Mask operator
$\mathbf{S} \in \mathbb{C}^{3f\times 3n}$: Polarized Spherical harmonic transform (TEB)
$\boldsymbol{\Sigma} \in \mathbb{R}^{3f\times 3f}$: TEB power spectra

- Inpainting of (T, Q, U) or (T,E,B) not independently (cross-correlations)
  $\longrightarrow$ Correlated Multichannel Inpainting
- Require knowledge of the power-spectra (pure or pseudo-cl estimators): cosmology assumption

Multichannel Inpainting with Correlations

$$\text{find} \quad \text{argmin}_{\mathbf{x}\in\mathbb{R}^{3n}} \mathcal{R}(\mathbf{x}) \;\; s.t. \; \mathbf{My} = \mathbf{Mx}, \quad \mathcal{R}(\mathbf{x}) = \{\beta||\mathbf{Sx}||^2_{2,\Sigma^{-1}}\}$$

- Solve constrained form using Douglas-Rachford algorithm
- Technical point: computing prox of non-diagonally weighted ℓ_2 norm
  - Separability: $\text{prox}_{\beta||.||^2_{2,\mathbf{\Sigma}^{-1}}} \rightarrow \left\{\text{prox}_{\beta||.||^2_{2,\mathbf{\Sigma}_\ell^{-1}}}\right\}_{\ell\leq\ell_{max}}$
  - SVD Decomposition: $\mathbf{\Sigma}_\ell = \mathbf{U}_\ell\mathbf{D}_\ell\mathbf{U}_\ell^T$
  - Decomposition in orthonormal basis: $\text{prox}_{\beta||.||^2_{2,\mathbf{\Sigma}_\ell^{-1}}}(\mathbf{x}_{\ell m}) = \mathbf{U}_\ell \, \text{prox}_{\beta||.||^2_{2,\mathbf{D}_\ell^{-1}}}(\mathbf{U}_\ell^T\mathbf{x}_{\ell m}), \quad \mathbf{x}_{\ell m} = [x_{T,\ell m}, x_{E,\ell m}, x_{B,\ell m}]^T$

Algorithm

$$\text{find} \quad \text{argmin}_{\mathbf{x} \in \mathbb{R}^{3n}} \mathcal{R}(\mathbf{x}) \;\; s.t. \; \mathbf{My} = \mathbf{Mx}, \quad \mathcal{R}(\mathbf{x}) = \{\beta ||\mathbf{Sx}||^2_{2,\Sigma^{-1}}\}$$

Polarized CMB Inpainting

1- Given $\{\mathbf{\Sigma}_\ell\}_{\ell \leq \ell_{max}}$, perform SVD decomposition $\mathbf{\Sigma}_\ell = \mathbf{U}_\ell \mathbf{D}_\ell \mathbf{U}_\ell^{\mathbf{T}}$ to obtain $\{\mathbf{D}_\ell, \mathbf{U}_\ell\}_{\ell < \ell_{max}}$

2- Choose $\mathbf{a}^0 = \mathbf{SMy}$ and the parameters β, n_{max} .

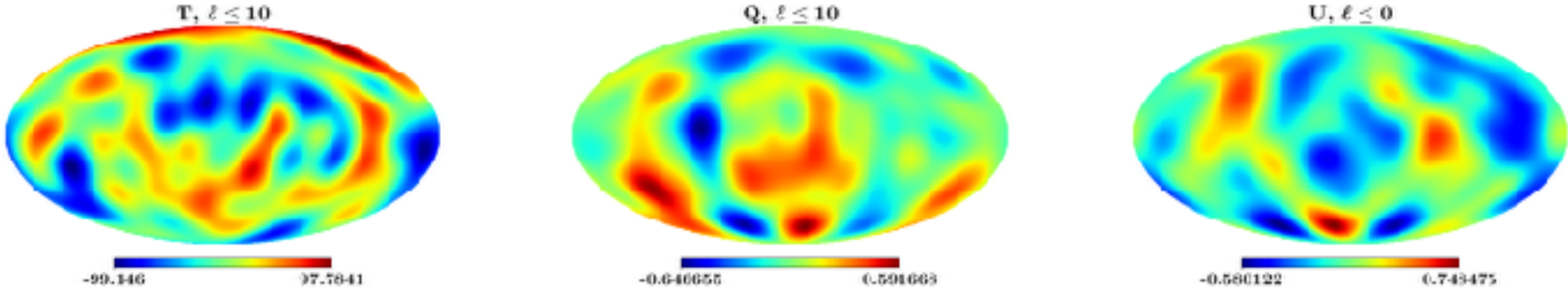
3- Iterate ($n_{max} \geq n \geq 0$):

$$\begin{cases} \mathbf{a}^{n+1/2} = \{\mathbf{S}(\mathbf{a}^n + \mathbf{M}(\mathbf{y} - \mathbf{MS}^*\mathbf{a}^n))\} \\ \forall \{\ell \leq \ell_{max}, m \in (-\ell, .., \ell)\} : \\ \qquad \mathbf{a}^{n+1}_{\ell m} = \mathbf{a}^n_{\ell m} - \mathbf{a}^{n+1/2}_{\ell m} + \mathbf{U}_\ell \, \mathrm{prox}_{\beta ||.||^2_{2,\mathbf{D}_\ell^{-1}}} \left[\mathbf{U}_\ell^{\mathbf{T}} (2\mathbf{a}^{n+1/2}_{\ell m} - \mathbf{a}^n_{\ell m}) \right] \end{cases}$$

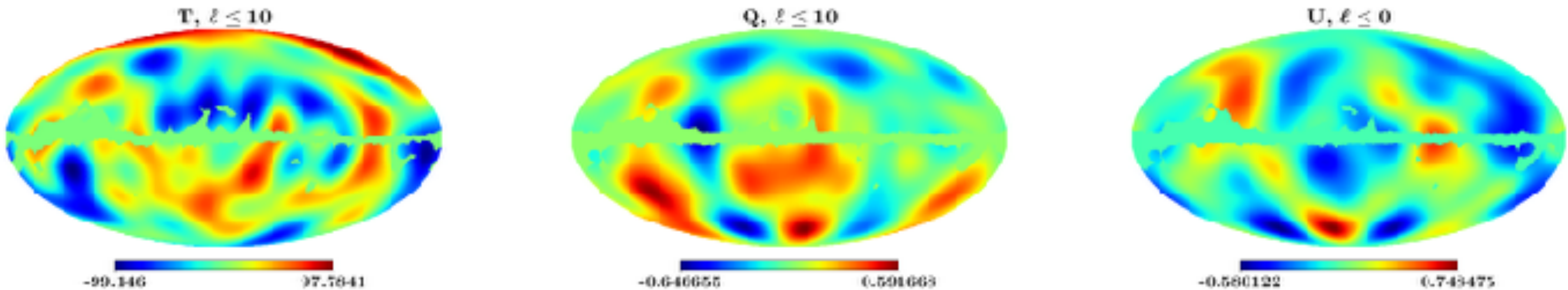
with $\mathbf{a}^n_{\ell m} = [a^n_{T,\ell m}, a^n_{E,\ell m}, a^n_{B,\ell m}]^T$

Inpainting Results (fskyT=0.9, fskyP=0.89)

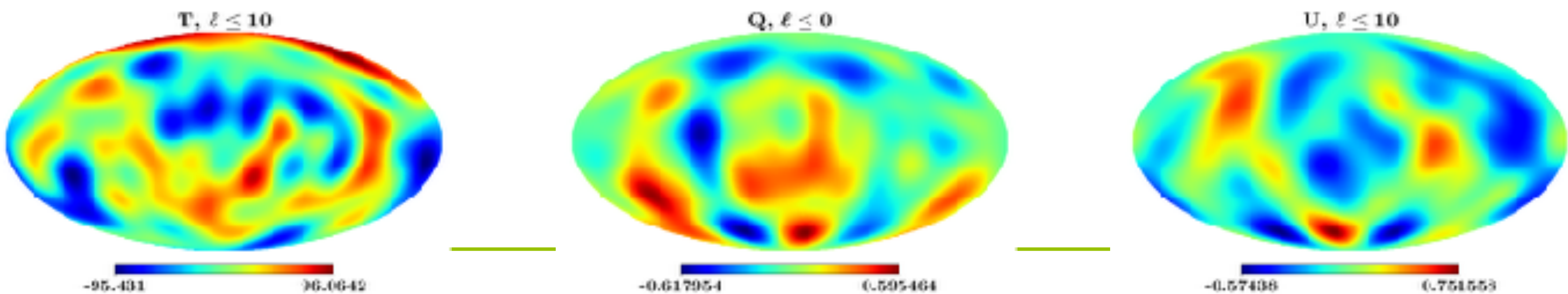
Input Maps



Masked Maps

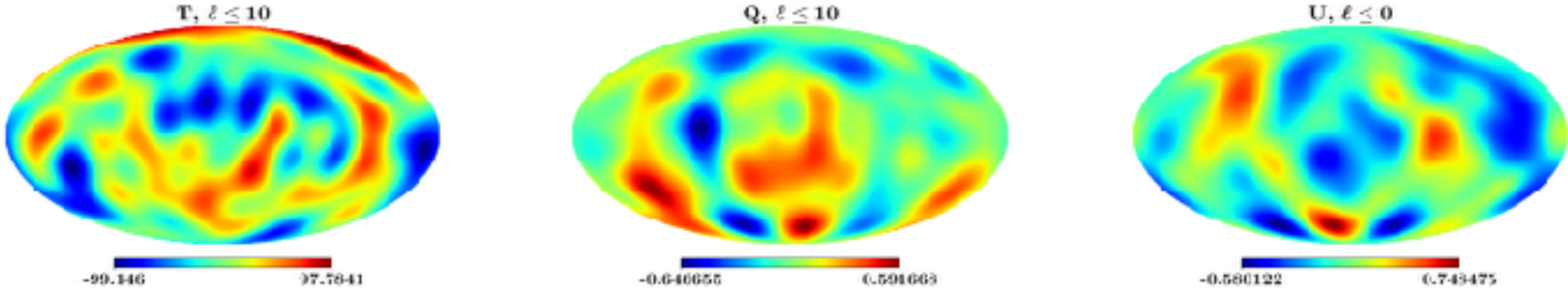


Inpainted Maps

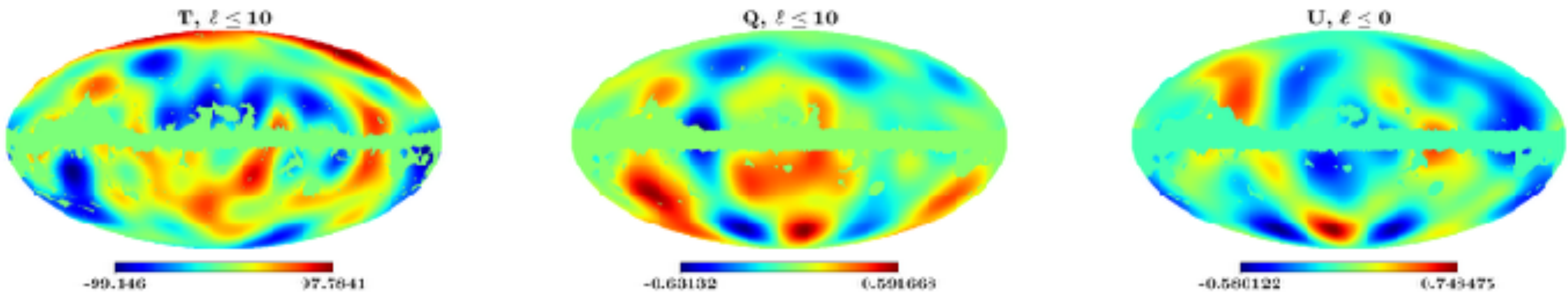


Inpainting Results (fskyT=0.8, fskyP=0.8)

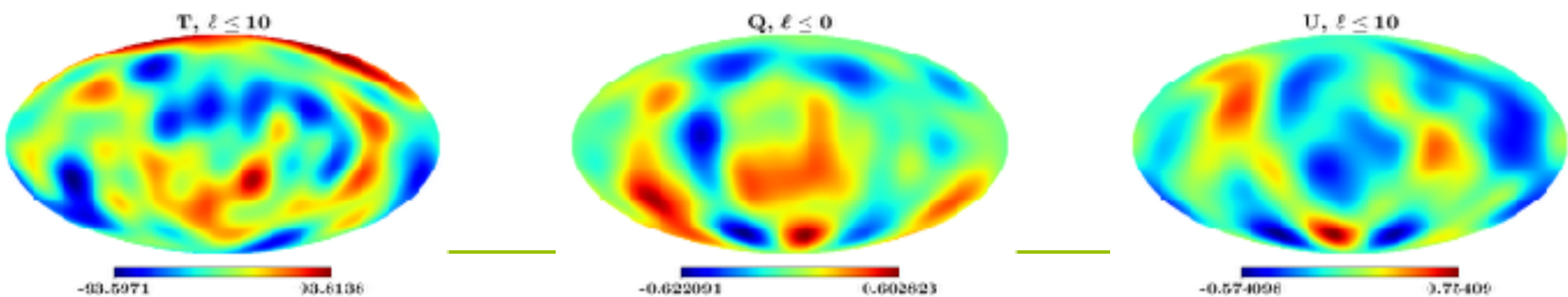
Input Maps



Masked Maps

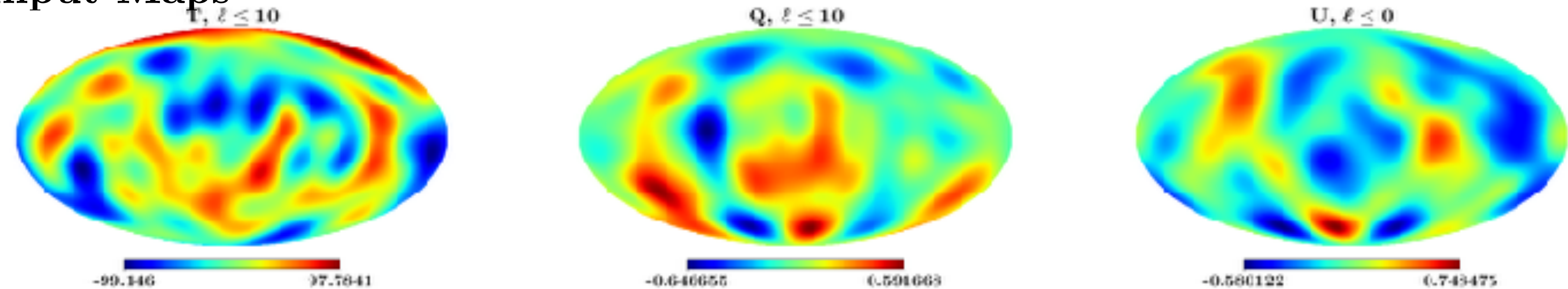


Inpainted Maps

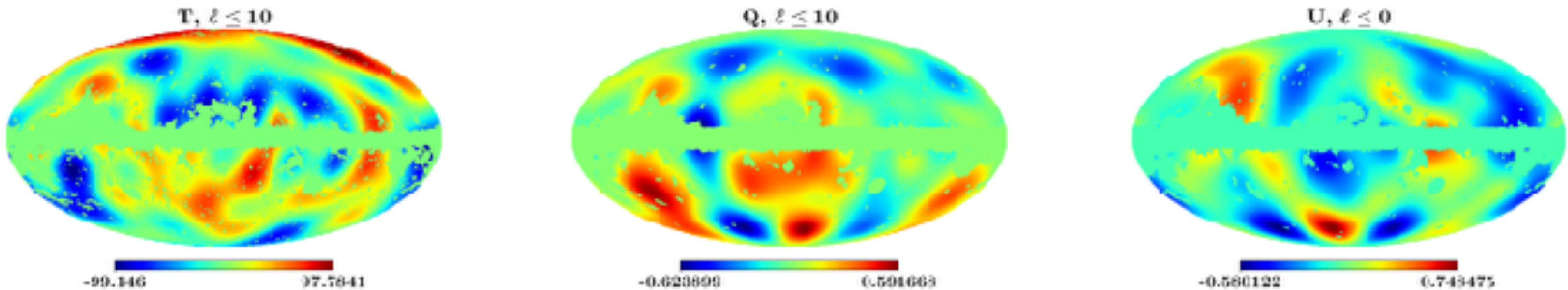


Inpainting Results (fskyT=0.72, fskyP=0.75)

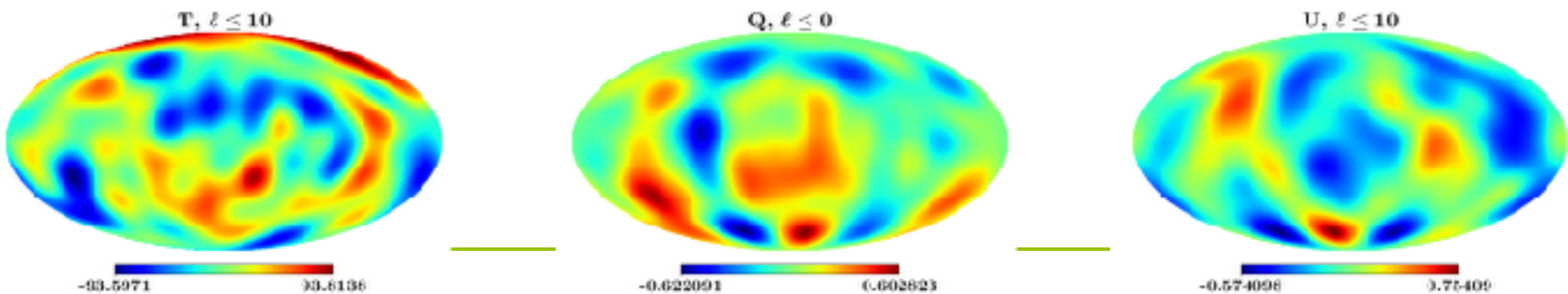
Input Maps



Masked Maps



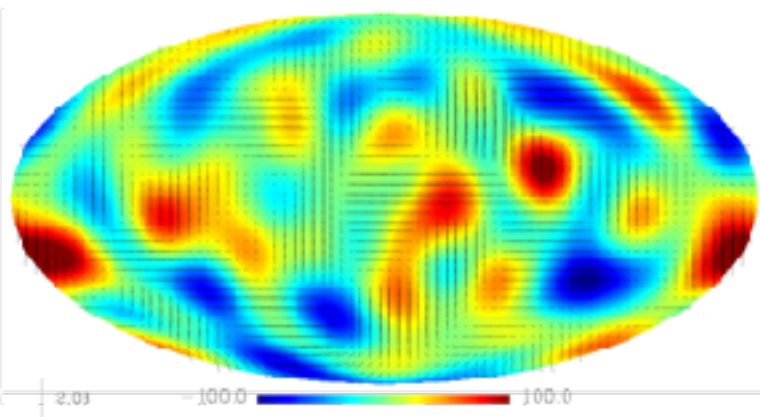
Inpainted Maps



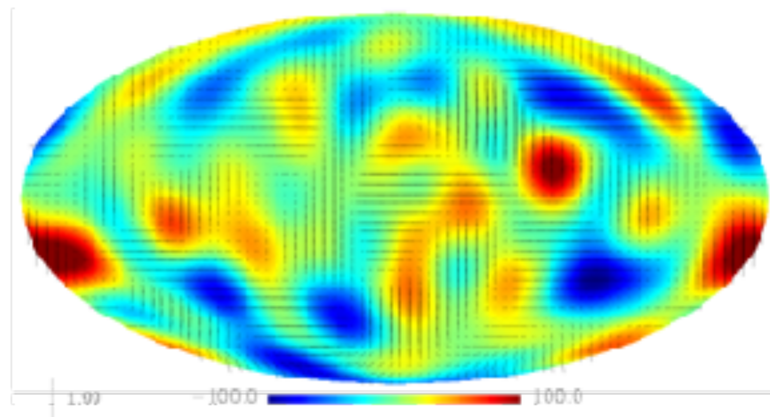
Impact of Mask Size on Recovery

- Similarly to what performed for temperature only inpainting:
  - Impact of Mask Size on recovery of large scale CMB coefficients
  - Statistics on error per ℓ measured on 100 simulations per mask configuration:

$$E_X[\ell] = 100 \times \left\langle \left\langle \frac{\left|a^i_{X,\ell m} - a^{ref}_{X,\ell m}\right|^2}{C_{X,\ell}} \right\rangle_m \right\rangle_{i\in\{1,..,100\}}, X \in \{T, E, B\}$$

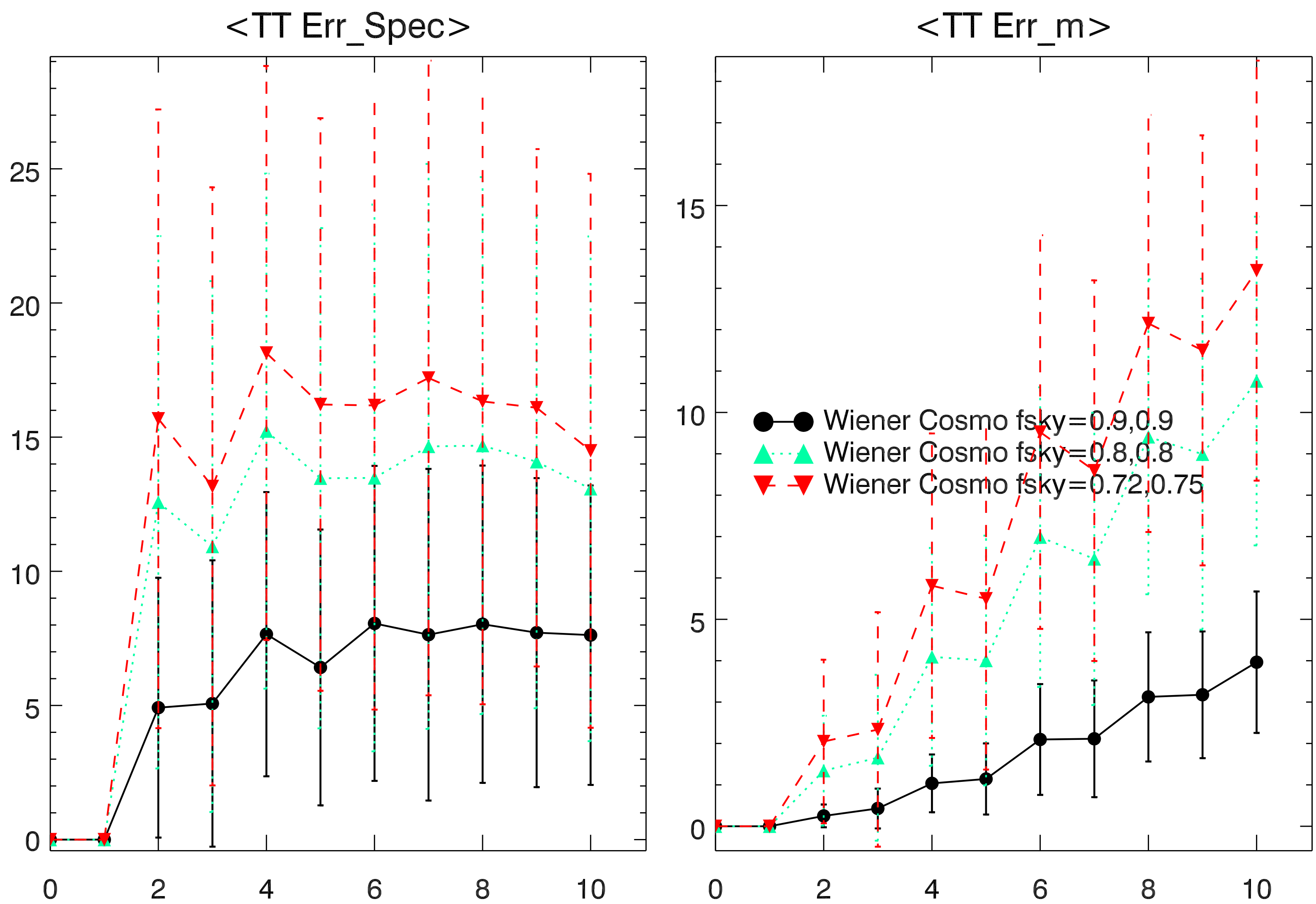


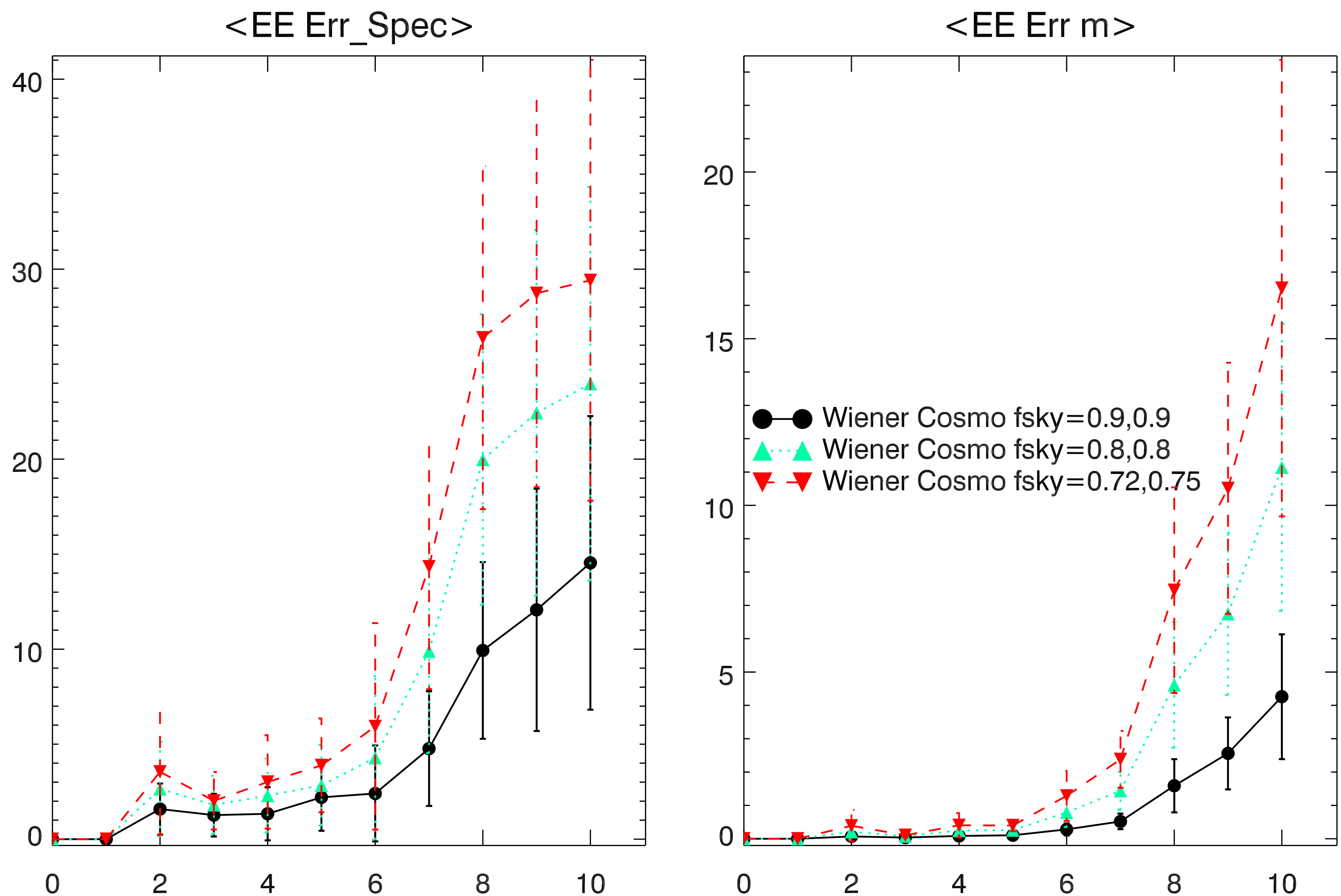
INPUT



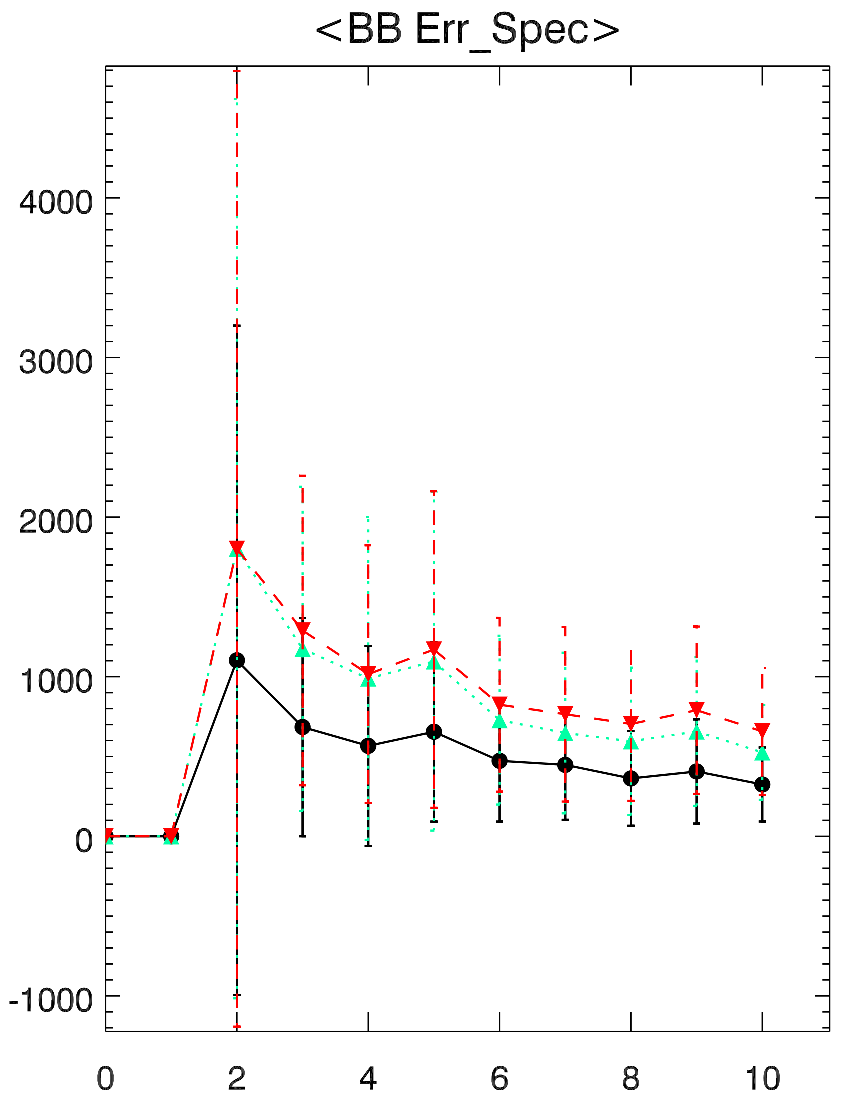
INPAINTED

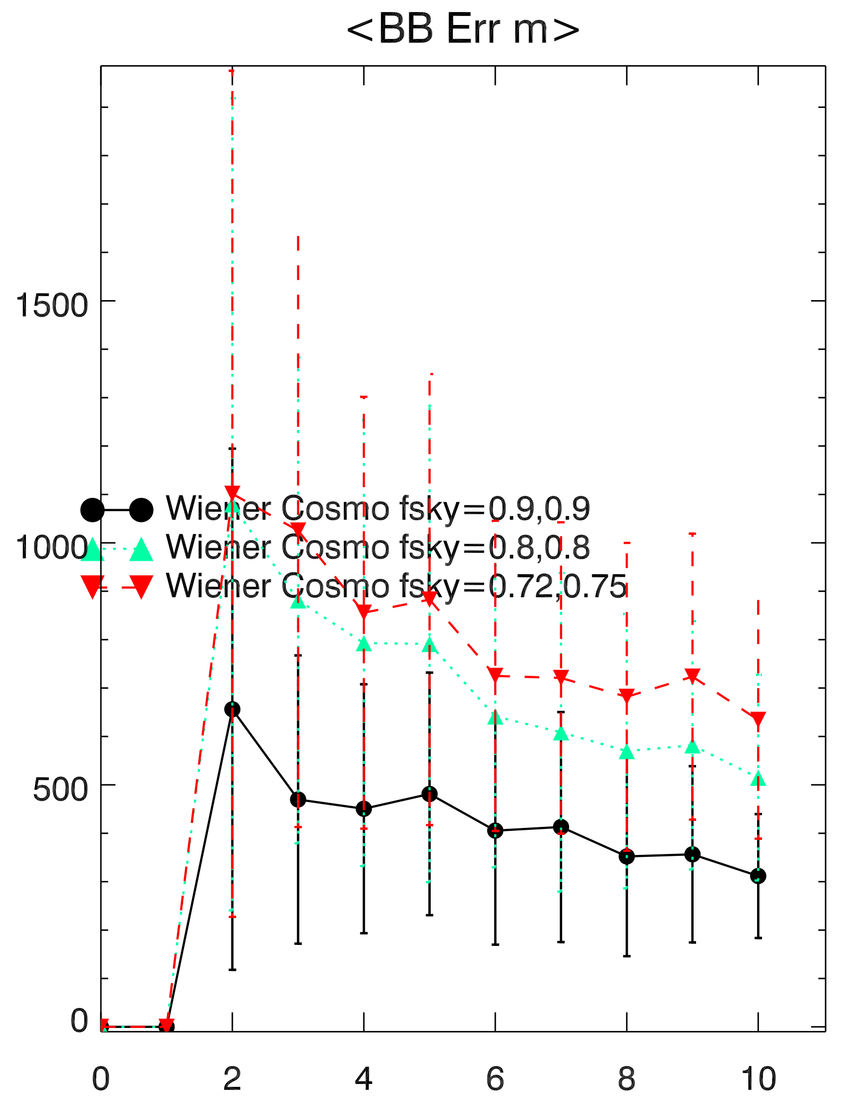
$fsky_T = 0.84$, $fsky_P = 0.76$


<TT Err_Spec>
<TT Err_m>
Wiener Cosmo fsky=0.9,0.9
Wiener Cosmo fsky=0.8,0.8
Wiener Cosmo fsky=0.72,0.75
0
5
10
15
20
25
2
4
6
8
10



<EE Err_Spec>
<EE Err m>
Wiener Cosmo fsky=0.9,0.9
Wiener Cosmo fsky=0.8,0.8
Wiener Cosmo fsky=0.72,0.75


<BB Err_Spec>
4000
3000
2000
1000
0
-1000
0
2
4
6
8
10


<BB Err m>
1500
1000
500
0
0
2
4
6
8
10
Wiener Cosmo fsky=0.9,0.9
Wiener Cosmo fsky=0.8,0.8
Wiener Cosmo fsky=0.72,0.75

Conclusions

- ✓ **Extensions of Wiener-inpainting for polarised CMB**
  - ➡ Can be used for any correlated multichannel data set
  - ➡ Require knowledge of the auto and cross-spectra.
  - ➡ Douglas-Rachford algorithm adapted to this inpainting task.

- ✓ **Results using inpainting on polarised data.**
  - ➡ T and E well reconstructed using inpainting.
  - ➡ Similar behaviour relative to Fsky for T and E.
  - ➡ Inpainting cannot be used for the reconstruction of the B mode.

- ✓ **Reproducible Research** http://www.cosmostat.org/software.html