Weak Lensing on the Ball

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What is Gravitational Lensing?

- Propagating photons follow geodesics in space
- The geodesics are distorted from 'straight' lines by the presence of massive objects
- Amount of deflection depends on
  - Geometry of the lens-observer-source set up
  - Mass of the lensing object

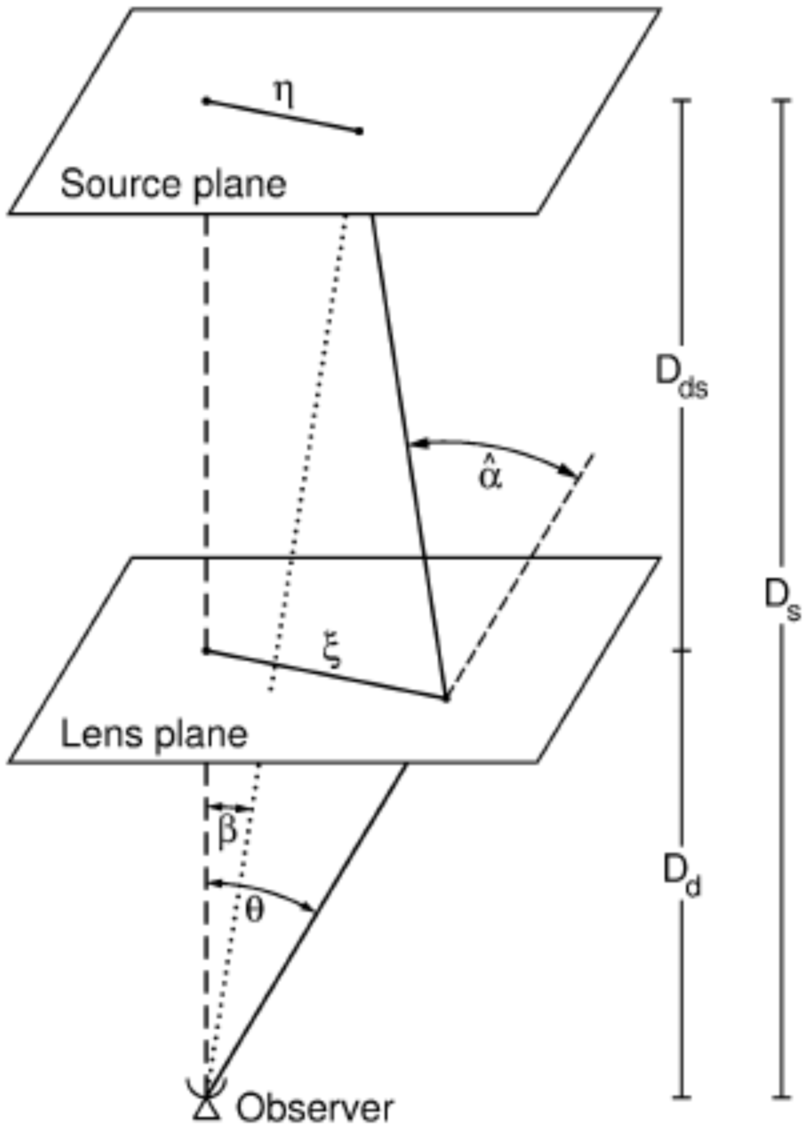
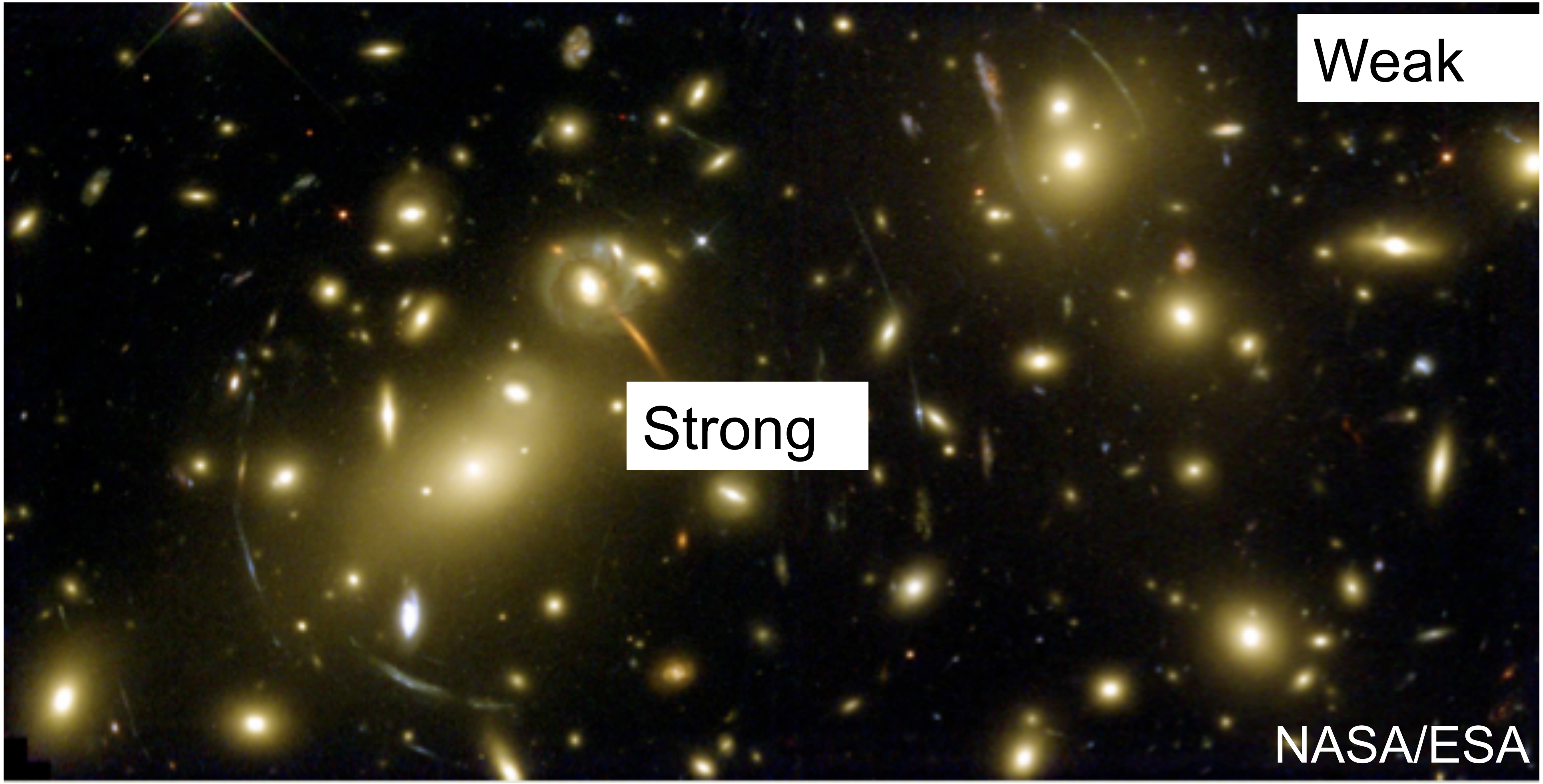
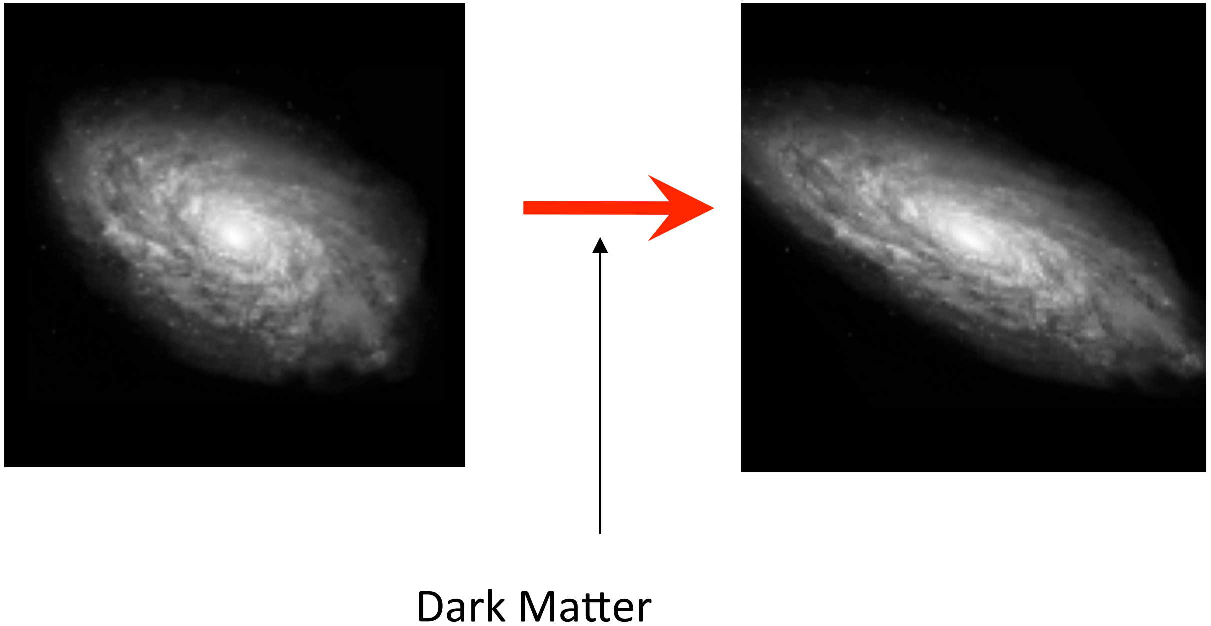


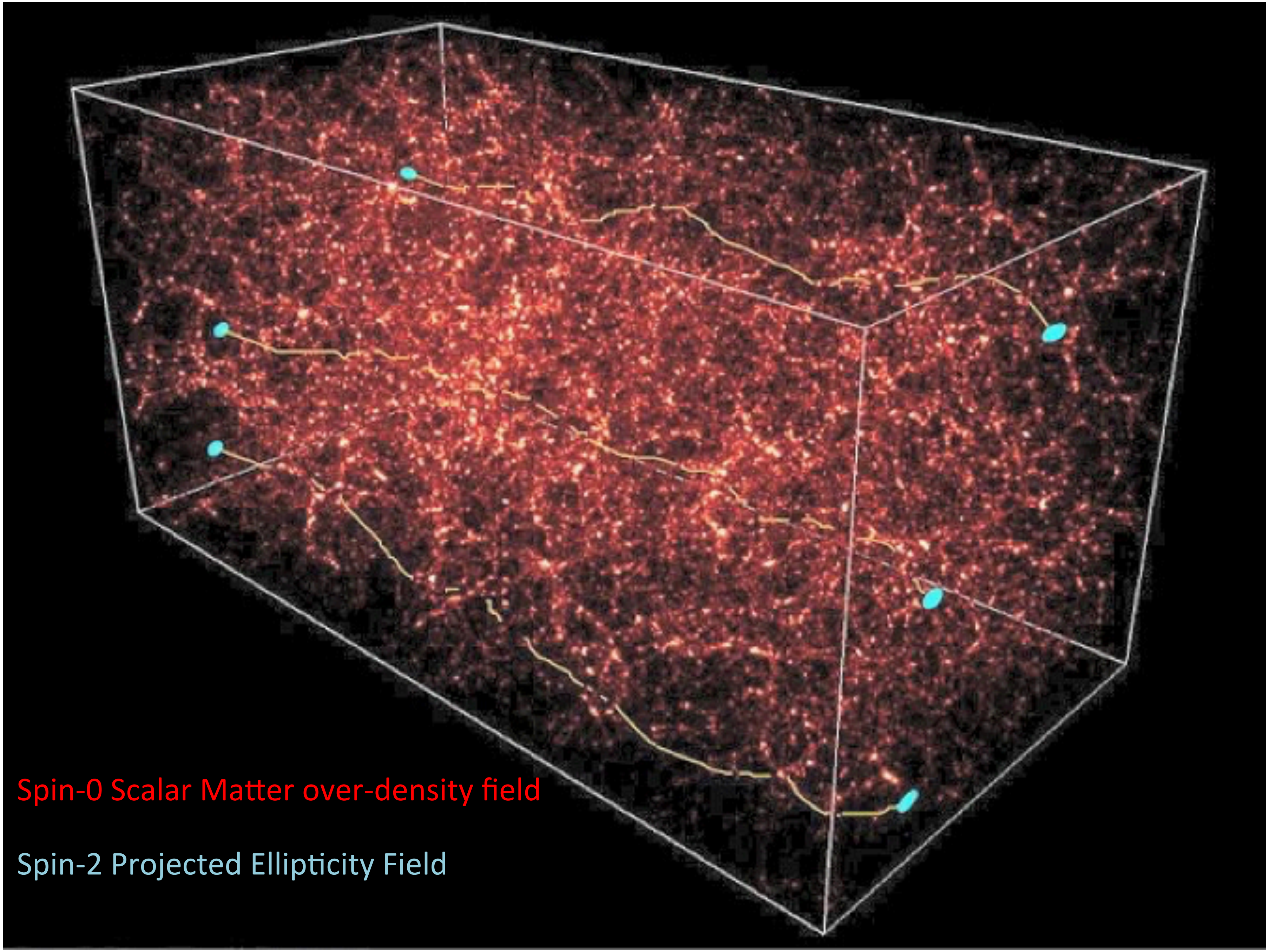
Figure from Bartelmann & Schneider 2003



- Abell 2218 (Draco)

The weak distortion is simply a (very small) change in ellipticity of a galaxy




Spin-0 Scalar Matter over-density field
Spin-2 Projected Ellipticity Field

spectroscopic

✓ types + redshifts

✗ no lensing

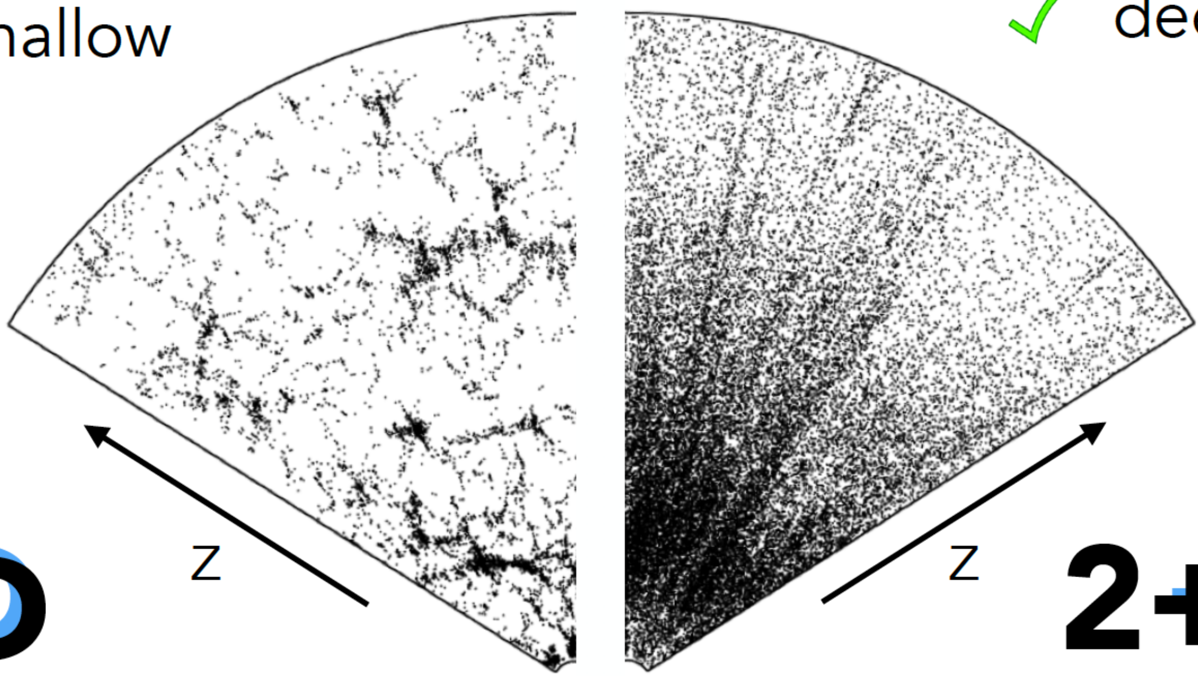
✗ shallow

photometric

✗ no types / redshifts

✓ lensing

✓ deep

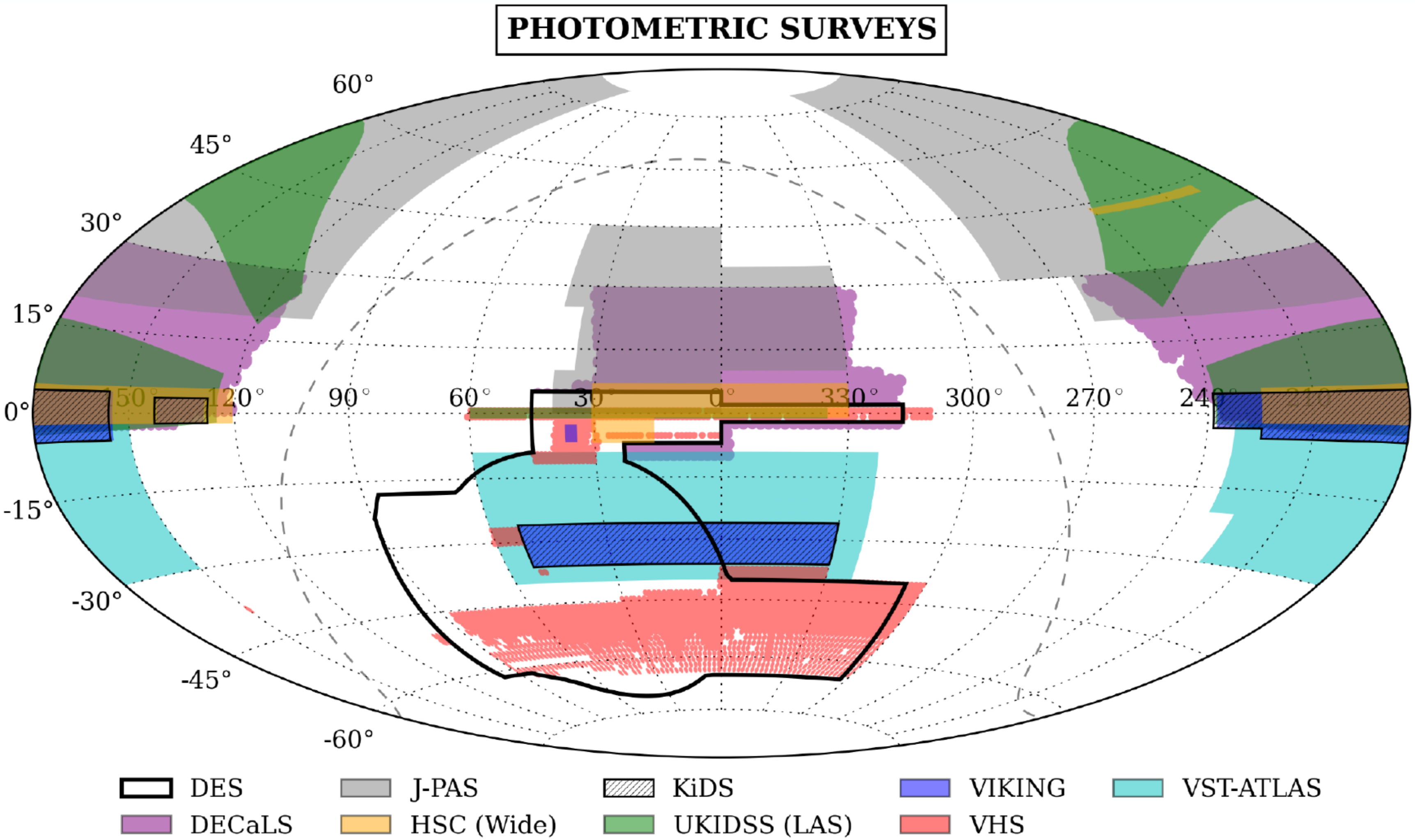


3D

2+1D

credit: Aragon-Calvo et al (2014)

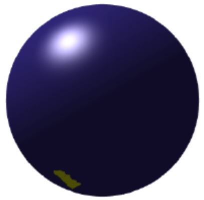
When? Now! (and only getting better)



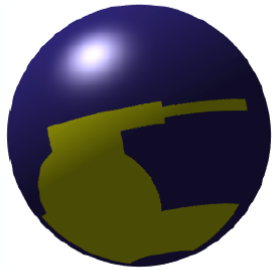
Credit: A. Merson and M. Soumagnac

Spherical Sky

- Shape and size area of sky

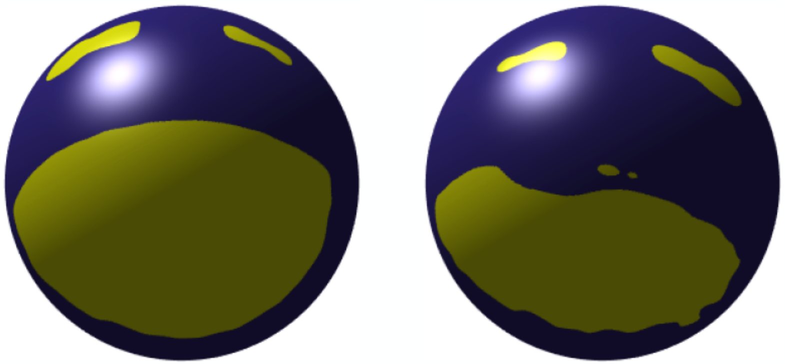


DES SV



DES full

Euclid



$$A(\mathbf{k}; r(t)) = \int d^3\mathbf{x}\ e^{-i\mathbf{k}\cdot\mathbf{x}} A(\mathbf{x}; r(t)),$$

3D random field at hypersurface r(t) and Fourier transform

$$\langle A(\mathbf{k}; r(t_1)) A^*(\mathbf{k}'; r(t_2)) \rangle = (2\pi)^3 \delta^3(\mathbf{k} - \mathbf{k}') P(k; r(t_1), r(t_2)),$$

Power Spectrum

$$\tilde{A}(\hat{\mathbf{n}}; r) = \int_0^r dr_1\ F_A(r, r_1) A(r_1 \hat{\mathbf{n}}; r_1),$$

Projected Field

Projected Power Spectrum

$$\begin{aligned} C_\ell^{AB}(r, r') &\equiv \langle \tilde{A}_{\ell m}(r) \tilde{B}^*_{\ell m}(r') \rangle \\ &= \int_0^r dr_1 \int_0^r dr_2 F_A(r, r_1) F_B(r', r_2) \int \frac{d^3\mathbf{k}}{(2\pi)^3} P(k; r_1, r_2) (4\pi)^2 j_\ell(kr_1) j_\ell(kr_2) Y_{\ell m}(\hat{\mathbf{k}}) Y^*_{\ell m}(\hat{\mathbf{k}}) \\ &= \int_0^r dr_1 \int_0^{r'} dr_2 F_A(r, r_1) F_B(r', r_2) \int \frac{2dk k^2}{\pi} P(k; r_1, r_2) j_\ell(kr_1) j_\ell(kr_2) \end{aligned}$$

(

Equal-Time Ansatz $$P(k; r_1, r_2) \simeq [P(k; r_1)P(k; r_2)]^{1/2}.$$

- assuming that the correlation of the underlying field is restricted to small-scales
- over such scales the look-back time is approximately equal (r1 ≈ r2)
- therefore either P (k ; r1) or P (k ; r2) could be used instead of P (k ; r1, r2)
- The geometric mean approximation is then used as an algebraic convenience such that the integrals can be separated

$$\delta(\mathbf{k}, t) = \sum_{n=1}^{\infty} D^n(t) f_n(\mathbf{k}),$$

$$\begin{aligned} P^{\mathrm{UETC}}(k; r, r') &= \langle \delta(\mathbf{k}, t)\delta^*(\mathbf{k}, t')\rangle \\ &= \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} D^n(t)D^m(t')\langle f_n(\mathbf{k}) f_m^*(\mathbf{k})\rangle \end{aligned}$$

$$\begin{aligned} P^{\mathrm{ETC}}(k; r(t)) &= \langle \delta(\mathbf{k}, t)\delta^*(\mathbf{k}, t)\rangle \\ &= \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} D^n(t)D^m(t)\langle f_n(\mathbf{k}) f_m^*(\mathbf{k})\rangle \end{aligned}$$

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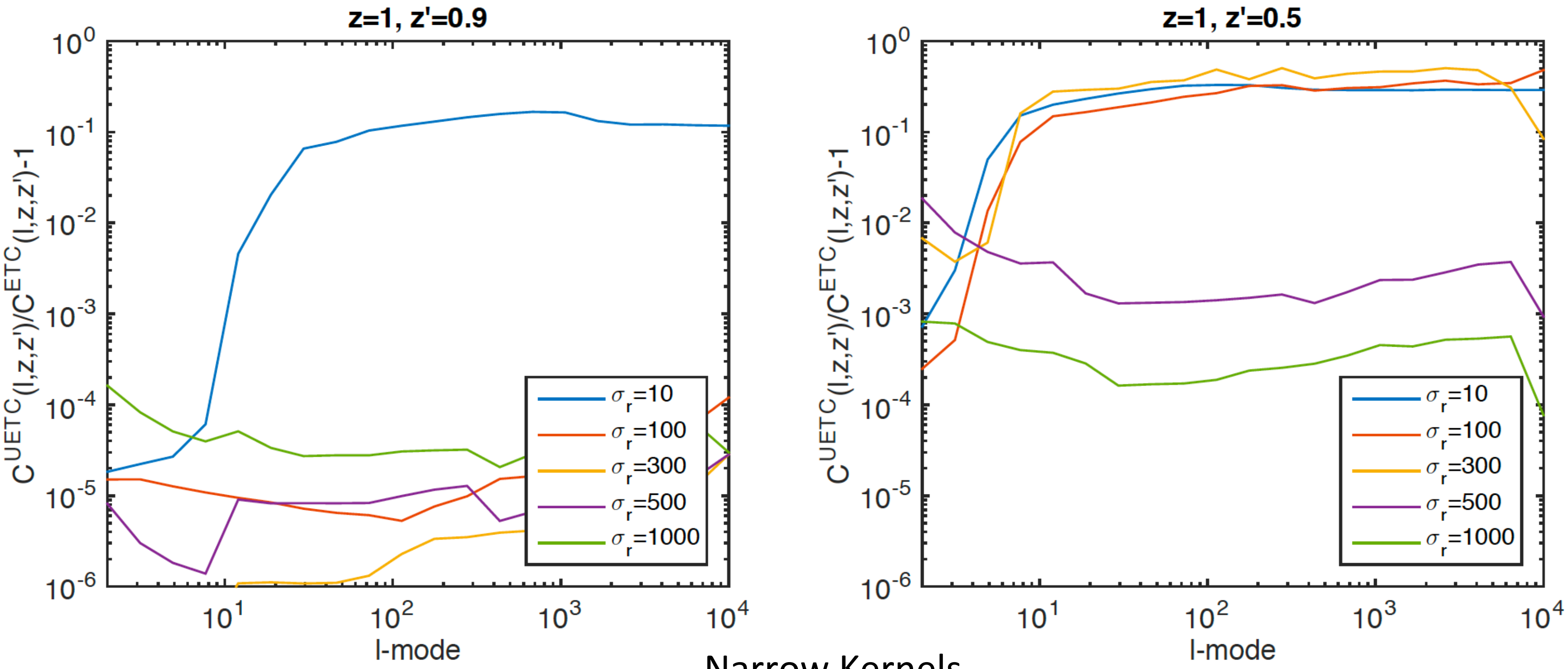
Perturbatively expanded δ at order nm

$$\begin{aligned} P^{\mathrm{ETC}}(k;r(t)) &= \langle \delta(\mathbf{k},t)\delta^*(\mathbf{k},t)\rangle \\ &= \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} D^n(t)D^m(t)\langle f_n(\mathbf{k}) f_m^*(\mathbf{k})\rangle \\ &= \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} D^n(t)D^m(t)P_{nm}(k) \\ &= D^2(t)P_{11}(k) + D^4(t)P_{22}(k) + 2D^4(t)P_{13}(k) + \ldots \end{aligned}$$

$$\delta(\mathbf{k},t) \simeq \exp\left[\int^t \mathrm{d}t' \int \frac{\mathrm{d}^3\mathbf{k}'}{(2\pi)^3}\frac{\mathbf{k}.\mathbf{k}'}{k'^2}\delta_{\mathrm{L}}(\mathbf{k},t')\right] \times \delta_{\mathrm{S}}(\mathbf{k},t),$$

Eikonal phase approximation

Gaussian Projection Kernel



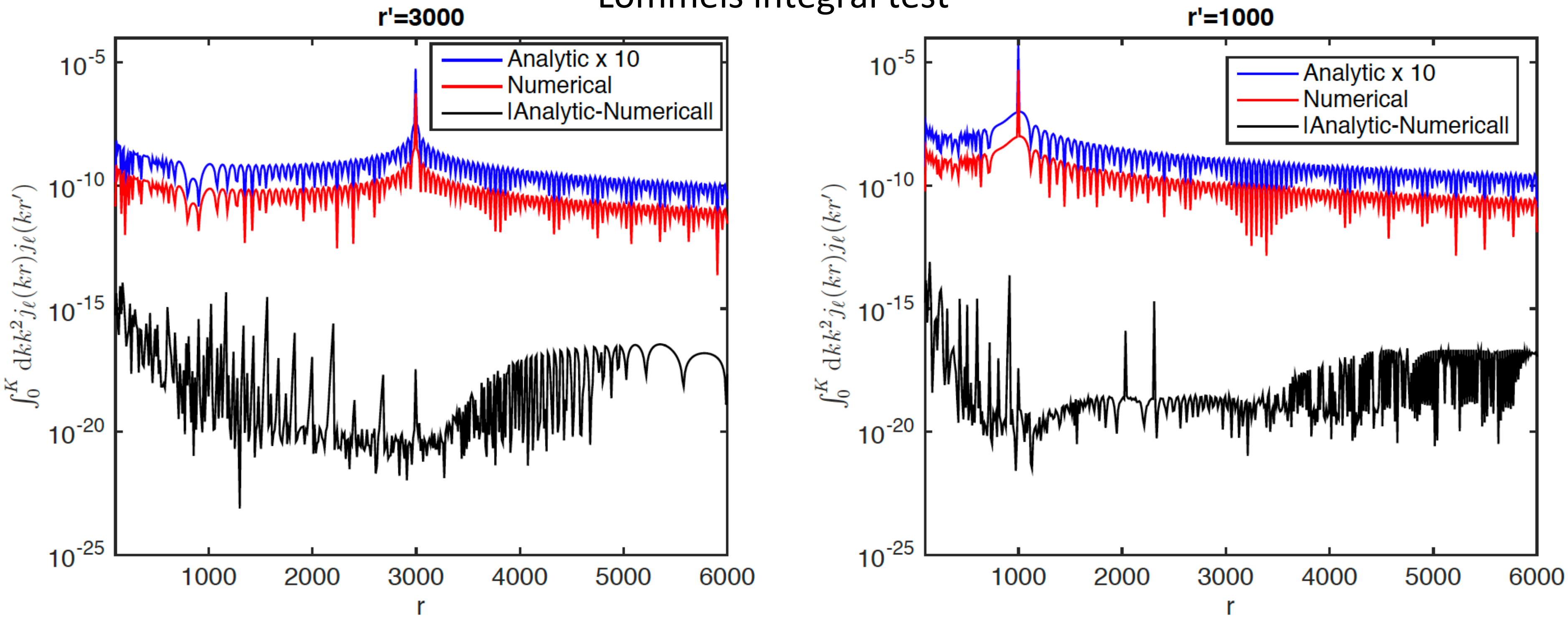
Narrow Kernels
and/or close redshift bins

may (or may not) be a major issue – more work needed

$$\int \frac{2dkk^2}{\pi} P^{\rm UETC}(k;r_1,r_2) j_\ell(kr_1) j_\ell(kr_2)$$

$$\int \frac{2dkk^2}{\pi} [P^{\rm ETC}(k;r_1) P^{\rm ETC}(k,r_2)]^{1/2} j_\ell(kr_1) j_\ell(kr_2),$$

Lommels integral test



Yes, some approximations are available − more on these later − but full case is hard

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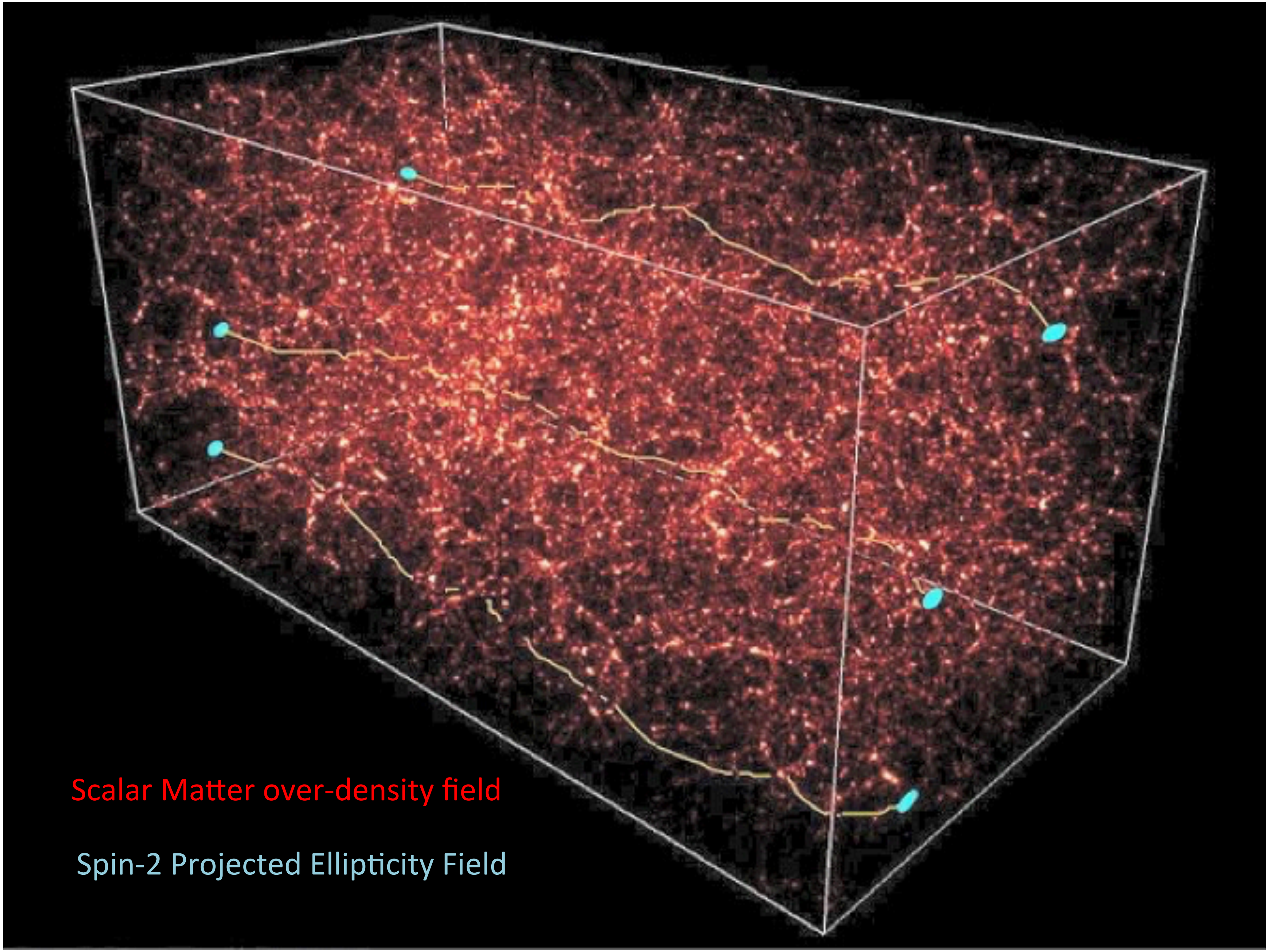
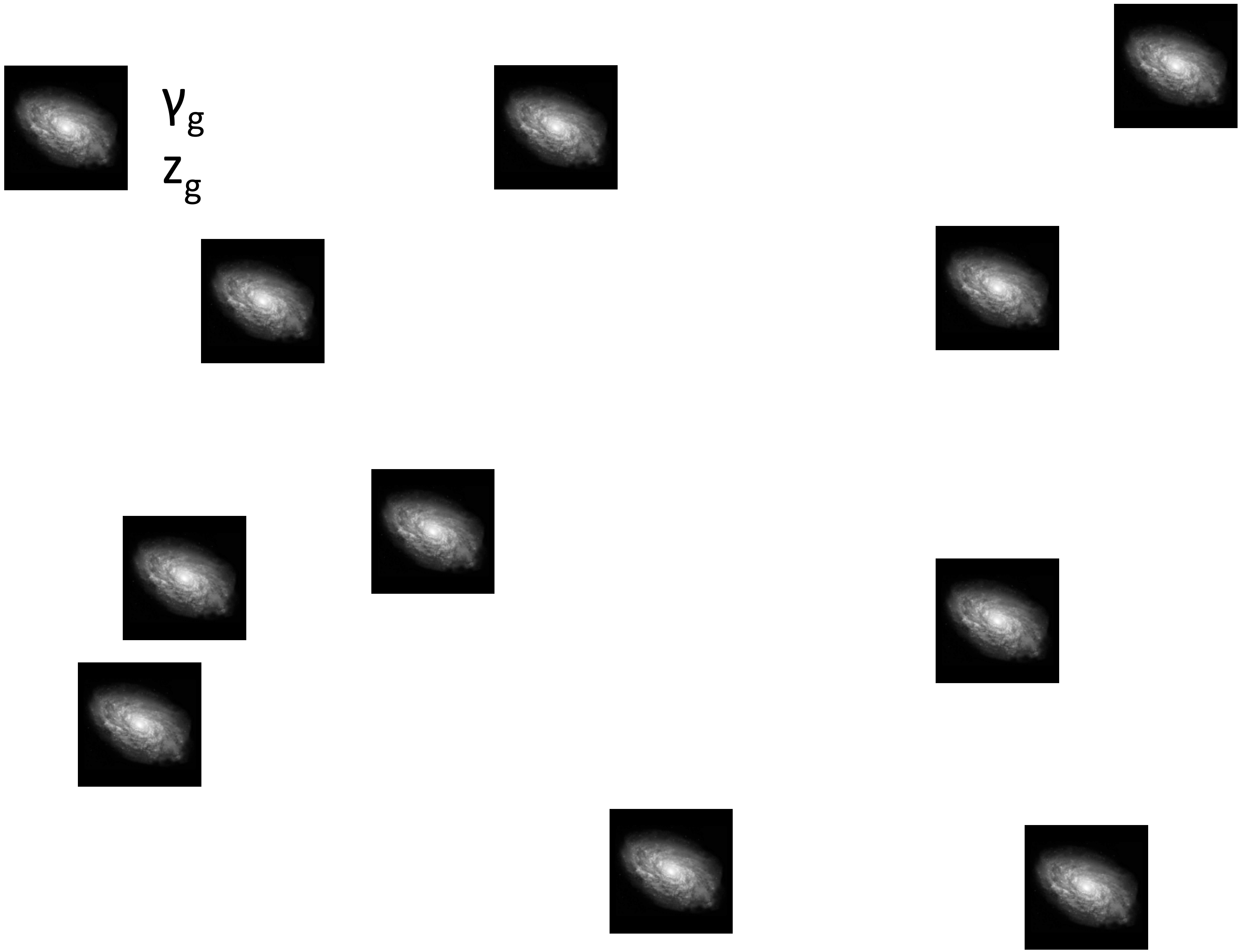

Scalar Matter over-density field
Spin-2 Projected Ellipticity Field


γ_g
z_g

Spherical-Bessel

$$\gamma_\ell^m(k) = \left(\frac{2}{\pi}\right)^{1/2} \sum_g \gamma_g(r_g, \theta_g) j_\ell(kr_g)\,{}_2Y_\ell^m(\theta_g)$$

Complex Field　　Spin-2 Spherical Harmonics

$$\langle \gamma_\ell^m(k) \gamma_{\ell'}^{m'*}(k') \rangle = C_\ell^{SB}(k, k') \delta_{\ell\ell'} \delta_{mm'}$$

Power Spectrum

$$\gamma_\ell^m(k) = \left(\frac{2}{\pi}\right)^{1/2} \sum_g \gamma_g(r_g,\theta_g) j_\ell(kr_g) {}_2Y_\ell^m(\theta_g) \quad \langle \gamma_\ell^m(k) \gamma_{\ell'}^{m'*}(k') \rangle = C_\ell^{SB}(k,k')\delta_{\ell\ell'}\delta_{mm'}$$

$$C_\ell^{SB}(k,k') = |D_\ell|^2 \mathcal{A}^2 \left(\frac{2}{\pi}\right) \int \frac{\mathrm{d}\tilde{k}}{\tilde{k}^2} G_\ell^{SB}(k,\tilde{k}) G_\ell^{SB}(k',\tilde{k}),$$

$$G_\ell^{SB}(k,\tilde{k}) = \int \mathrm{d}z_p j_\ell(kr(z_p)) n(z_p) \times \int \mathrm{d}z' p(z'|z_p) U_\ell(r[z'],\tilde{k}),$$

$$U_\ell(r[z],k) = \int_0^{r[z]} \mathrm{d}r' \frac{F_K(r,r')}{a(r')} j_\ell(kr') P^{1/2}(k,r'),$$

$$|D_\ell| = \sqrt{(\ell+2)!/(\ell-2)!}$$

Spherical-Radial

$$\gamma_\ell^m(z) = \left(\frac{2}{\pi}\right)^{1/2} \sum_{g \in z} \gamma_g(r_g, \theta_g)\, {}_2Y_\ell^m(\theta_g)$$

Complex Field   Spin-2 Spherical Harmonics

$$\langle \gamma_\ell^m(z)\, \gamma_{\ell'}^{m'\,*}(z') \rangle = C_\ell^{SB}(z, z)\, \delta_{\ell\ell'}\, \delta_{mm'}$$

$$\gamma_\ell^m(z) = \left(\frac{2}{\pi}\right)^{1/2} \sum_{g\in z} \gamma_g(r_g, \theta_g)\, {}_2Y_\ell^m(\theta_g) \quad \langle \gamma_\ell^m(z)\gamma_{\ell'}^{m'*}(z')\rangle = C_\ell^{SB}(z, z')\delta_{\ell\ell'}\delta_{mm'}$$

$$C_\ell^{SR}(z, z') = |D_\ell|^2 \mathcal{A}^2 \left(\frac{2}{\pi}\right) \int \frac{\mathrm{d}k}{k^2} G_\ell^{SR}(z, k) G_\ell^{SR}(z', k),$$

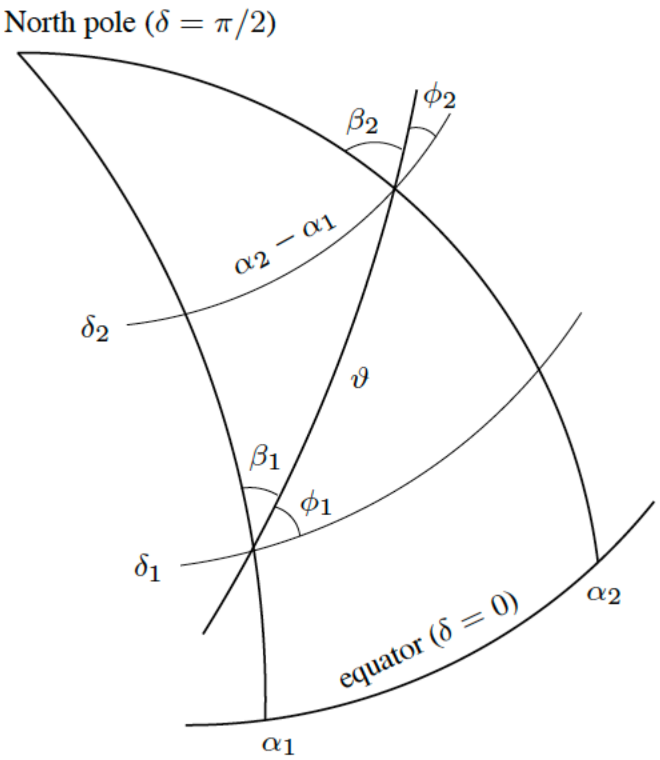
$$\begin{aligned} G_\ell^{SR}(z, k) &= \int \mathrm{d}z_p W^{SR}(z, z_p) n(z_p) \\ &\times \int \mathrm{d}z' p(z'|z_p) U_\ell(r[z'], k), \end{aligned}$$

$$U_\ell(r[z], k) = \int_0^{r[z]} \mathrm{d}r' \frac{F_K(r, r')}{a(r')} j_\ell(kr') P^{1/2}(k, r'),$$

$$|D_\ell| = \sqrt{(\ell+2)!/(\ell-2)!}$$

Real/Configuration Space in Angular Direction

$$\hat{\xi}_{\pm}(\vartheta) = \frac{\sum_{ij} w_i w_j [\varepsilon_{\rm t}(\boldsymbol{\vartheta}_i)\,\varepsilon_{\rm t}(\boldsymbol{\vartheta}_j) \pm \varepsilon_{\times}(\boldsymbol{\vartheta}_i)\,\varepsilon_{\times}(\boldsymbol{\vartheta}_j)]}{\sum_{ij} w_i w_j}$$



$$\xi_+(\theta, z, z') = \frac{1}{2\pi}\sum_\ell (\ell + 0.5) d^\ell_{22}(\theta) [C_\ell^{SR,E}(z,z') + C_\ell^{SR,B}(z,z')]$$

$$\xi_-(\theta, z, z') = \frac{1}{2\pi}\sum_\ell (\ell + 0.5) d^\ell_{-22}(\theta) [C_\ell^{SR,E}(z,z') - C_\ell^{SR,B}(z,z')].$$

Some Approximations

- All main weak lensing results use approximations of the full case

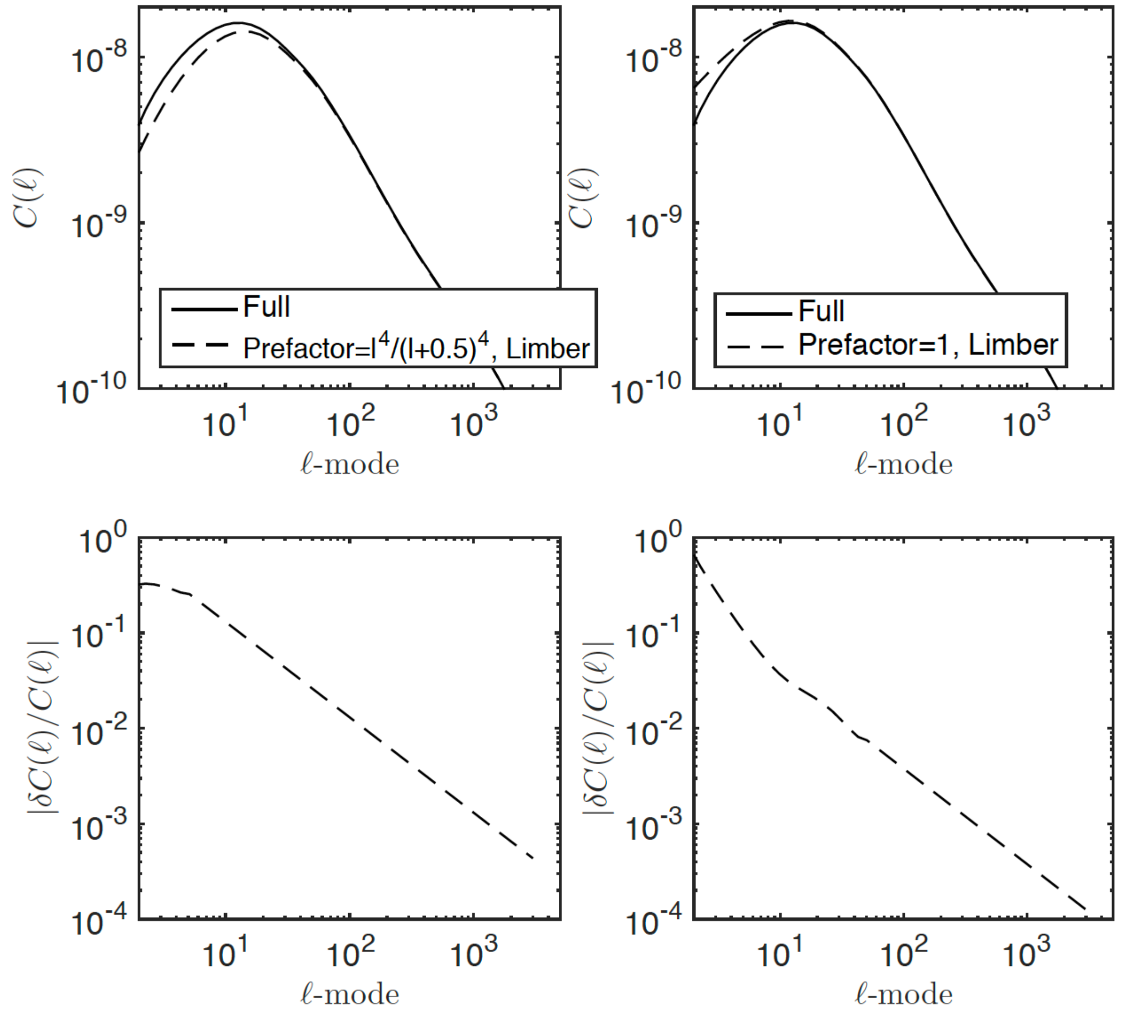
- Main approximations **all used in combination**:
  - Limber and Pre-factor Unity
  - Hankel transforms
  - Binning in redshift, 'tomography'
- i.e. binned, high-ell, small theta, correlation functions

- Approximations (probably) ok for current surveys
- But (probably) not for next-generation (Euclid, LSST)

Limber (1953) - Kaiser (1998) Approximation $\lim_{\epsilon \to 0} \int_0^\infty e^{-\epsilon(x-\nu)} f(x) J_\nu(x) dx = f(\nu) - \frac{1}{2} f''(\nu) - \frac{\nu}{6} f'''(\nu) + \ldots$

$$j_\ell(kr) \to \sqrt{\frac{\pi}{2\ell+1}} \delta^D(\ell + 1/2 - kr)$$

$$|D_\ell| = \sqrt{(\ell+2)!/(\ell-2)!} \longrightarrow 1$$



- Replace Bessel functions with delta functions
- Additional assumption of no l-dependence leads to lucky cancellation

l>>2 / Hankel Transform Approximation

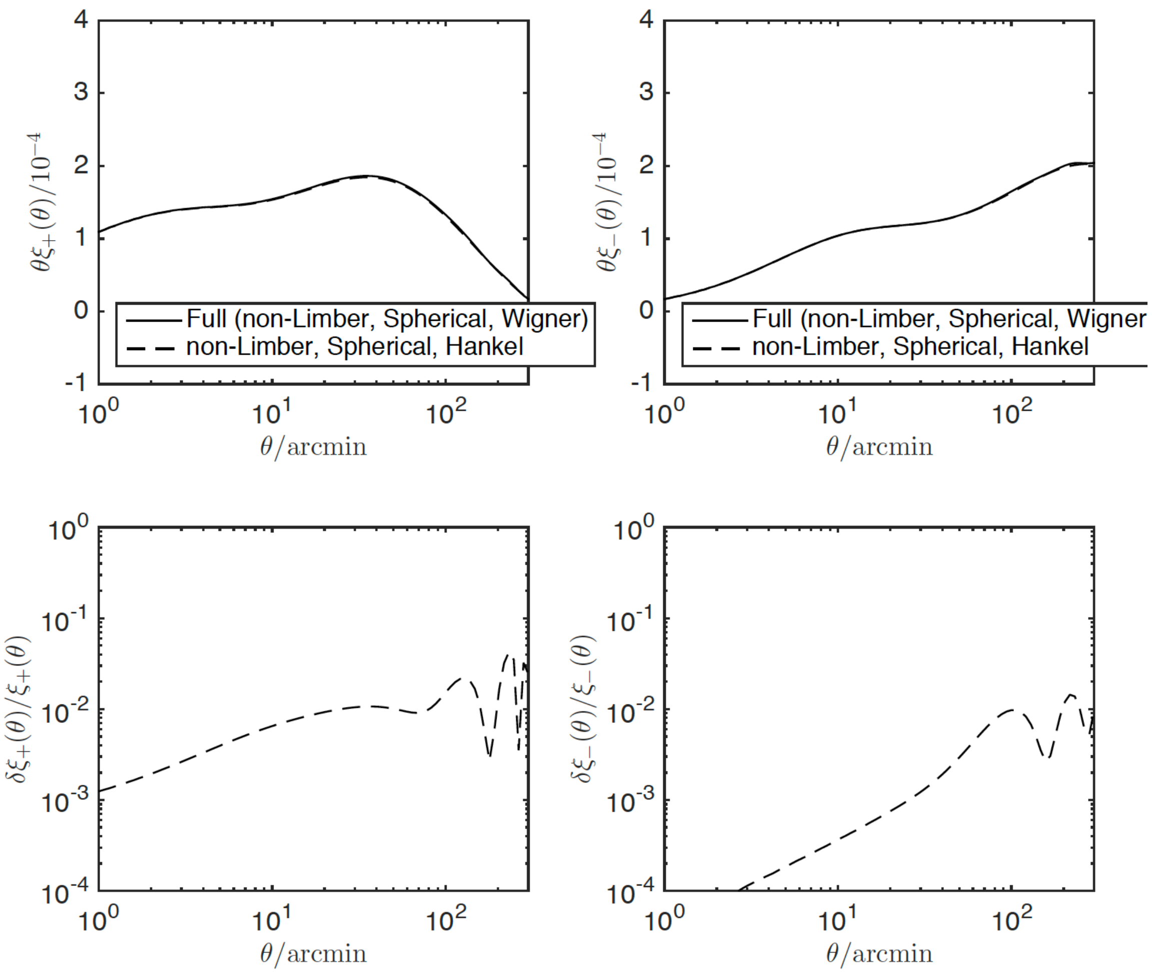
$$\xi_+(\theta, z, z') = \frac{1}{2\pi}\sum_\ell (\ell+0.5) d^\ell_{22}(\theta) \left[C_\ell^{SR,E}(z,z') + C_\ell^{SR,B}(z,z')\right]$$

$$\xi_-(\theta, z, z') = \frac{1}{2\pi}\sum_\ell (\ell+0.5) d^\ell_{-22}(\theta) \left[C_\ell^{SR,E}(z,z') - C_\ell^{SR,B}(z,z')\right].$$

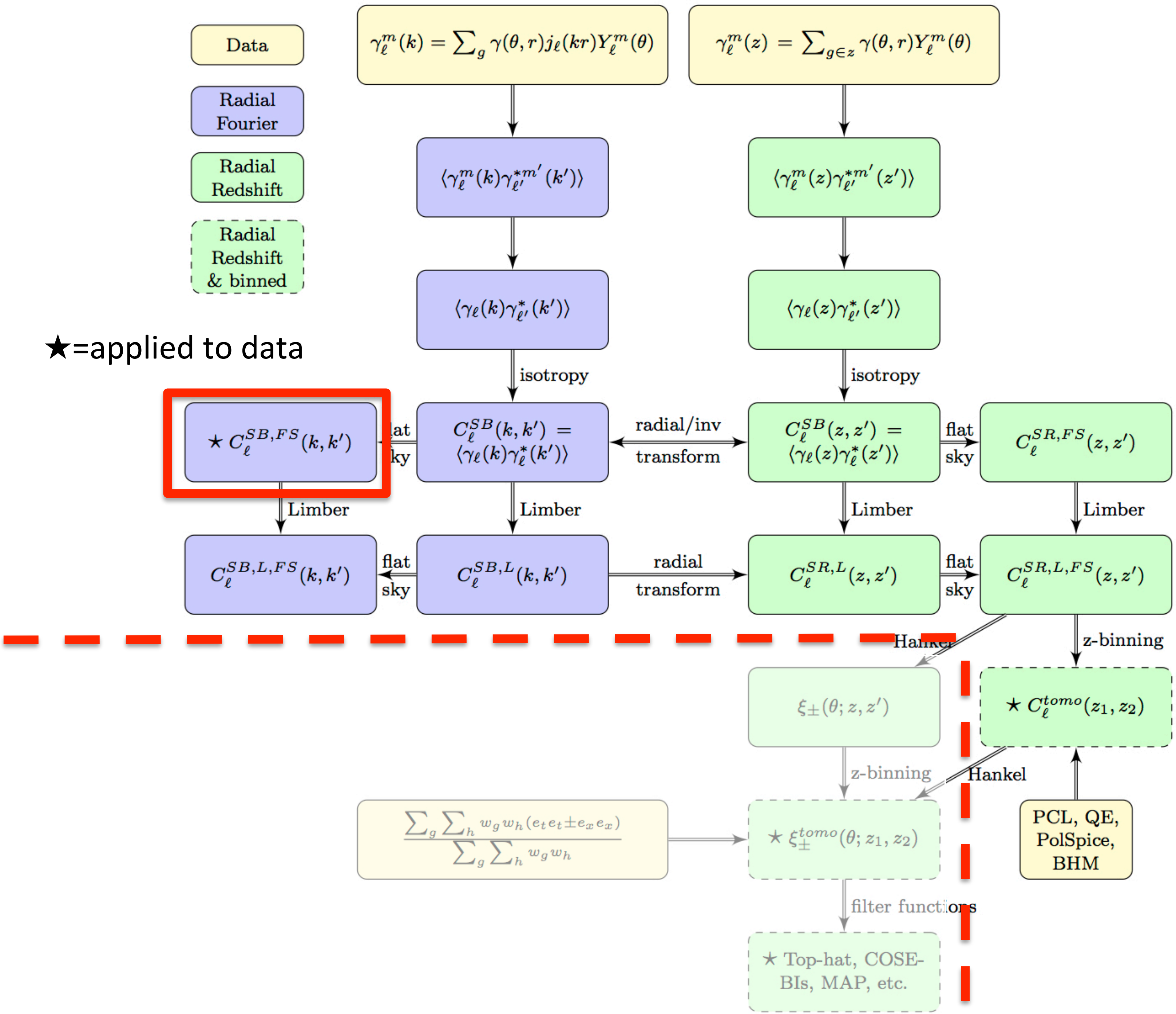
↓

$$\xi_+(\theta, z, z') = \frac{1}{2\pi}\sum_\ell \ell J_0(\ell\theta) \left[C_\ell^{SR,E}(z,z') + C_\ell^{SR,B}(z,z')\right]$$

$$\xi_-(\theta, z, z') = \frac{1}{2\pi}\sum_\ell \ell J_4(\ell\theta) \left[C_\ell^{SR,E}(z,z') - C_\ell^{SR,B}(z,z')\right]$$



- In large l-mode case l>>2 Wigner matrices are approximated by Bessel Functions of the 1st kind


Data
Radial Fourier
Radial Redshift
Radial Redshift & binned
★=applied to data
$\gamma_\ell^m(k) = \sum_g \gamma(\theta, r) j_\ell(kr) Y_\ell^m(\theta)$
$\gamma_\ell^m(z) = \sum_{g\in z} \gamma(\theta, r) Y_\ell^m(\theta)$
$\langle \gamma_\ell^m(k) \gamma_{\ell'}^{*m'}(k') \rangle$
$\langle \gamma_\ell^m(z) \gamma_{\ell'}^{*m'}(z') \rangle$
$\langle \gamma_\ell(k) \gamma_{\ell'}^{*}(k') \rangle$
$\langle \gamma_\ell(z) \gamma_{\ell'}^{*}(z') \rangle$
isotropy
isotropy
★ $C_\ell^{SB,FS}(k,k')$
flat sky
$C_\ell^{SB}(k,k') = \langle \gamma_\ell(k) \gamma_\ell^{*}(k') \rangle$
radial/inv transform
$C_\ell^{SB}(z,z') = \langle \gamma_\ell(z) \gamma_\ell^{*}(z') \rangle$
flat sky
$C_\ell^{SR,FS}(z,z')$
Limber
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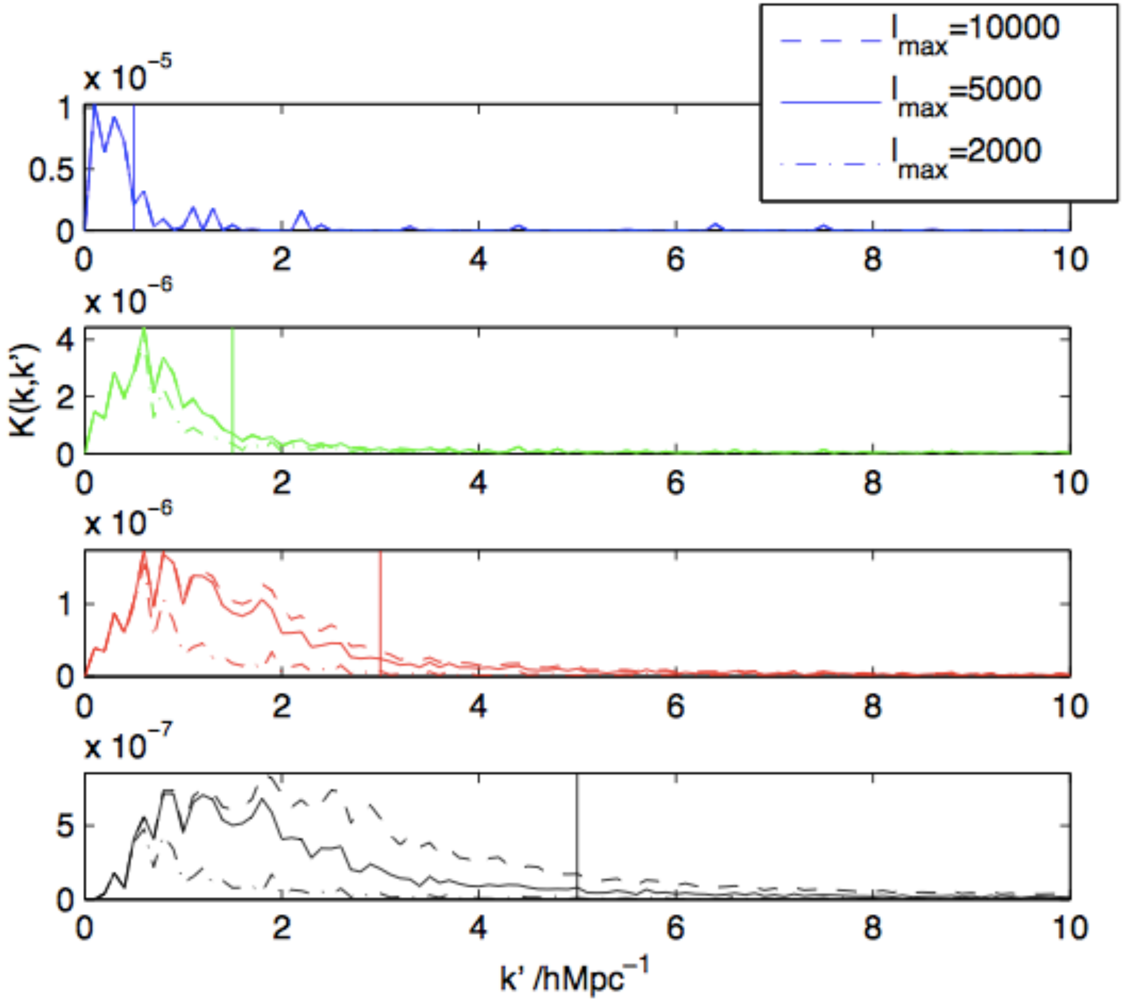
Hankel
z-binning
$\xi_\pm(\theta; z, z')$
★ $C_\ell^{tomo}(z_1, z_2)$
z-binning
Hankel
$\frac{\sum_g \sum_h w_g w_h (e_t e_t \pm e_x e_x)}{\sum_g \sum_h w_g w_h}$
★ $\xi_\pm^{tomo}(\theta; z_1, z_2)$
PCL, QE, PolSpice, BHM
filter functions
★ Top-hat, COSEBIs, MAP, etc.

Good Behavior in wavenumbers

$$C_\ell^S(k,k) = \int P(k';z)K(k',k)\mathrm{d}k'$$

For all Redshifts



k_{max}=1.5 h/Mpc
Peak k~0.5 h/Mpc

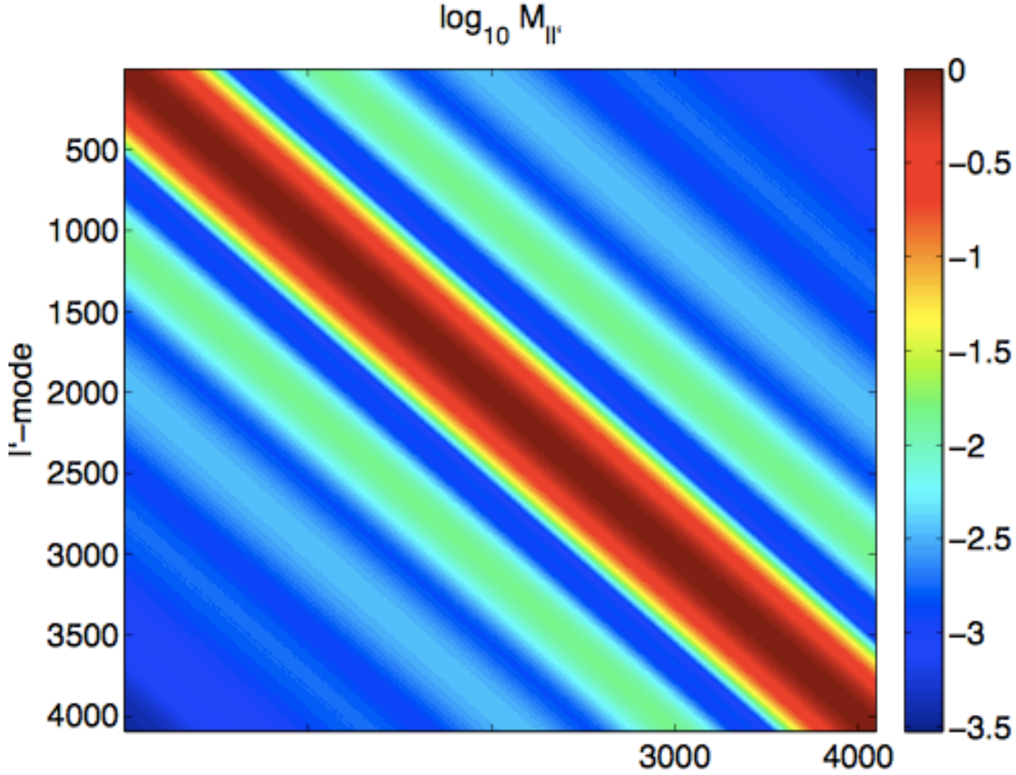
k_{max}=5.0 h/Mpc
Peak k~2.0 h/Mpc

Kitching et al. (2014)

Masks mix modes

$$\widetilde{C}_\ell^{EE}(k_1,k_2) = \left(\frac{\pi}{2}\right)^2 \sum_{\ell'} \left(\frac{\ell'}{\ell}\right) M^{3D}_{\ell\ell'} C^S_\ell \left(k_1\frac{\ell'}{\ell}, k_2\frac{\ell'}{\ell}\right)$$

- Can use Pseudo-Cl approach
  - Note slightly different from CMB – need to generalise to 3D case
  - (k and l-modes mix)



CFHTLenS W1

Kitching et al. (2014)

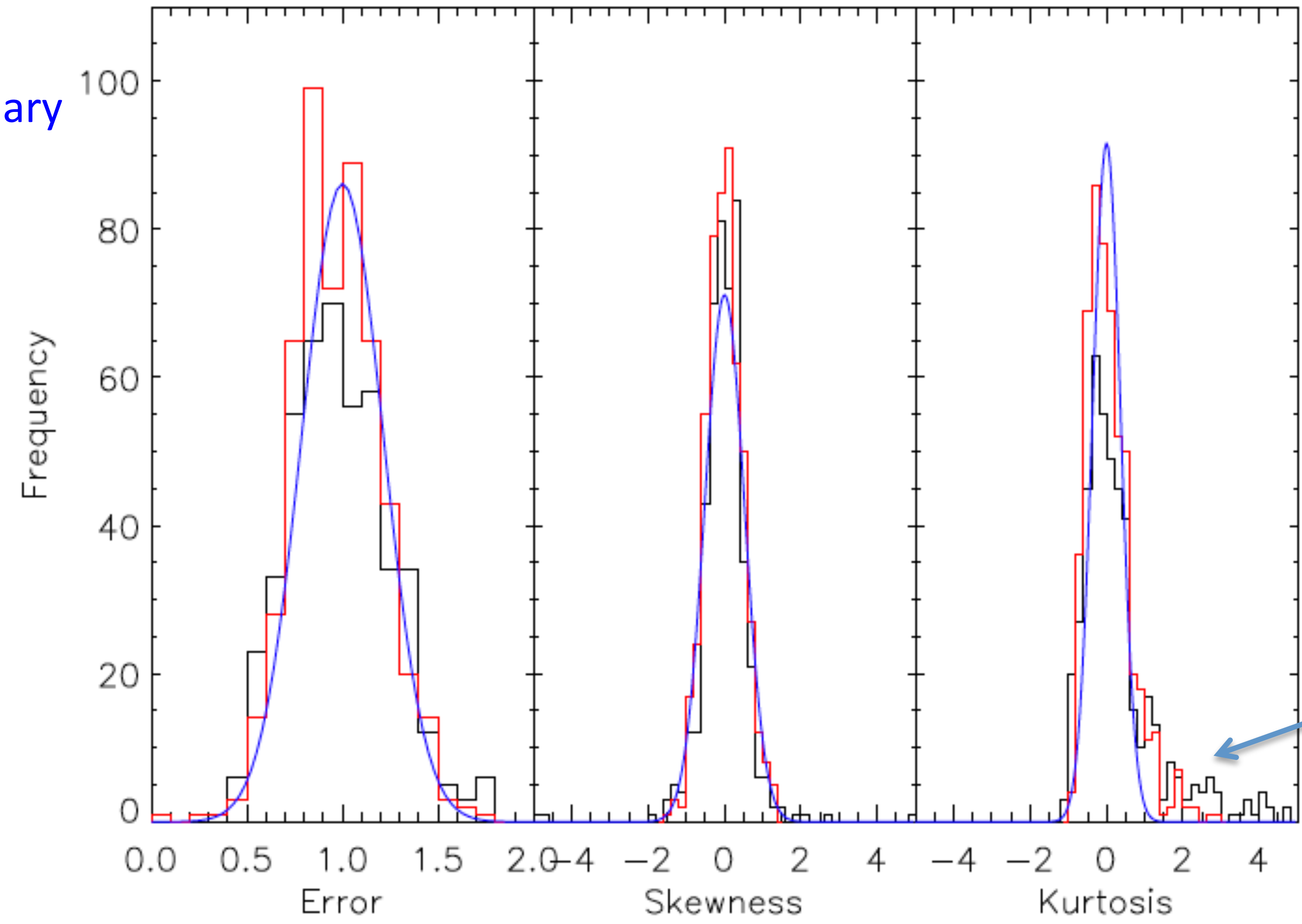
$$\gamma_\ell^m(k) = \left(\frac{2}{\pi}\right)^{1/2} \sum_g \gamma_g(r_g,\theta_g)\, j_\ell(kr_g)\, {}_2Y_\ell^m(\theta_g) \qquad p(e_\ell[k]) = \bigotimes_g [p_g(e) * p(\gamma)]\, j_\ell(kr)\, e^{-i\ell.\theta}$$

Series Convolution

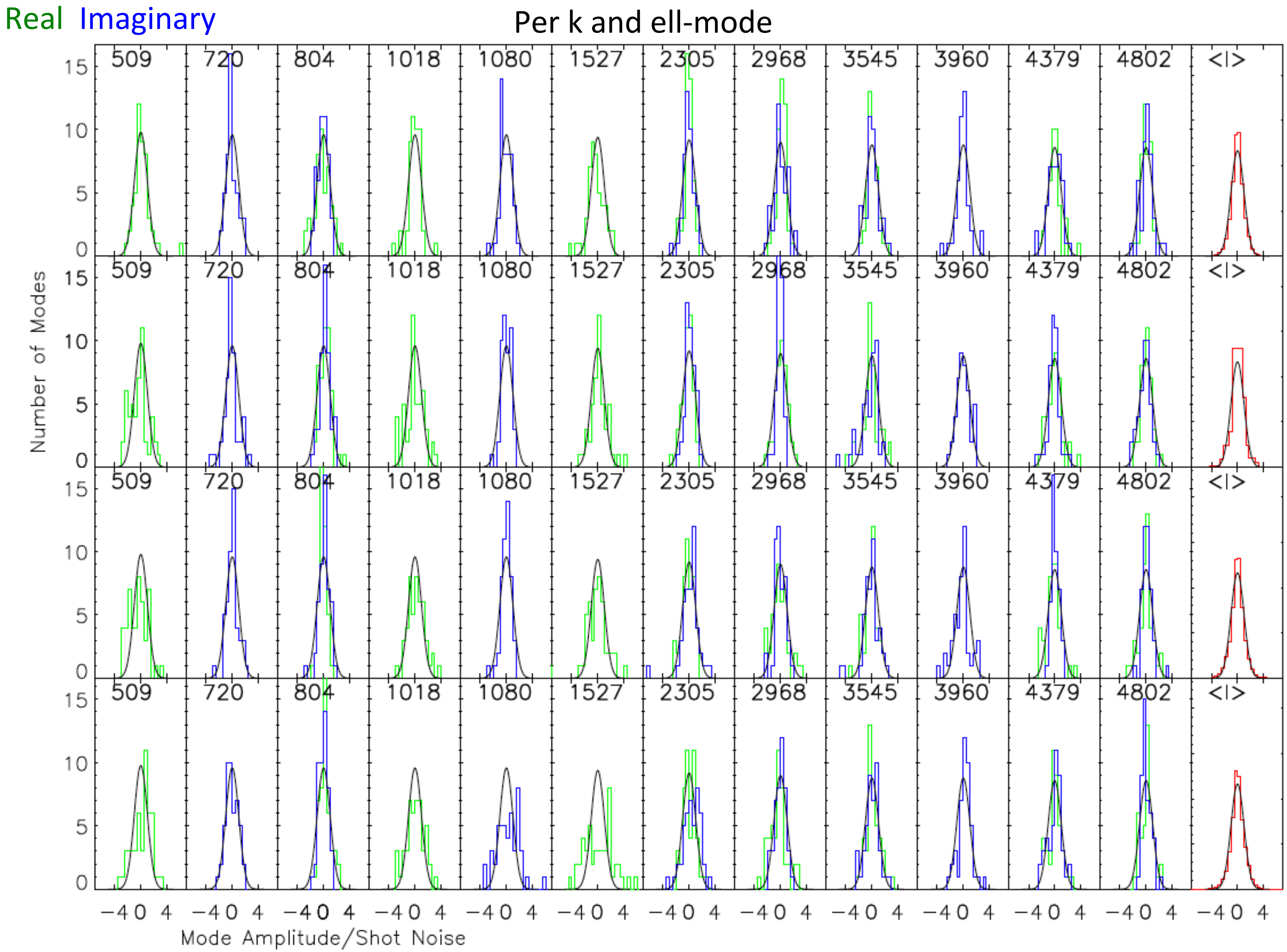
From Current Data

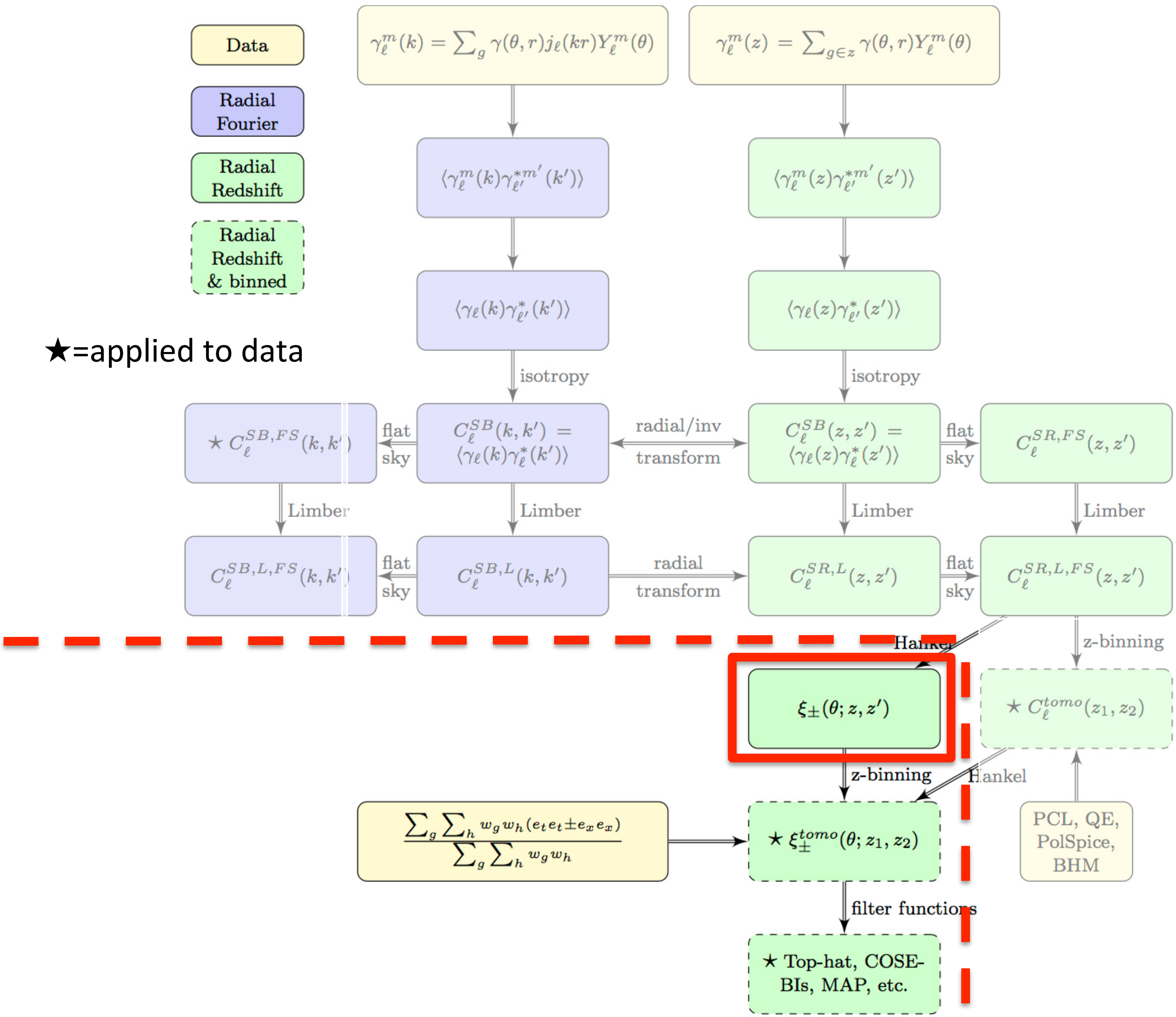
Real

Imaginary



Some Kurtosis
But not statistically significant


Real Imaginary
Per k and ell-mode
509
720
804
1018
1080
1527
2305
2968
3545
3960
4379
4802
<l>
Number of Modes
Mode Amplitude/Shot Noise
0
5
10
15
−4 0 4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PCL, QE, PolSpice, BHM
filter functions
★ Top-hat, COSEBIs, MAP, etc.

Mask doesn't mix
Modes Fourier modes

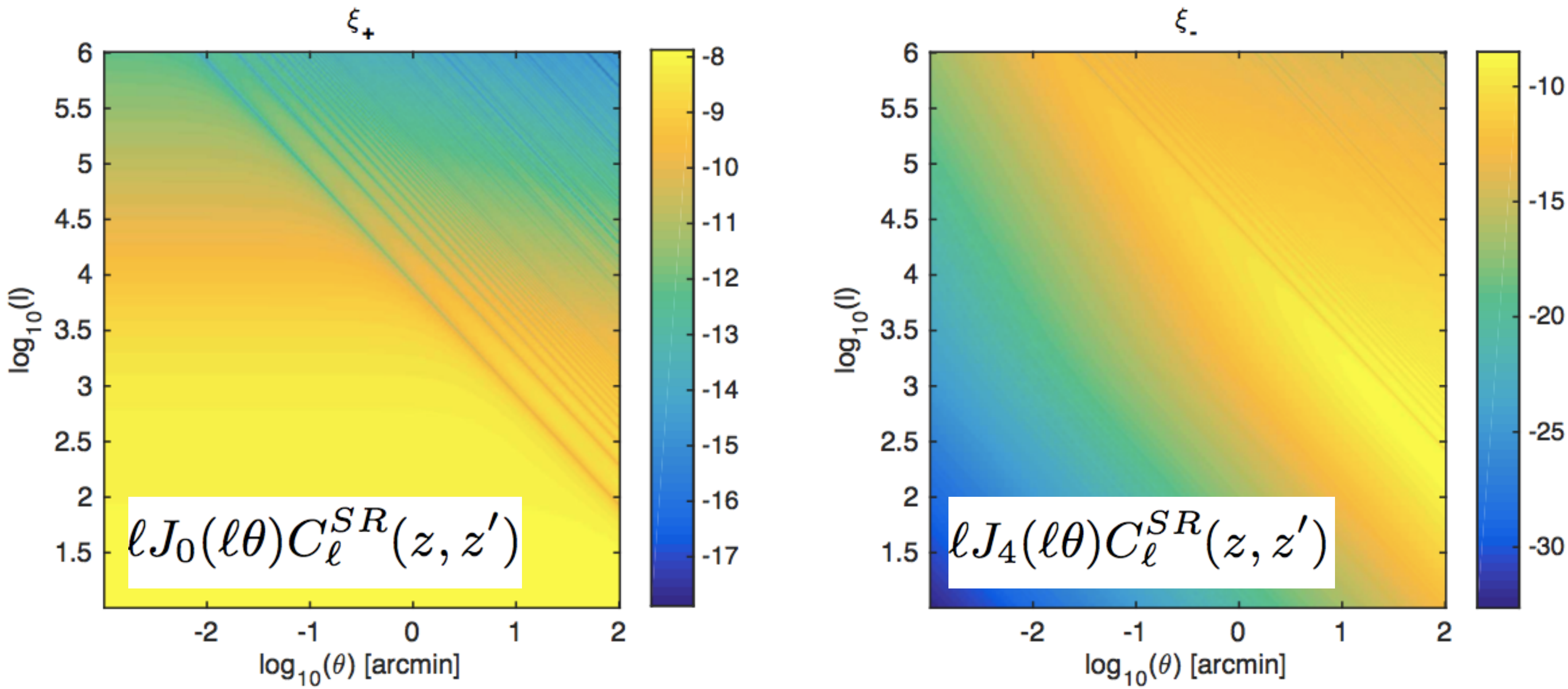
Can do a direct transform on the masked data

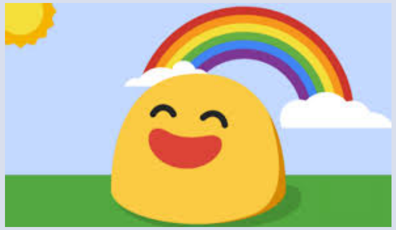
$$\frac{\sum_g \sum_h w_g w_h (e_t e_t \pm e_x e_x)}{\sum_g \sum_h w_g w_h}$$

Statistic mixes Fourier Modes via Hankel transform

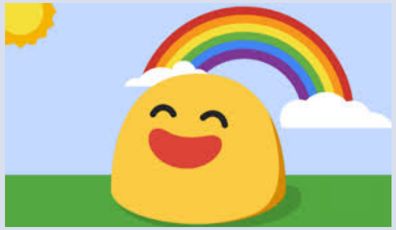
$$\xi_+(\theta, z, z') = \frac{1}{2\pi}\int d\ell \ell J_0(\ell\theta) C_\ell^{SR}(z, z')$$

$$\xi_-(\theta, z, z') = \frac{1}{2\pi}\int d\ell \ell J_4(\ell\theta) C_\ell^{SR}(z, z')$$



Statistics	Angular Scales	Fourier scales	Ease of Estimation from Data*
Configuration-Space Correlation functions	✔		
Fourier-Space Power Spectra		✔	

*Subjective estimates

Statistics	Angular Scales	Fourier scales	Ease of Estimation from Data*
Configuration-Space Correlation functions	✔		
Fourier-Space Power Spectra		✔	

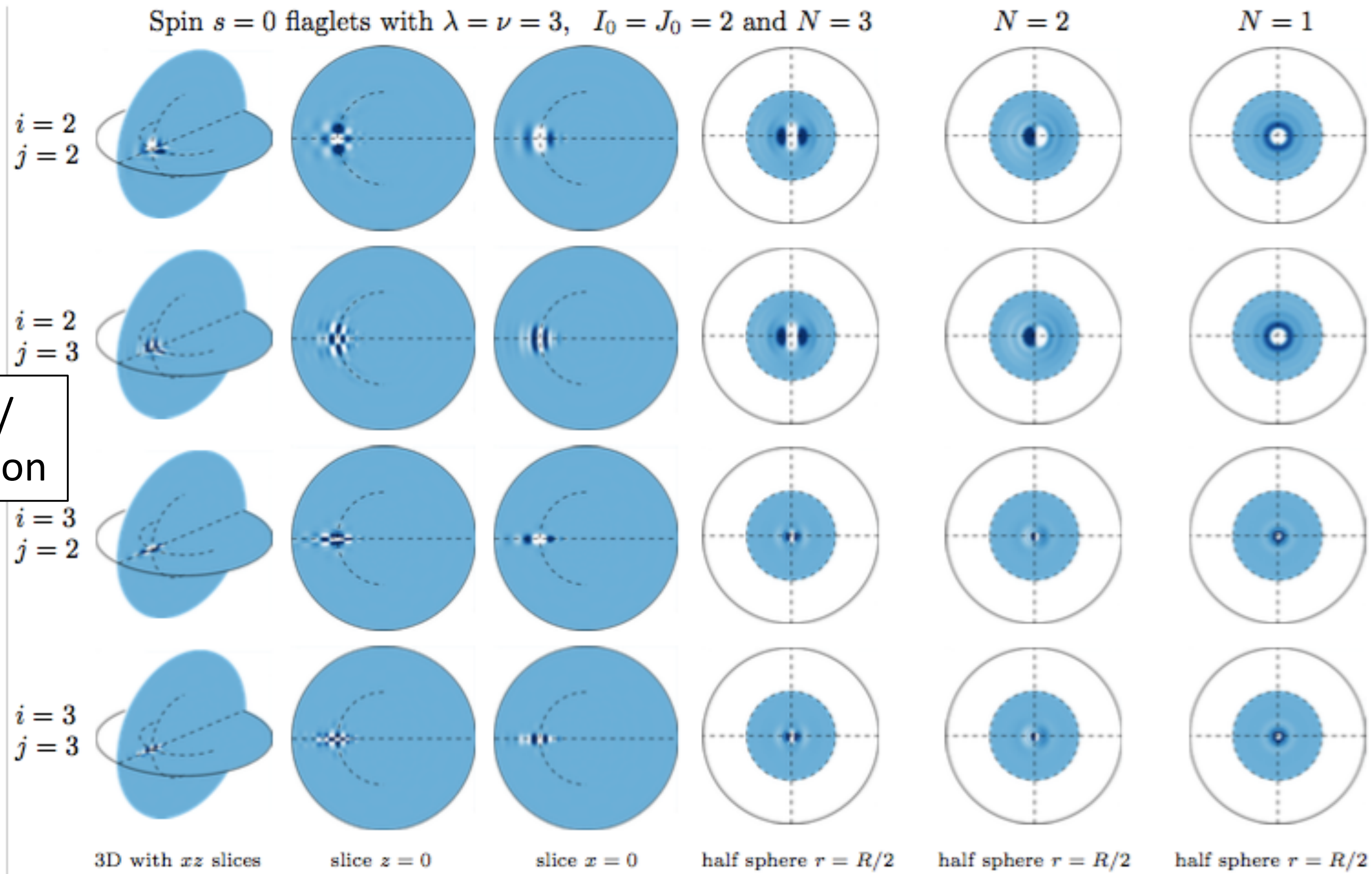
*Subjective estimates

Fourier-Laguerre basis: ${}_sY_{\ell m}(\theta,\phi)\ K_p(r)$

Separable basis on $\mathbb{S}^2 \times \mathbb{R}^+$ with measure $\mathrm{d}^3\vec{r} = r^2 \sin\theta \mathrm{d}\theta\mathrm{d}\phi\mathrm{d}r$

Spin-Flaglets

Stretching/
Compression



Leistedt, McEwen, Kitching & Peiris (PRD, 2015)

Flaglet transform

3D translations

Stretching

$$ {}_sW^{{}_s\Psi^{ij}}(\theta,\phi,\gamma,r) \equiv {}_sf \star {}_s\Psi^{ij} = \langle {}_sf | \mathcal{R}_{(\theta,\phi,\gamma)}\, {}_s\Psi^{ij} \rangle $$

Construct flaglets ${}_s\Psi^{ij}$ to:

- probe well defined radial & angular scales
- achieve exact reconstruction & multi resolution

Leistedt & McEwen (IEEE, 2012)

Leistedt, McEwen, Kitching & Peiris (PRD, 2015)

3D Cosmic shear with flaglets

Flaglet transform: $${}_sW^{\Psi^{ij}}(\theta,\phi,r) = ({}_2\gamma \star {}_2\Psi^{ij})(\theta,\phi,r)$$

Covariance of flaglet coefficients:

Data

$$\langle\, {}_2W^{\psi^{ij}}(\theta,\phi,r)\; {}_2W^{\psi^{i'j'}*}(\theta',\phi',r')\,\rangle \;=\; \frac{2}{\pi}\sum_{\ell}\frac{(N_{\ell,2})^2}{4}$$

$$\times \int \mathrm{d}k\; k^2 \int \mathrm{d}k'\; k'^2\, C_\ell^{\phi\phi}\, P_\ell(\Delta\theta)\; {}_2\mathcal{H}_\ell^{ij}(k,r)\; {}_2\mathcal{H}_\ell^{i'j'*}(k',r')$$

Theory

$$=\; C^{ij,i'j'}(\cos\Delta\theta, r, r')$$

Leistedt, McEwen, Kitching & Peiris (PRD, 2015)

Optimal Expansion? Malyarenko can help?

$$\gamma_\ell^m(k) = \left(\frac{2}{\pi}\right)^{1/2} \sum_g \gamma_g(r_g, \theta_g) j_\ell(kr_g)\, {}_2Y_\ell^m(\theta_g)$$

?

$${}_sX(r, \mathbf{n}) = \sum_{\ell=s}^{\infty} \sum_{m=-\ell}^{\ell} \sum_{j=1}^{\infty} {}_sc_{\ell j}\; {}_sf_{\ell j}(r)\; {}_sX_{\ell m j}\; {}_sY_{\ell m}(\mathbf{n}).$$

- What does "optimal" mean?
  - Highest signal-to-noise
  - Best/optimal/minimal sampling
  - Smallest parameter error for particular cosmology
  - Fastest in terms of computation

- ***How*** to optimise?
  - How to determine optimal basis expansion

Conclusions

- Can generate $S^2 x R$ “3D” [sic] “Ball” [sic] spin-2 power spectra in a number of ways
  – Wavenumber in Angle and Radius: Spherical Bessel
  – Wavenumber in Angle Real-Space in Radius: Spherical Radial “Tomography”
  – Real-Space in Angle and Radius: Correlation Functions

- Several Approximations
  – Limber, Hankel : mostly benign
  – Equal-time ansatz : maybe benign, not clear at the current time (needs much more work)

- Open Questions/Things to do
  – What is the ***optimal*** combination of power spectra/correlation function for any given science objective
  – What is the ***optimal*** way to combine with galaxy clustering measurements
  – What is the ***optimal*** way to combine with CMB measurements
  – What is the ***optimal*** expansion (see Malyarenko)
  – Fast 3D pixelisation scheme/code aka “3D Healpix” currently missing

- More: the shear field is not a Gaussian random field or spin-2
  – Higher order statistics
  – Minkowski functionals
  – Full weak lensing case also has spin-3 contribution (very small, but nonetheless there)

References

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