

# Joint constraints on galaxy bias and $\sigma_8$ through the N-pdf of the galaxy number density

based on Arnalte-Mur et al. (2016) JCAP, 3, 5

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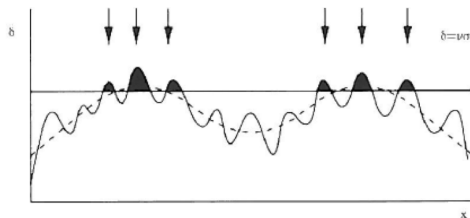


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# Definitions

- Bias parameter  $b$ :  $\delta_g = b\delta_m$  where

$$\delta_g(\vec{x}) = \frac{\rho_g(\vec{x}) - \bar{\rho}_g}{\bar{\rho}_g} \qquad \delta_m(\vec{x}) = \frac{\rho_m(\vec{x}) - \bar{\rho}_m}{\bar{\rho}_m}$$



- $\sigma_8^2$  corresponds to the matter density variance in spheres of radius  $R = 8h^{-1}$  Mpc, when using a linear model extrapolated to  $z = 0$ .

$$\sigma_8^2 = 4\pi \int_0^\infty \widetilde{W}_{TH}^2(k, R = 8h^{-1} \text{ Mpc}) P_{\text{lin}}(k) \frac{k^2 dk}{(2\pi)^3},$$

where  $\widetilde{W}_{TH}(k, R)$  is the Fourier transform of a spherical top-hat filter of radius  $R$ . The accepted value for this parameter from observations is  $\sigma_8 \sim 0.8$ .

# Introduction: Measuring bias

In most of the literature, the bias has been estimated using:

- **the 3D-correlation function of galaxies:** (Fry & Gaztanaga, 1993; Gaztanaga & Frieman, 1994; Croft et al., 1999);
- **counts-in-cells statistics:** Sigad et al. (2000); Colless et al. (2001); Marinoni et al. (2005); Kovač et al. (2011); Di Porto et al. (2014);
- **the projected correlation function:** Norberg et al. (2002); Zehavi et al. (2005, 2011); Arnalte-Mur et al. (2014);
- **the bi-spectrum:** Guo & Jing (2009); Pollack et al. (2014);
- **higher order moments of the galaxy distribution:** Szapudi (1998); Verde et al. (2002); Swanson et al. (2008); McBride et al. (2011);
- **the use of multivariate probability distributions, typically Gaussian or lognormal distributions, which are well suited priors for Bayesian analyses:** Jasche & Wandelt (2013); Ata et al. (2015); Granett et al. (2015); Jasche et al. (2015).

## Analysis of non-Gaussian cosmic microwave background maps based on the N-pdf. Application to *Wilkinson Microwave Anisotropy Probe* data

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In this paper, we will focus on a parametric non-Gaussian model, that accounts for a small and local (i.e. point-to-point) perturbation of the CMB temperature fluctuations, around its intrinsic Gaussian distribution:

$$\Delta T_i = (\Delta T_i)_G + a [(\Delta T_i)_G^2 - \langle (\Delta T_i)_G^2 \rangle] + b (\Delta T_i)_G^3, \quad (1)$$

The linear term is the Gaussian part, whose N-point probability density function (N-pdf) can be easily described in terms of the standard inflationary model.

# Introduction: Our goal

Our method relies on the use of the whole set of N-pdf of the galaxy number density fluctuations:

This multivariate probability density function depends on the bias parameter and the correlations of the underlying cold dark matter fluctuations.

## The estimation

- The N-pdf of the galaxy number density field as a function of the bias and the  $\sigma_8$  parameters is, in fact, the likelihood of the data (i.e., the galaxy catalogue) given these parameters.
- Therefore optimal parameter estimation can be performed via the maximum-likelihood.

# The galaxy density N-pdf method

## Observable

Galaxy overdensity field in  $N$  cubic cells  $\rightarrow \{\Delta_{g,i}\}_{i=1}^N \rightarrow \Delta_{g,i} = \frac{\rho_{g,i}}{\bar{\rho}_g}$

## Model: probability distribution function of $\vec{\Delta}_g$

- Constant, linear bias:  $\delta_g = b\delta_m$
- Log-normal distribution model for the matter density field (Coles & Jones, 1991; Kayo et al., 2001) [See Peter Coles' blog "In the Dark" May 10, 2016]]
- Difference w.r.t. "traditional" counts-in-cells: we take into account the cells' positions and the corresponding correlations between them

# Model: probability distribution function of $\vec{\Delta}_g$

$$f(\vec{\Delta}_g) = \frac{1}{(2\pi)^{N/2} |\mathbf{H}_0|^{1/2}} \frac{\exp\left(-\frac{1}{2}\vec{y}^t \cdot \mathbf{H}_0^{-1} \cdot \vec{y}\right)}{\prod_{j=1}^N (\Delta_{g,j} + b - 1)}, \text{ where:}$$

$$y_i = \log \left[ \frac{\sqrt{1 + \sigma_m^2}}{b} (\Delta_{g,i} + b - 1) \right]$$

$$\mathbf{H}_0 = \log(1 + \mathbf{C}_m), \quad C_{m,ij} = \xi_m(|\vec{r}_i - \vec{r}_j|)$$



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## Parameters of the model

- Galaxy bias  $\rightarrow b$
- Cosmological parameters ( $\theta_{\text{cosmo}}$ )  $\rightarrow \xi_m(r) \rightarrow \mathbf{C}_m$

# Parameter estimation

We use a Bayesian approach, knowing that the model distribution function is the *likelihood of the data* ( $\vec{\Delta}_g$ ) given a model:

$$\mathcal{L}(\vec{\Delta}_g|b, \theta_{\text{cosmo}}) = f(\vec{\Delta}_g)$$

- Fixing a fiducial cosmological model:

$$p(b|\vec{\Delta}_g) \propto \mathcal{L}(\vec{\Delta}_g|b)p(b)$$

- Allowing  $\sigma_8$  to vary  $\rightarrow \mathbf{C}_m = \left(\frac{\sigma_8}{\sigma_8^{\text{fid}}}\right)^2 \mathbf{C}_m^{\text{fid}}$ :

$$p(b, \sigma_8|\vec{\Delta}_g) \propto \mathcal{L}(\vec{\Delta}_g|b, \sigma_8)p(b, \sigma_8)$$

$\rightarrow$  We assume flat priors in all cases,  $p(b)$ ,  $p(b, \sigma_8)$

$\rightarrow$  We use a Fisher matrix approach to estimate parameters' uncertainties

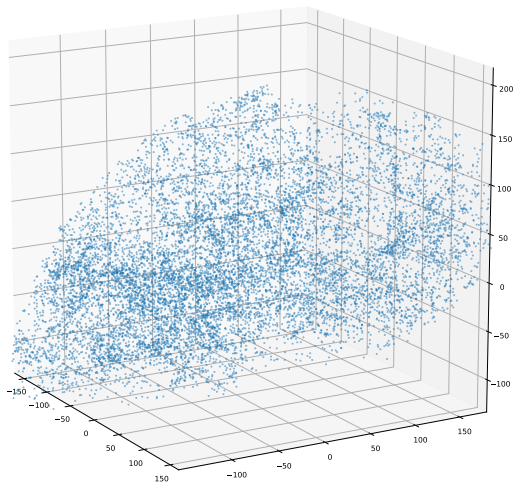
# SDSS main galaxy sample

- We used the data provided by the New York University Value Added Catalogue [Blanton et al. (2005)]. We have selected a volume limited sample with  $M_r \leq -20$  in the redshift range  $0.033 < z < 0.106$ , where  $M_r$  is the  $K + E$  corrected  $r$ -band absolute magnitude.
- We study this field using a grid of cubic cells with a side of  $30 h^{-1} \text{Mpc}$ .
- We compute the completeness for each of the cells ( $c_i$ ) as the combination of the radial and the angular selection functions and keep only those cells with  $c_i \geq 0.8$ .
- We obtain the number of galaxies  $n_i$  in each of these accepted cells, and estimate the galaxy number density for each cell as

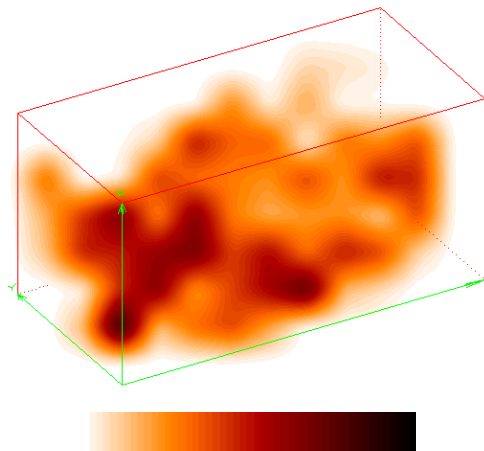
$$\rho_{g,i} = \frac{n_i}{c_i V_c}$$

where  $V_c = (30 h^{-1} \text{Mpc})^3$  is the volume of a cell.

# SDSS main galaxy sample



# SDSS main galaxy sample

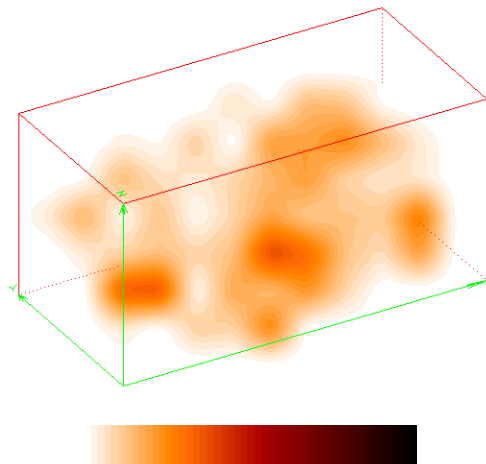


3D projection of the galaxy number density field corresponding to the SDSS catalogue. Colour palette used in the projection corresponds to densities  $1 \leq \Delta_g \leq 2$  from left to right.

# Lognormal simulations

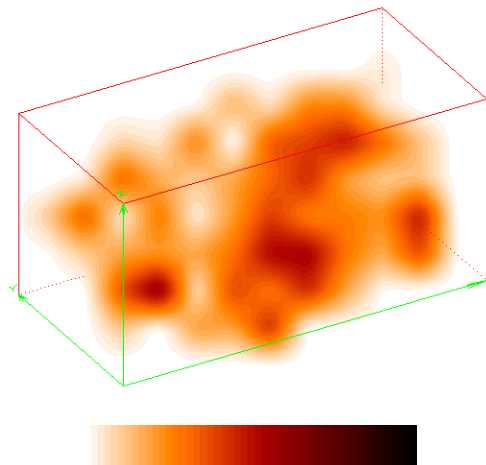
- We generate the lognormal simulations taking into account that a lognormal random field is a local transformation of a Gaussian field.
- We compute the matter correlation function  $\xi_m(r)$  via a Fourier transform of  $P_m(k)$ .
- We then generate the Gaussian random field  $\delta_{0,i}$  in a cubic grid using the standard method of generating Gaussian Fourier modes with variances given by  $P_0(k)$  and then performing a FFT.
- Finally, we obtain the corresponding matter density field  $\Delta_{m,i}$ .
- Given a value for the galaxy bias  $b$ , we can generate the corresponding galaxy density field  $\Delta_{g,i}$ . We have explored four input values of the bias:  $b = 0.5, 1.0, 1.5, 2.0$ .

# Lognormal simulations



Bias  $b = 0.5$

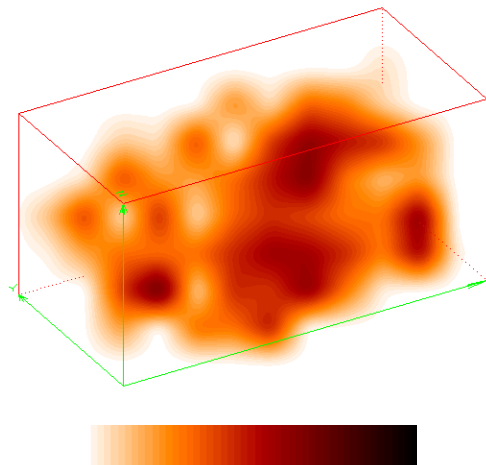
# Lognormal simulations



Bias  $b = 1.0$

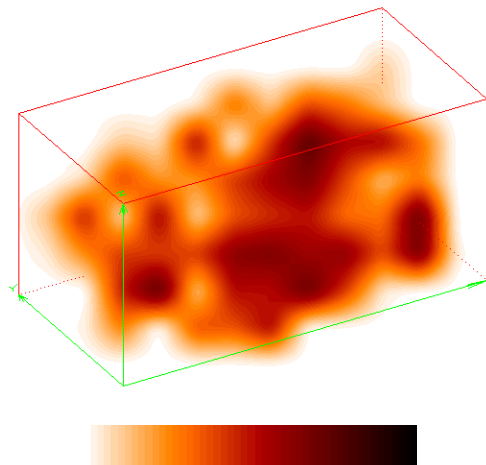


# Lognormal simulations



Bias  $b = 1.5$

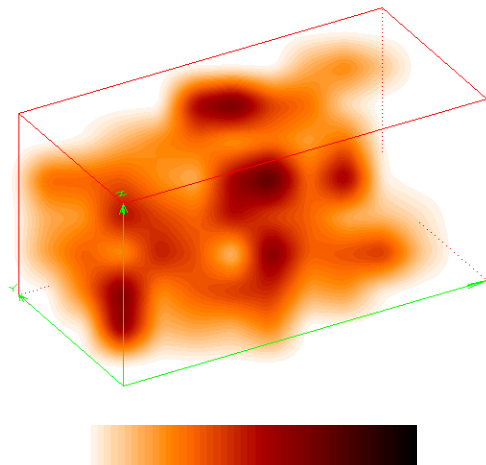
# Lognormal simulations



Bias  $b = 2.0$

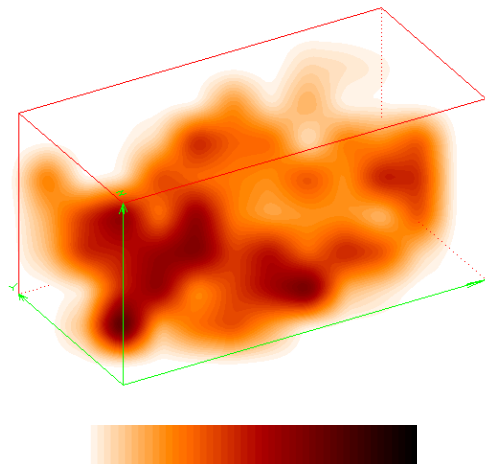
- We use the set of galaxy mocks obtained from the Las Damas simulations <http://lss.phy.vanderbilt.edu/lasdamas> [McBride et al. (2009)], and in particular the *gamma* release.
- The simulations are populated by galaxies using the halo occupation distribution (HOD) formalism, with the HOD parameters tuned to reproduce the observed number density and projected correlation function  $w_p(r_p)$  (at scales  $r_p \in [0.3, 30] h^{-1} \text{ Mpc}$ , as studied in Zehavi et al. (2011)) of the corresponding SDSS catalogues.
- We end up with a total of 120 mock galaxy catalogues and we compute the galaxy density field  $\Delta_{g,i}$ .

# Las Damas mock catalogues



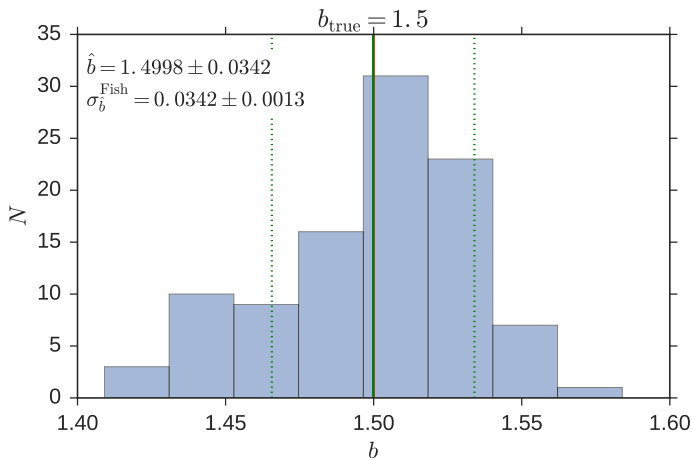
Las Damas

# SDSS main galaxy sample



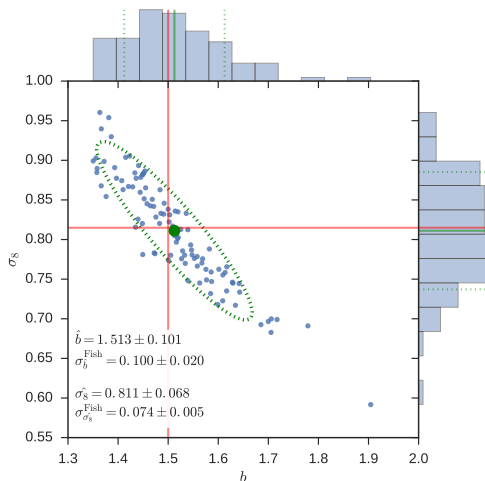
SDSS

# Test on lognormal simulations



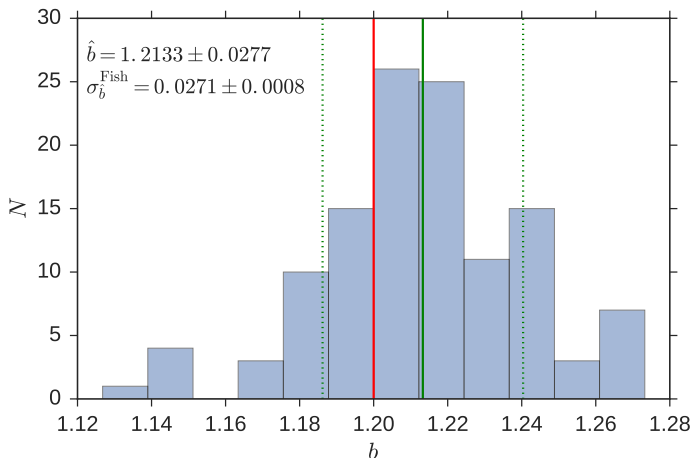
Distribution of the maximum-likelihood estimate of the bias  $\hat{b}$ . The amplitude of the matter power spectrum is fixed.

# Test on lognormal simulations



Distribution of the maximum-likelihood estimates of the parameters  $b$ ,  $\sigma_8$ , for the case in which we allow both parameters to change.

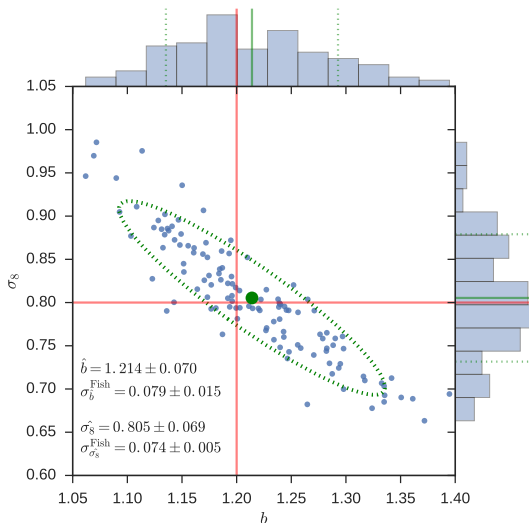
# Test on Las Damas simulations



Distribution of the maximum-likelihood estimates of the bias, for the 120 realizations of the Las Damas mocks, for the case in which  $\sigma_8$  is fixed.

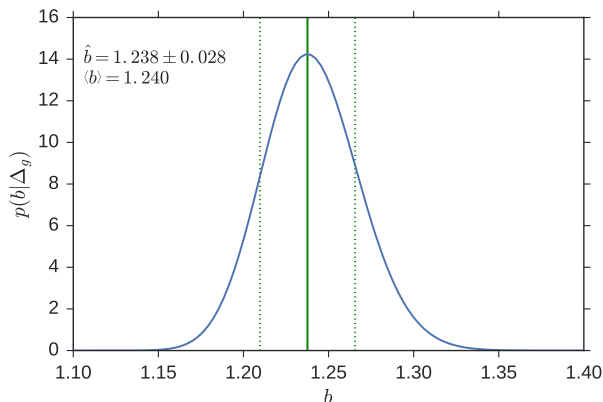


# Test on Las Damas simulations



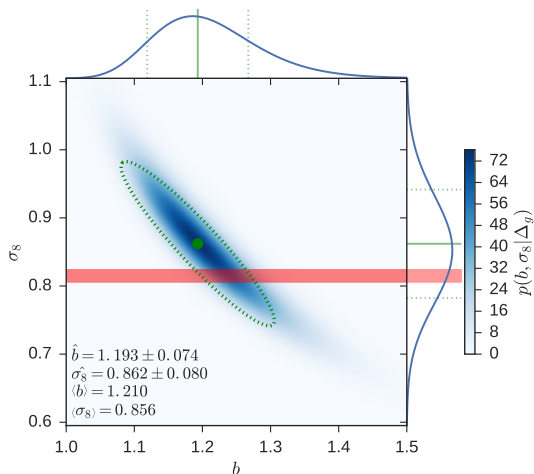
Distribution of the maximum-likelihood joint estimates of  $b$  and  $\sigma_8$ .

# Results for the SDSS data



Posterior probability distribution of the bias parameter obtained by analysing the SDSS catalogue for the case in which  $\sigma_8$  is fixed at its fiducial value,  $\sigma_8^{\text{fid}} = 0.8149$ .

# Results for the SDSS data



Joint posterior probability distribution of  $b$  and  $\sigma_8$  for the SDSS. Top and right sub-panels are marginalized probability distributions.

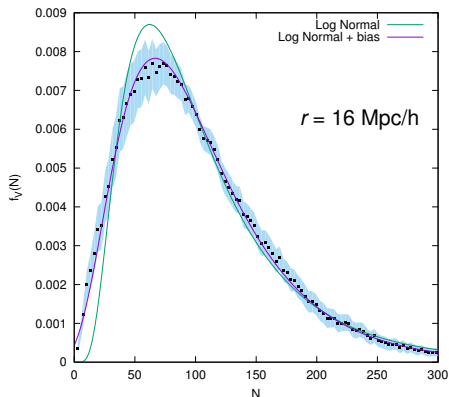
# Results for the SDSS data. Model selection

- *Null hypothesis  $H_0$* : bias is fixed at  $b \equiv 1$ , only  $\sigma_8$  is allowed to vary
- *Alternative hypothesis  $H_1$* : Both bias  $b$  and  $\sigma_8$  are allowed to vary.
  - *AIC: Akaike information criterion*
  - *BIC: Bayesian information criterion*
  - *GLRT: Generalized likelihood ratio test*

Criterion	
<b>AIC</b> ( $H_0$ ) – <b>AIC</b> ( $H_1$ )	12.6
<b>BIC</b> ( $H_0$ ) – <b>BIC</b> ( $H_1$ )	8.22
<b>GLRT</b> : $\log\left(\frac{L_1}{L_0}\right)$ at $\nu = 1$	7.29

The different criteria provide either ‘substantial’ or ‘strong’ evidence in favour of the biased model ( $H_1$ ).

# Results for the SDSS data. Counts-in-cells



For the counts-in-cells distribution function  $f_V(N)$  (1-pdf)  
[Hurtado-Gil, Martínez, Arnalte-Mur et al. A&A 601, A40 (2017)]

$$f(\Delta) = \frac{1}{\sqrt{2\pi H_0}} \frac{\exp\left(-\frac{1}{2} \frac{y^2}{H_0}\right)}{\Delta + b - 1} \quad \text{where} \quad H_0 = \log(1 + C)$$

# Conclusions

- We have presented a full description of the N-probability density function of the galaxy number density fluctuations in terms of the galaxy bias and  $\sigma_8$ .
- Parameter estimation can be performed via the maximum-likelihood, even more, Bayesian inference can be also performed providing a *prior*.
- The methodology has been tested with both ideal log-normal realizations and mocks derived from the Las Damas project, showing that the maximum-likelihood estimates are unbiased.
- We have applied our formalism to the SDSS main sample. For a volume-limited subset with magnitude  $M_r < -20$ . We obtain  $\hat{b} = 1.193 \pm 0.074$  and  $\hat{\sigma}_8 = 0.862 \pm 0.080$ .
- We show that the *alternative* hypothesis ( $H_1$  of a galaxy bias parameter  $b$  given by the maximum-likelihood estimator) is favoured with respect to a no-biasing scenario given by the *null* hypothesis  $H_0$  of  $b \equiv 1$ .

- Arnalte-Mur, P., Martínez, V. J., Norberg, P., et al. 2014, Mon. Not. Roy. Astron. Soc. , 441, 1783
- Ata, M., Kitaura, F.-S., & Müller, V. 2015, Mon. Not. Roy. Astron. Soc. , 446, 4250
- Blanton, M. R., Schlegel, D. J., Strauss, M. A., et al. 2005, AJ, 129, 2562
- Coles, P. & Jones, B. 1991, Mon. Not. Roy. Astron. Soc. , 248, 1
- Colless, M., Dalton, G., Maddox, S., et al. 2001, Mon. Not. Roy. Astron. Soc. , 328, 1039
- Croft, R. A. C., Dalton, G. B., & Efstathiou, G. 1999, Mon. Not. Roy. Astron. Soc. , 305, 547
- Di Porto, C., Branchini, E., Bel, J., et al. 2014, ArXiv e-prints
- Fry, J. N. & Gaztanaga, E. 1993, Astrophys. J. , 413, 447
- Gaztanaga, E. & Frieman, J. A. 1994, Astrophys. J. Let. , 437, L13
- Granett, B. R., Branchini, E., Guzzo, L., et al. 2015, Astron. Astrophys. , 583, A61
- Guo, H. & Jing, Y. P. 2009, Astrophys. J. , 702, 425
- Jasche, J., Leclercq, F., & Wandelt, B. D. 2015, JCAP, 1, 36

- Jasche, J. & Wandelt, B. D. 2013, *Astrophys. J.* , 779, 15
- Kayo, I., Taruya, A., & Suto, Y. 2001, *Astrophys. J.* , 561, 22
- Kovač, K., Porciani, C., Lilly, S. J., et al. 2011, *Astrophys. J.* , 731, 102
- Marinoni, C., Le Fèvre, O., Meneux, B., et al. 2005, *Astron. Astrophys.* , 442, 801
- McBride, C., Berlind, A., Scoccimarro, R., et al. 2009, in *Bulletin of the American Astronomical Society*, Vol. 41, American Astronomical Society Meeting Abstracts #213, 253
- McBride, C. K., Connolly, A. J., Gardner, J. P., et al. 2011, *Astrophys. J.* , 739, 85
- Norberg, P., Baugh, C. M., Hawkins, E., et al. 2002, *Mon. Not. Roy. Astron. Soc.* , 332, 827
- Pollack, J. E., Smith, R. E., & Porciani, C. 2014, *Mon. Not. Roy. Astron. Soc.* , 440, 555
- Sigad, Y., Branchini, E., & Dekel, A. 2000, *Astrophys. J.* , 540, 62
- Swanson, M. E. C., Tegmark, M., Blanton, M., & Zehavi, I. 2008, *Mon. Not. Roy. Astron. Soc.* , 385, 1635



Szapudi, I. 1998, Mon. Not. Roy. Astron. Soc. , 300, L35

Verde, L., Heavens, A. F., Percival, W. J., et al. 2002, Mon. Not. Roy. Astron. Soc. , 335, 432

Zehavi, I., Zheng, Z., Weinberg, D. H., et al. 2011, Astrophys. J. , 736, 59

Zehavi, I., Zheng, Z., Weinberg, D. H., et al. 2005, Astrophys. J. , 630, 1