Imperial College London

# Random Fields in Cosmological hierarchical models (& inflation)

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With code/ideas from *J Alsing, S Balan*

Isotropic Random Fields in Astrophysics
Cardiff, June 2017

*Alsing, Heavens, Jaffe, Kiessling, Wandelt, Hoffmann 2016*

*Alsing, Heavens & Jaffe 2017*

ICIC
Imperial Centre
for Inference & Cosmology

# Cosmological Random Fields

- Bayesian Hierarchical models for cosmological maps, spectra, and parameters
  - random fields
  - sampling techniques
  - applications to CMB and weak lensing

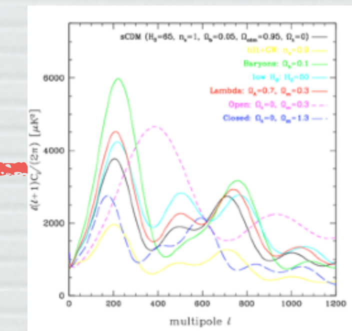
- Modelling inflationary potential as a random field

# Random fields in Cosmological Bayesian Hierarchical Models

cosmological parameters

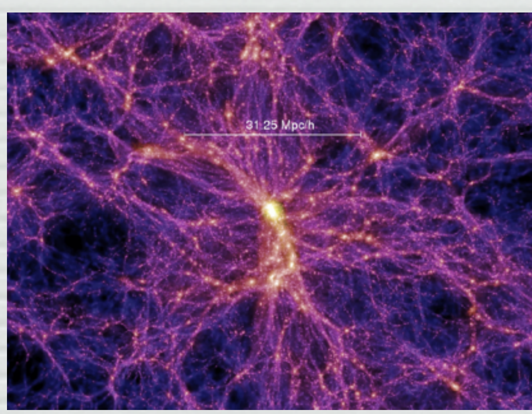
Power Spectra

Initial fields: $\delta_i(\boldsymbol{r})$, $\Phi(\boldsymbol{r})$

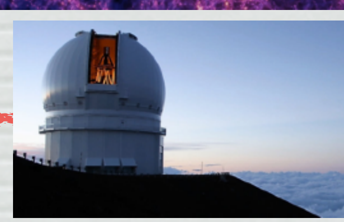


Deterministic evolution

effective parameters (since we don't know the full theory)

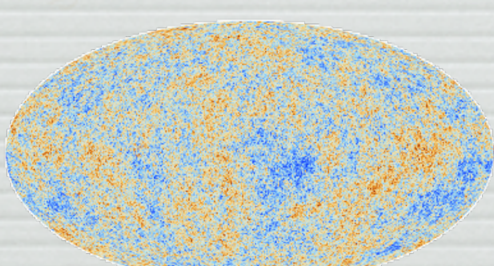


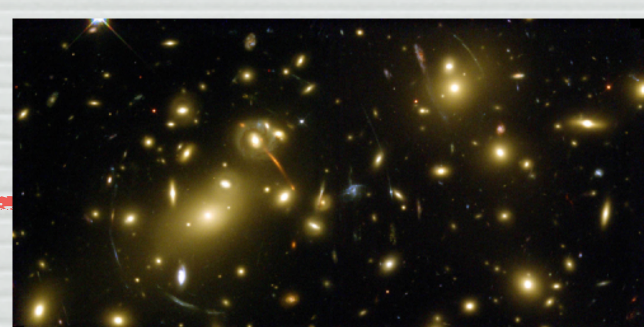
Astrophysical fields: slices/volumes/projections



Experimental apparatus

experimental parameters (some unknown)





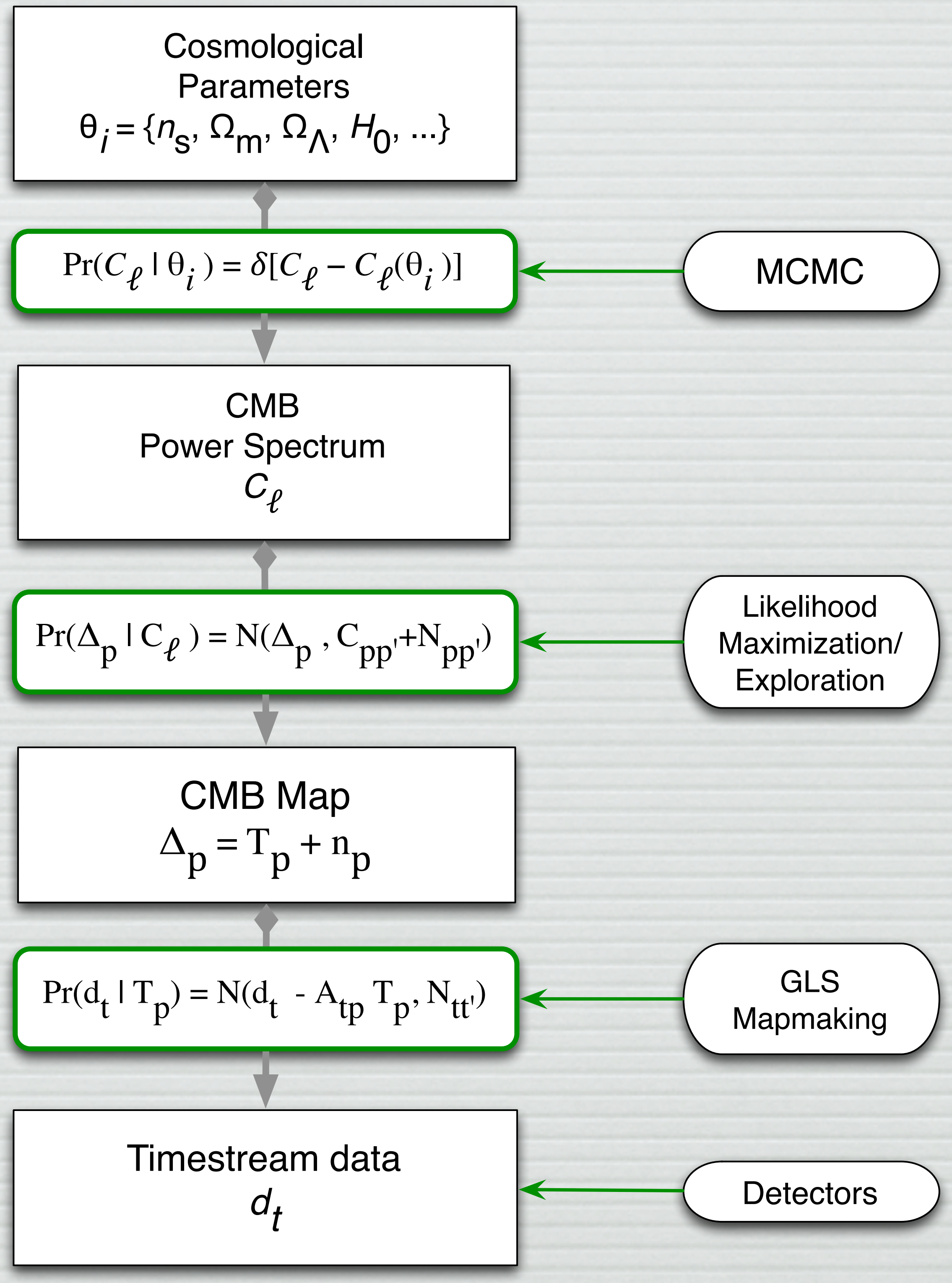
Data fields: maps, catalogs, timestreams

# Where are the cosmological random fields?

- Initial (post-inflation?) fluctuations may be only true "random field".
  - ~known to be approximately isotropic, Gaussian
- Unknowns in [actually deterministic] evolution & measurement *modelled* as further random fields or paramaterised processes
  - also may be some further "true" quantum randomness
  - e.g.
    - details of galaxy formation
    - properties of experimental noise

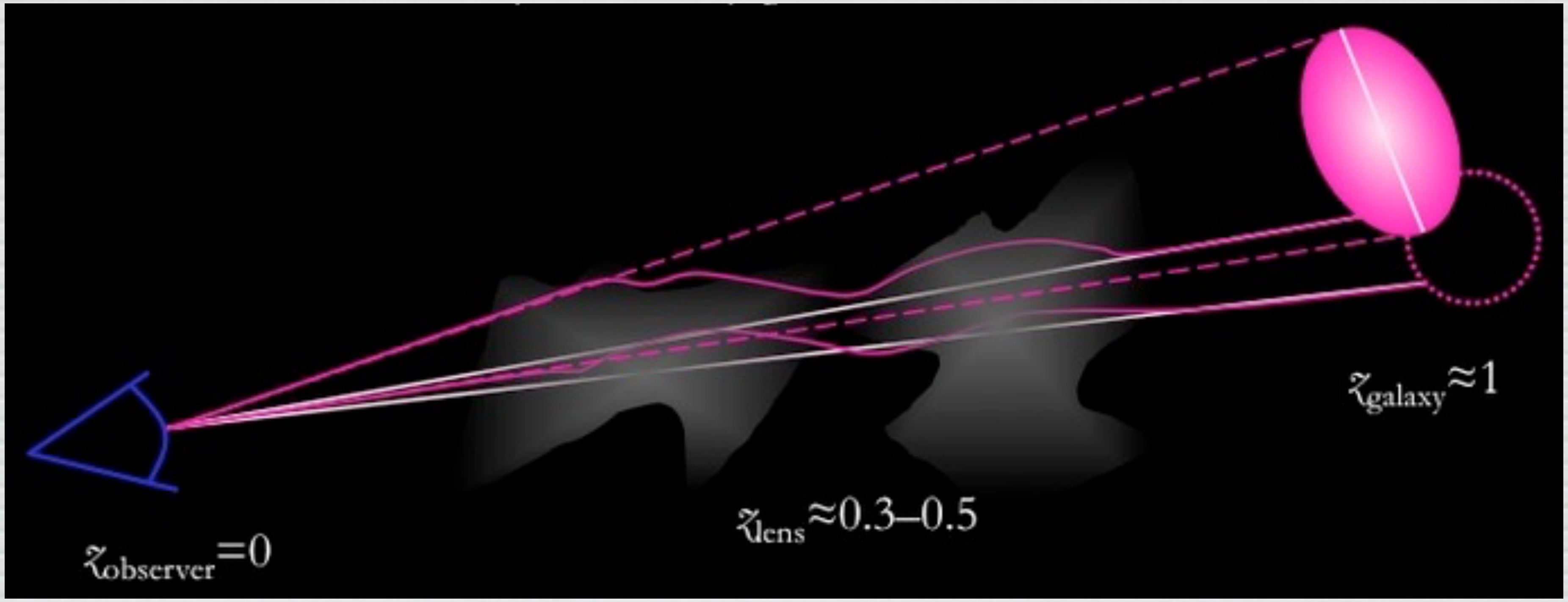
# Quick case study: Cosmostatistics of the CMB

- CMB as a hierarchical model
  - can be computed exactly using Gibbs methods, estimated w/ approximations for $P(\hat{C}_\ell | C_\ell)$
- Map and power spectrum are just (approximately) sufficient statistics
- Radical compression (~***sparsity***):
  - $\mathbf{10^{12}}$ samples → $\mathbf{10^{7}}$ pixels → $\mathbf{10^{3}}$ $C_\ell$ → **6** parameters
- This version assumes
  - isotropic Gaussian signal (no topology)
  - known & Gaussian noise properties
  - known (isotropic) beam shape
  - no foregrounds
  - no systematics
- Even so: compute-bound $O(N_{pix}^3)$:
  - covariance matrix in mapmaking
  - likelihood evaluation in $C_\ell$ step



# Weak Gravitational Lensing

- Intervening matter bends the path of light
  - results in distorted images
  - measures the line-of-sight density, suitably integrated
    - kernel depends on the distance to the source galaxy behind and the cosmology




Courtesy Euclid/Jason Rhode, JPL

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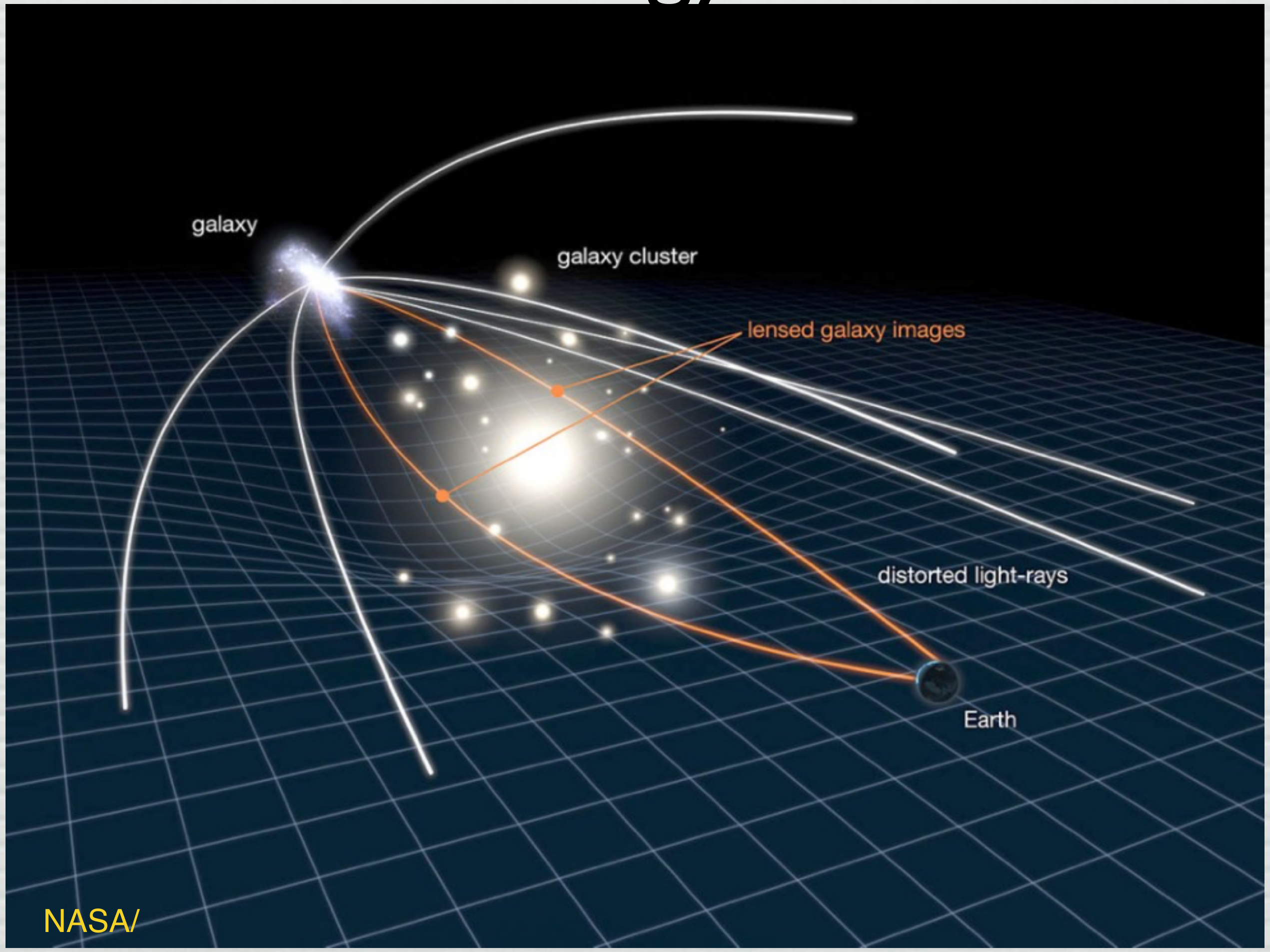
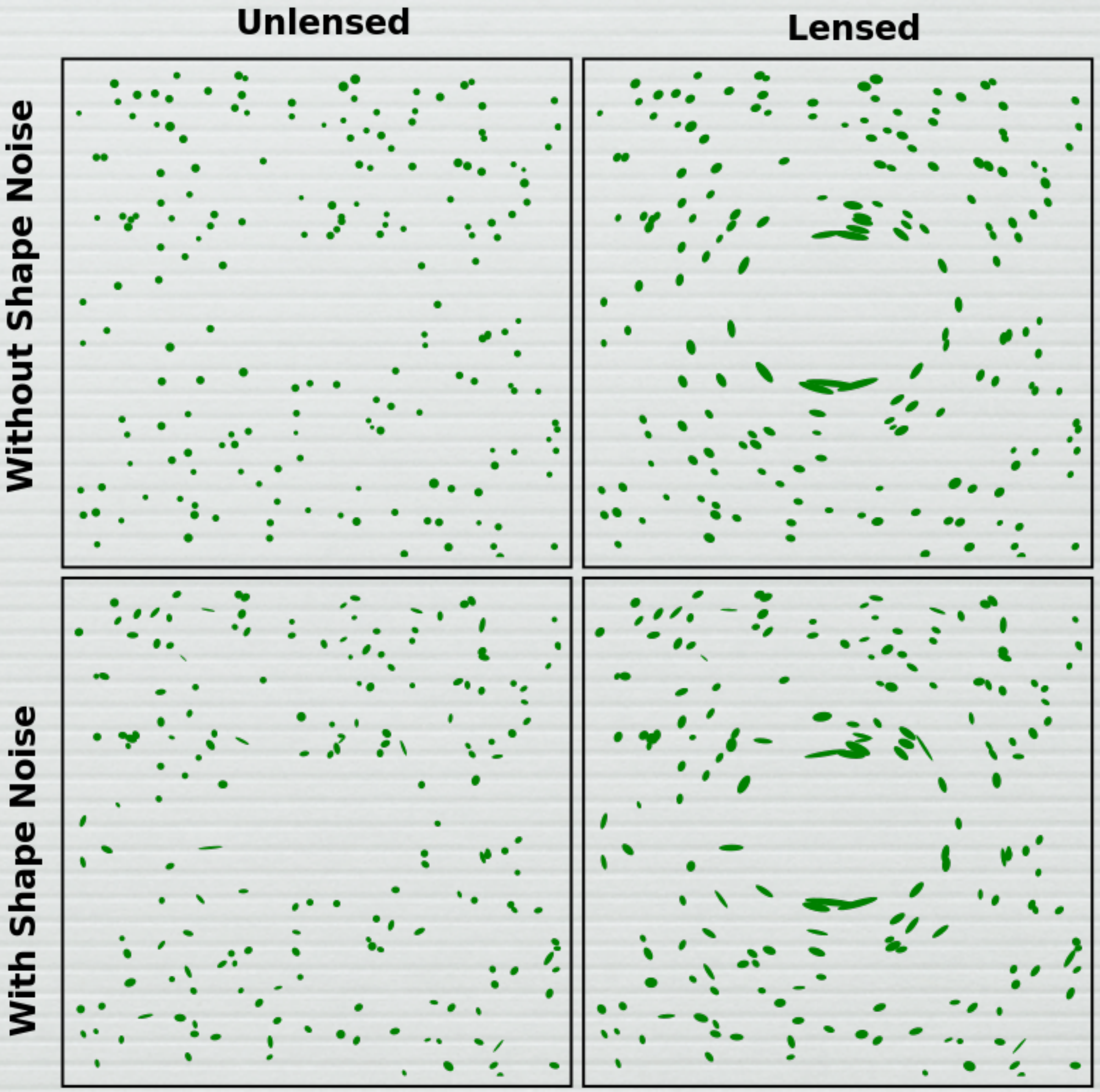


# Weak lensing of galaxies

- First analyzed/observed in clusters

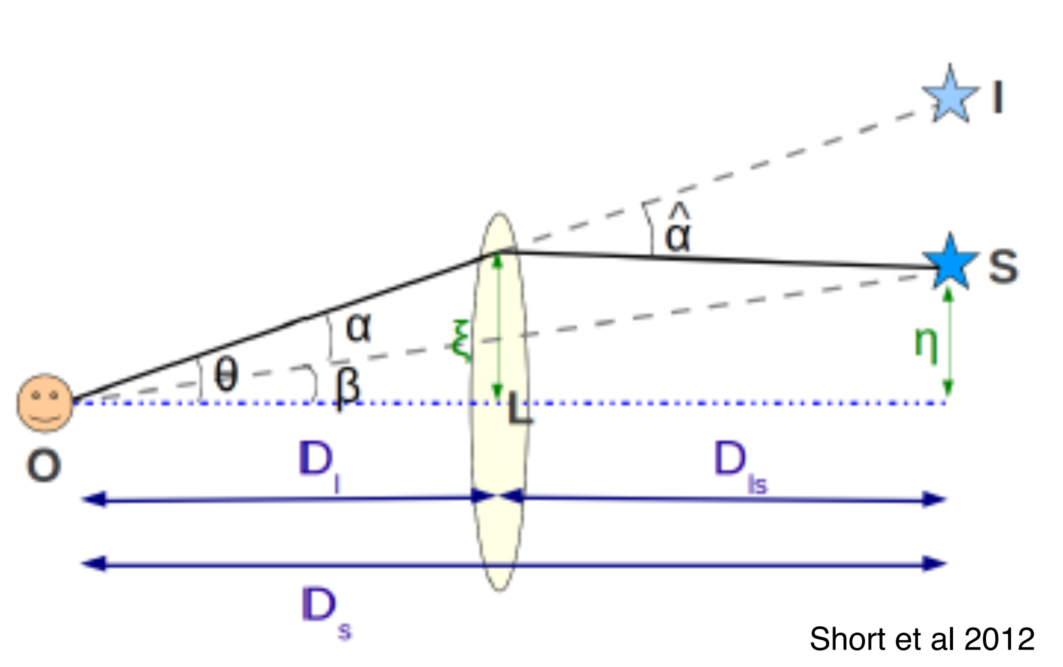
*Image: Composite Credit: X-ray: NASA/CXC/CfA/ M.Markevitch et al.; Lensing Map: NASA/STScI; ESO WFI; Magellan/U.Arizona/ D.Clowe et al. Optical: NASA/STScI; Magellan/U.Arizona/D.Clowe et al.*




Wikipedia

# Lensing formalism

- Map the unperturbed image to the lensed image:
- source position = image position − deflection
- $\boldsymbol{\beta} = \boldsymbol{\theta} - \boldsymbol{\alpha}$ (2D vectors on the sky)
- Born approx + small deflections:
  - linear mapping




$$A_{ij} = \frac{\partial \beta_i}{\partial \theta_j} = \delta_{ij} - \frac{\partial \alpha_i}{\partial \theta_j} \quad = \delta_{ij} - \partial_i \partial_j \psi,$$

lensing potential → metric perturbation ←

$$\psi(\boldsymbol{\theta}, \chi) = \frac{2}{c^2} \int_0^\chi \mathrm{d}\chi' \frac{f_K(\chi - \chi')}{f_K(\chi) f_K(\chi')} \, \Phi(f_K(\chi')\boldsymbol{\theta}, \chi').$$

$$\mathbf{A} = \begin{pmatrix} 1 - \kappa - \gamma_1 & -\gamma_2 \\ -\gamma_2 & 1 - \kappa + \gamma_1 \end{pmatrix}$$

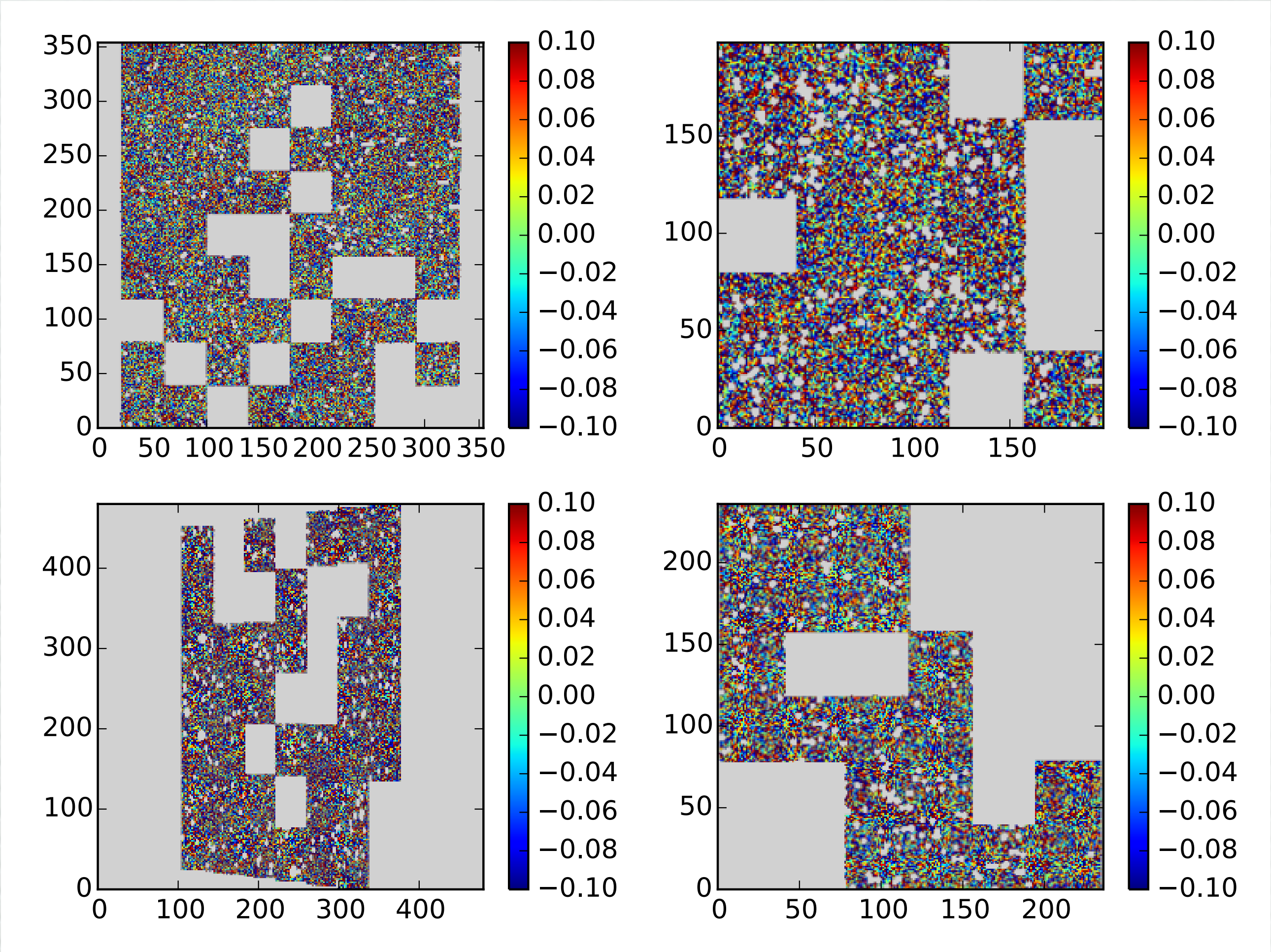
$$\kappa = \frac{1}{2}\left(\partial_1 \partial_1 + \partial_2 \partial_2\right) \psi = \frac{1}{2} \nabla^2 \psi;$$ convergence

$$\gamma_1 = \frac{1}{2}\left(\partial_1 \partial_1 - \partial_2 \partial_2\right) \psi; \quad \gamma_2 = \partial_1 \partial_2 \psi.$$ shear

- main point: these are all linear in the potential (by construction, but to an excellent approximation)

# CFHTLens

- 154 deg$^2$ *ugriz* multi-colour optical survey
- five years of data from the Wide, Deep and Pre-survey components of full CFHT Legacy Survey
- Optimised for weak lensing with deep *i*-band data taken in sub-arcsec seeing
- For general overview, see Erben et al 2012, Heymans et al 2012



# Worked example: Shear power spectra

- Shear: spin-2 (tensor), linearly related to density (potential)
- 2-point correlators encode cosmological information
  - motivates "quadratic estimators"
  - find quadratic combinations of data which give unbiased (and low variance) estimates of the underlying power spectra.
  - details sensitive to survey geometry (masks), noise, &c.
  - not quite "optimal" (Bayesian)
    - even when used in a likelihood (e.g., `CosmoMC`)
  - simple versions cheap & cheerful first steps

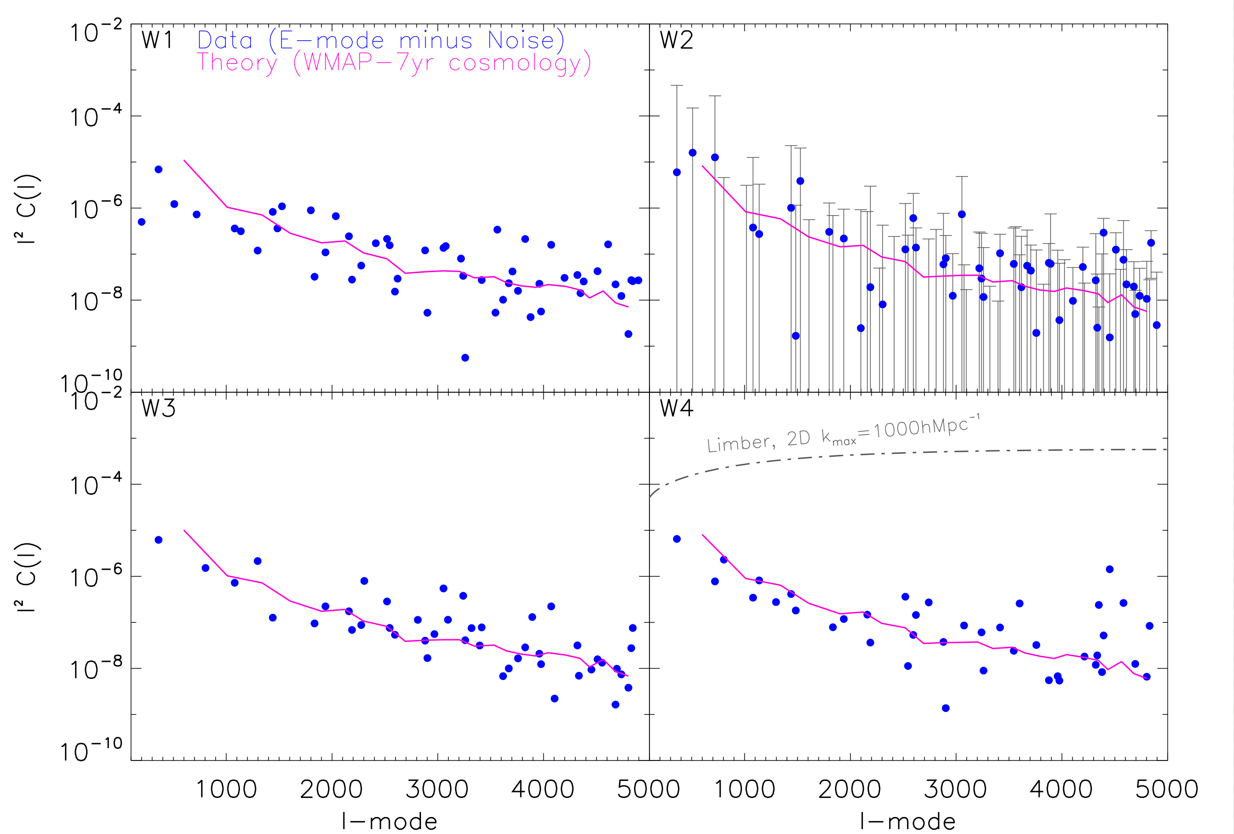
$$\gamma^{E(\alpha)}_{\ell m} = \frac{1}{2}\int \left[\gamma^{(\alpha)}(\phi)_2 Y^*_{\ell m}(\phi) + \gamma^{*(\alpha)}(\phi)_{-2}Y^*_{\ell m}(\phi)\right] d\Omega,$$

$$\gamma^{B(\alpha)}_{\ell m} = -\frac{i}{2}\int \left[\gamma^{(\alpha)}(\phi)_2 Y^*_{\ell m}(\phi) - \gamma^{*(\alpha)}(\phi)_{-2}Y^*_{\ell m}(\phi)\right] d\Omega,$$

$$\langle \gamma^{E(\alpha)*}_{\ell m}\gamma^{E(\beta)}_{\ell' m'}\rangle = C^{EE}_{\ell,\alpha\beta}\delta_{mm'}\delta_{\ell\ell'},$$

$$\langle \gamma^{E(\alpha)*}_{\ell m}\gamma^{B(\beta)}_{\ell' m'}\rangle = C^{EB}_{\ell,\alpha\beta}\delta_{mm'}\delta_{\ell\ell'},$$

$$\langle \gamma^{B(\alpha)*}_{\ell m}\gamma^{B(\beta)}_{\ell' m'}\rangle = C^{BB}_{\ell,\alpha\beta}\delta_{mm'},\delta_{\ell\ell'}, \quad (7)$$




Kitching et al 2015

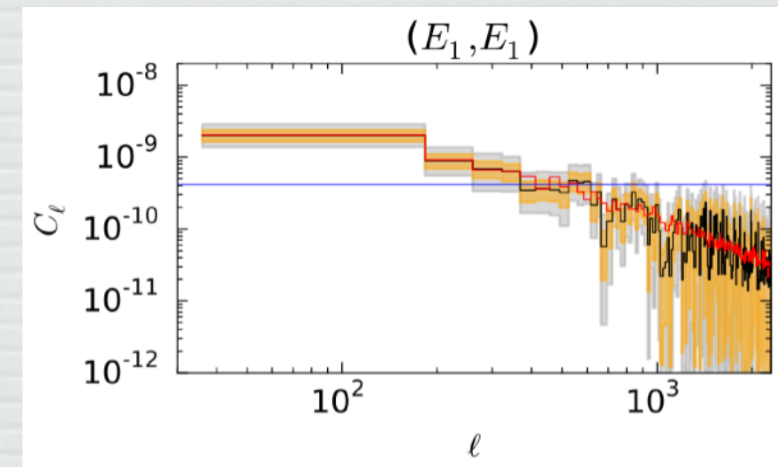
# Joint map/power spectrum inference

- Link between $d$ and $C$ is the true map $s$
- Natural to sample from $C$ and **s** jointly, conditioned on the data $d$: $P(C, s \mid d)$
- Marginalise over the map(s) **s** to get $P(C \mid d)$
- Assume Gaussian fields [large scales]

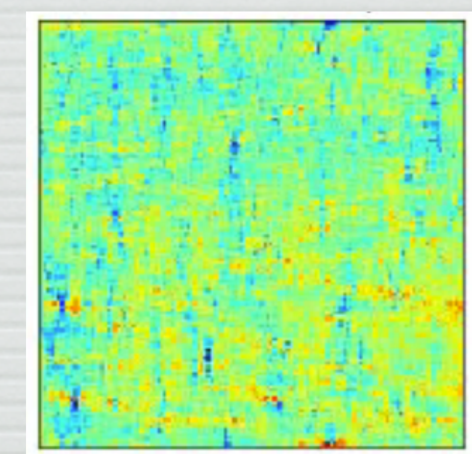
- How to do this inverse problem?
  - Instead, consider the forward model…

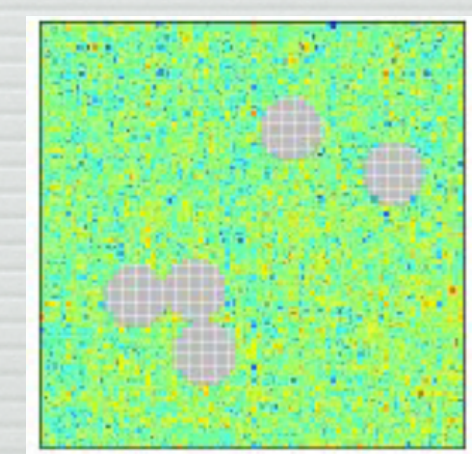
# Hierarchical Models for cosmology (maps & spectra)

- Break the problem into steps
- Parameters
  - $C$ = (various) power spectra
  - $s$ = true shear map
    - (many more parameters)
- Data: pixelised shear values
  - $d = s + n$ (noise)
- We typically want $P(C|d)$
- Conditional distributions, e.g., $P(s|C)$, are often known
  - (so **Gibbs sampling** can be used)



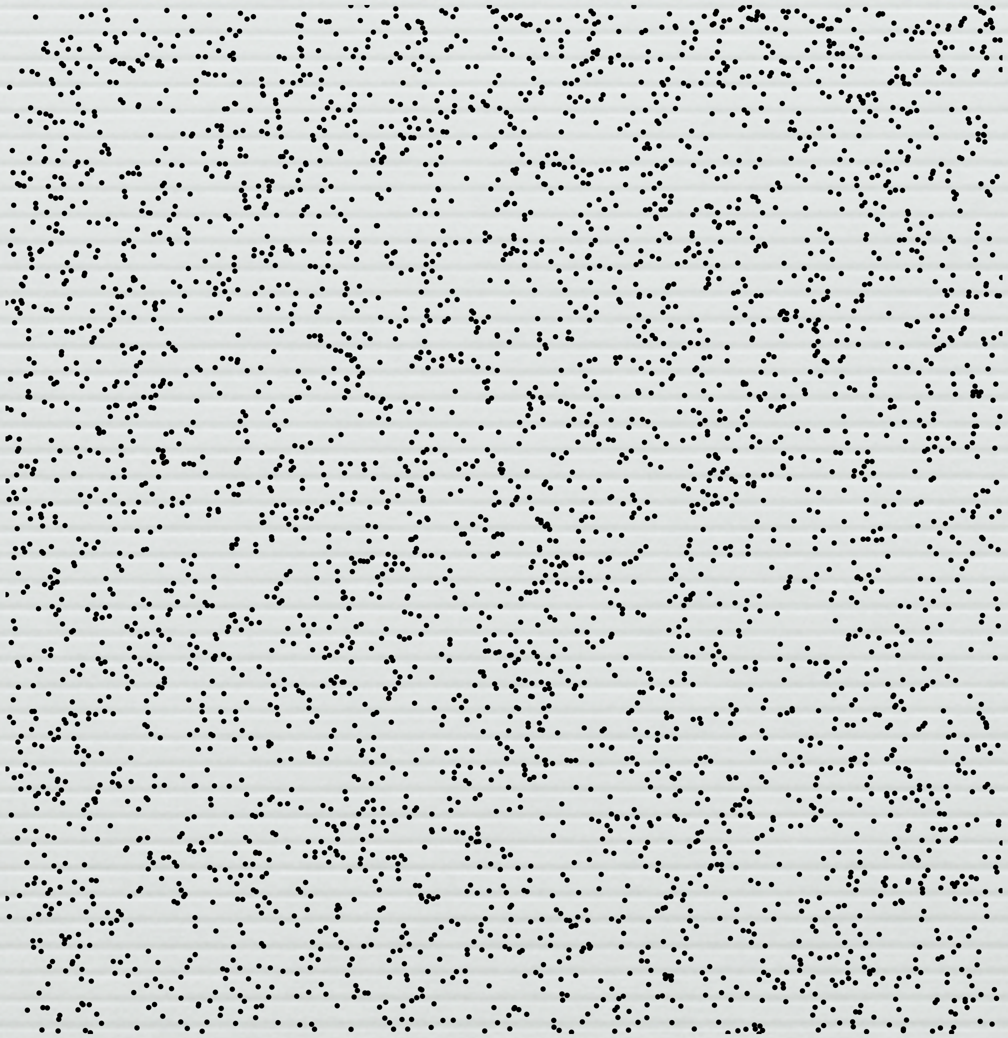

Joint estimate of map ($s$) & spectra ($C$)





# From data to model (and back)

- noisy, redshift-binned, masked data
- ⇒shear spectra
- ⇒cosmology



Credit: J. Alsing

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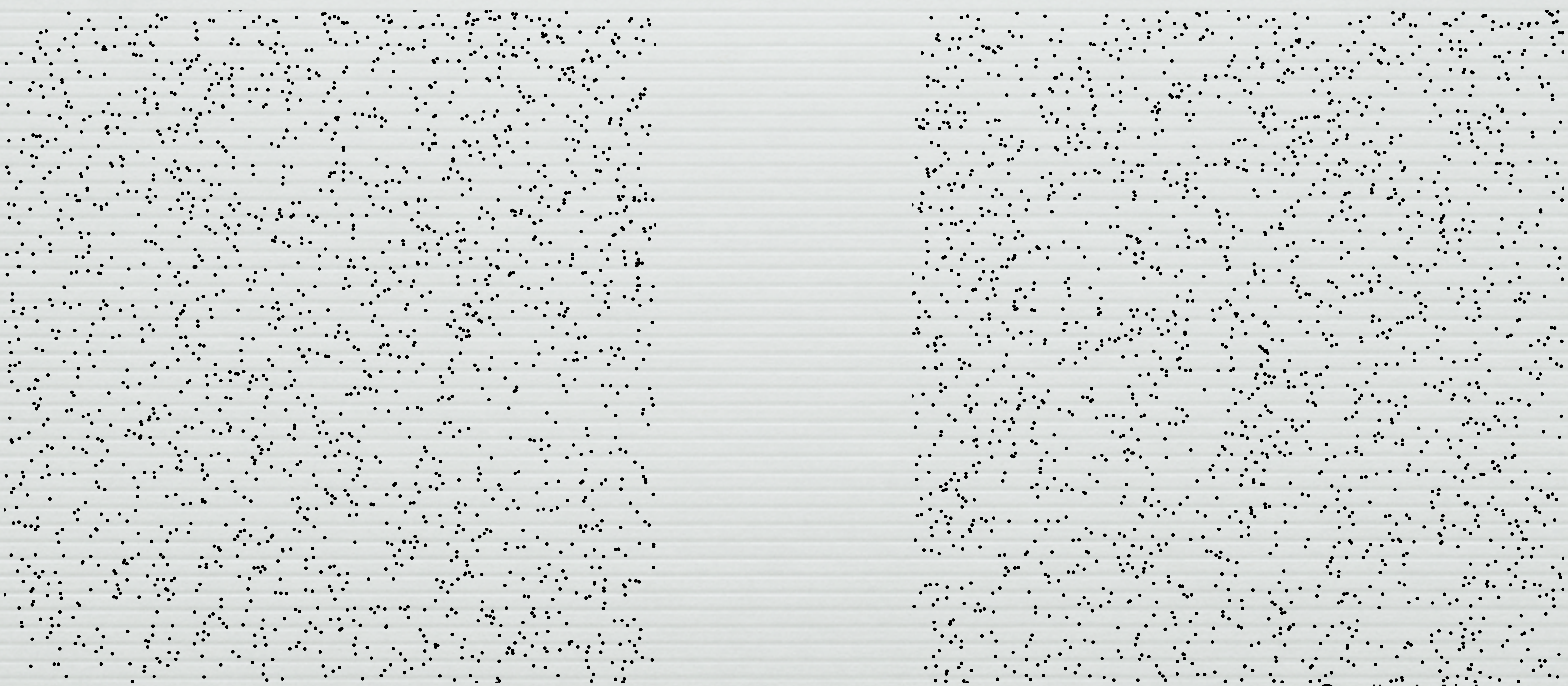

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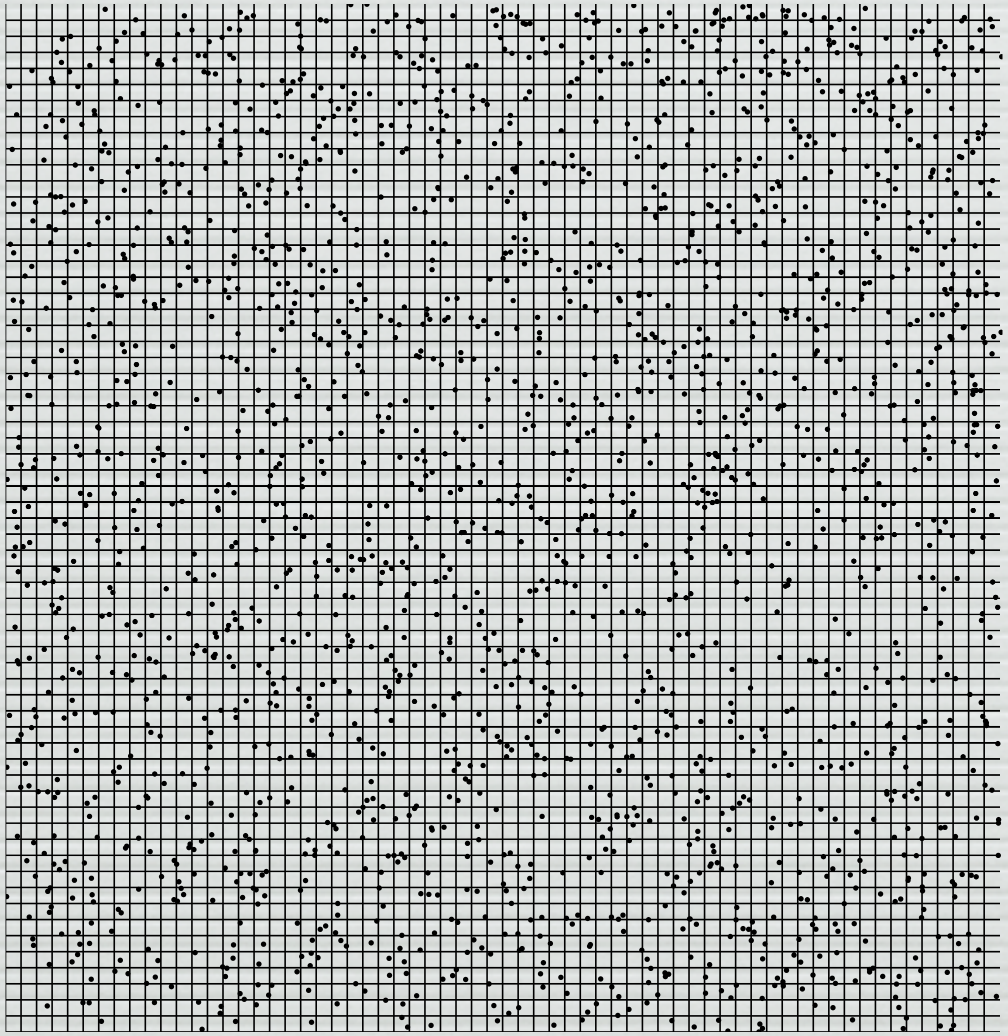

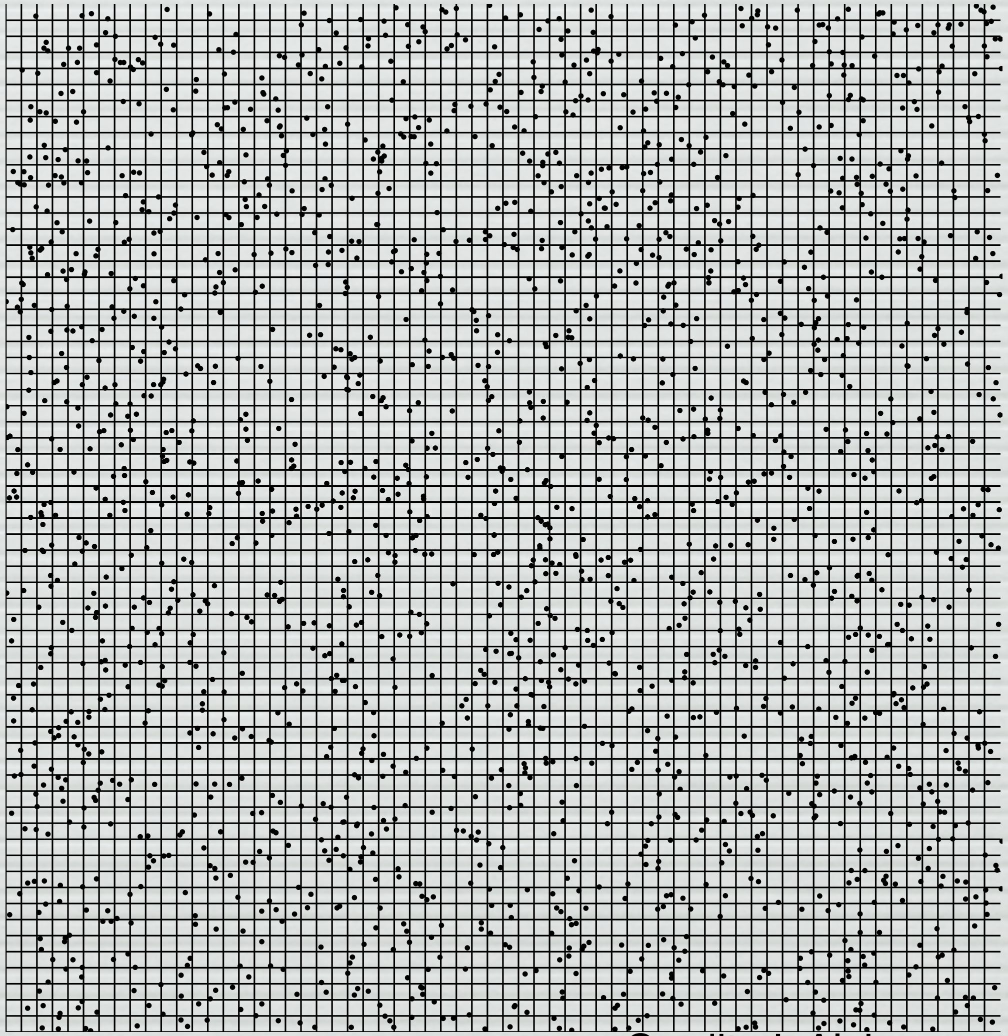

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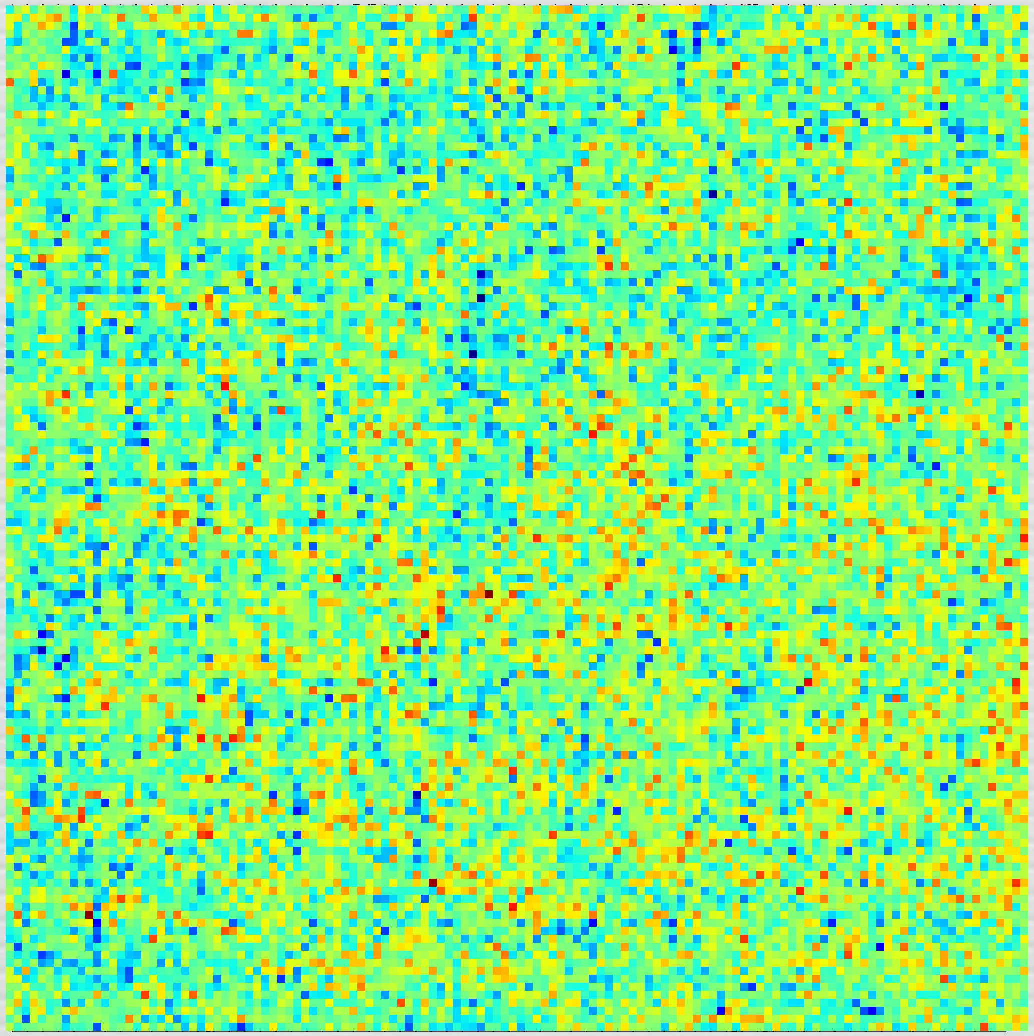

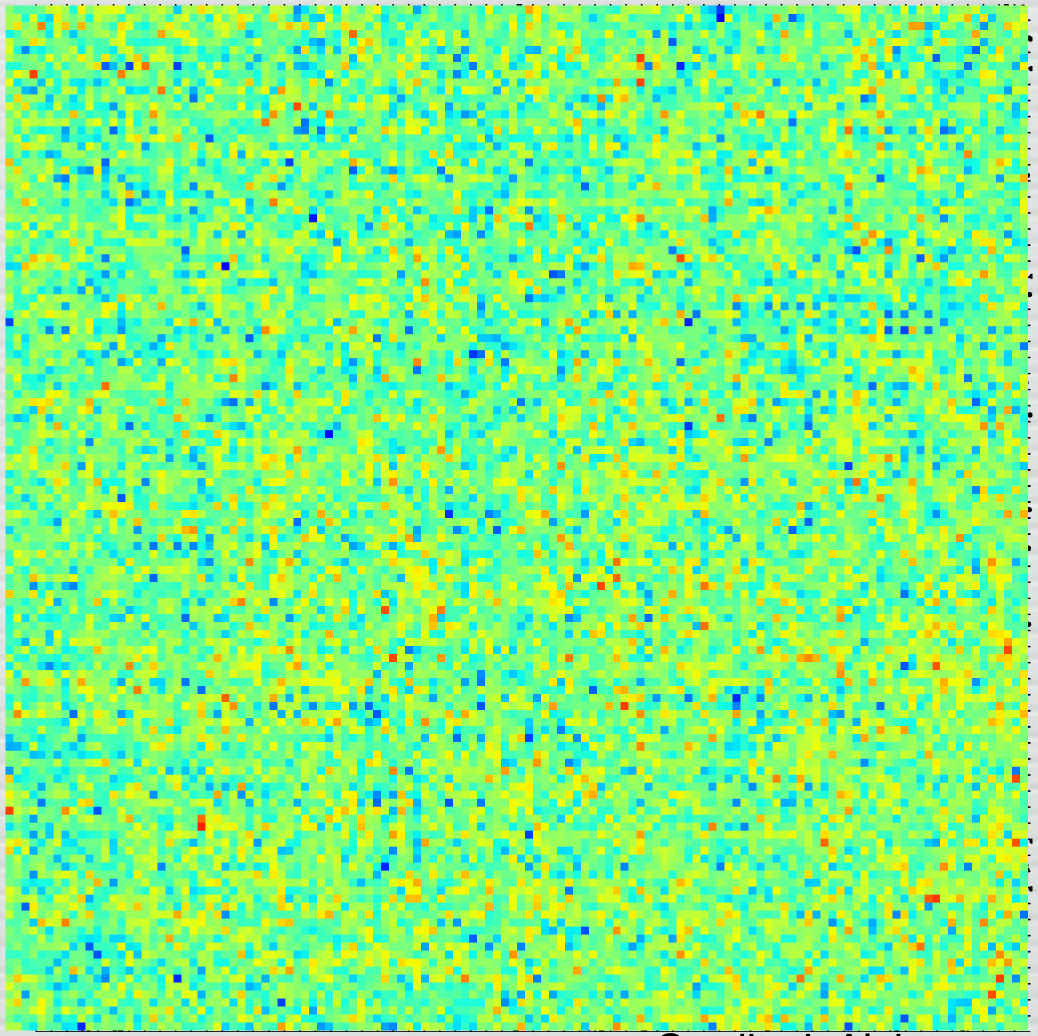

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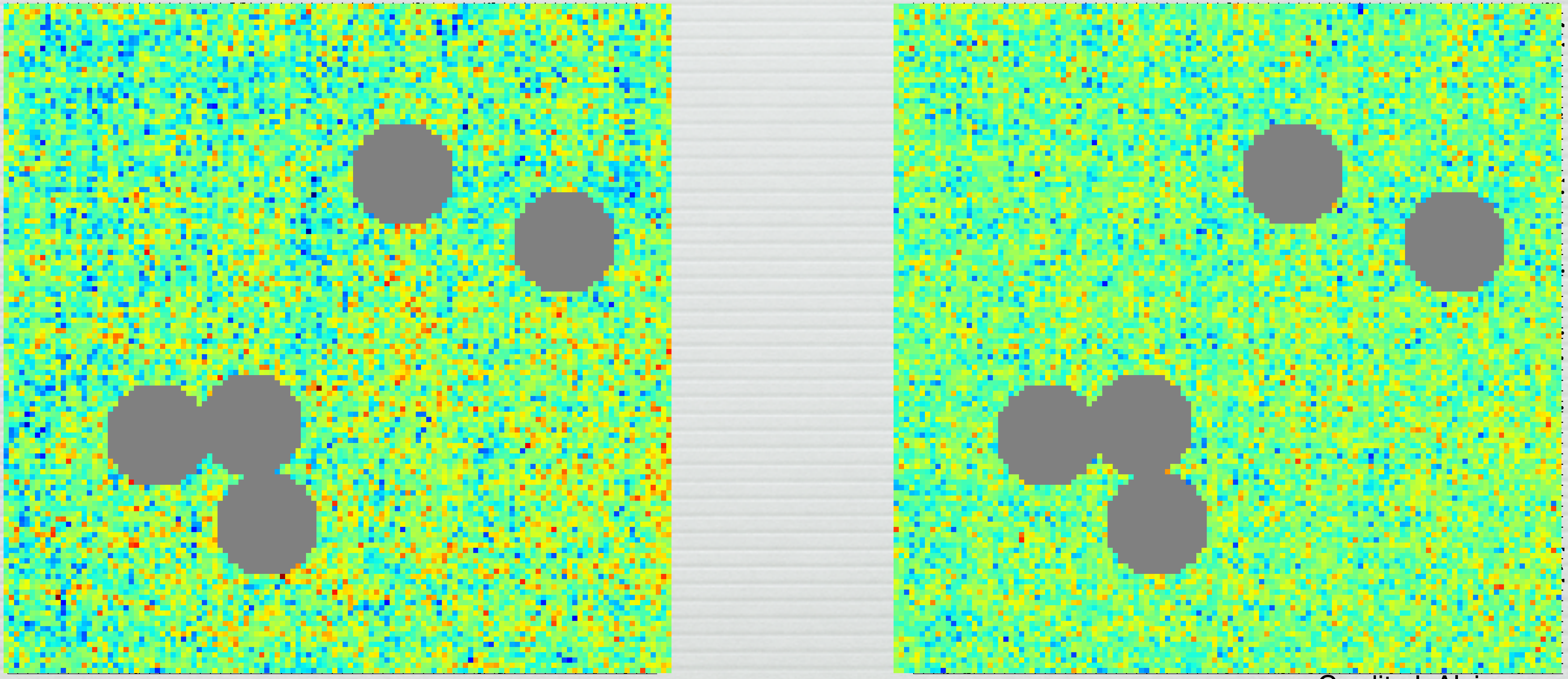

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*Gibbs sampling* naively requires $O(N^3)$

***Wiener filter***:

$$\mathbf{C}^{i+1} \leftarrow P(\mathbf{C}|\mathbf{s}^i) = \mathcal{W}^{-1}(\cdot)$$

$$\mathbf{s}^{i+1} \leftarrow P(\mathbf{s}|\mathbf{C}^{i+1}, \mathbf{N}, \mathbf{d}) = \mathcal{N}\left(\mathbf{d}_{\mathrm{WF}}, \mathbf{C}_{\mathrm{WF}}\right)$$

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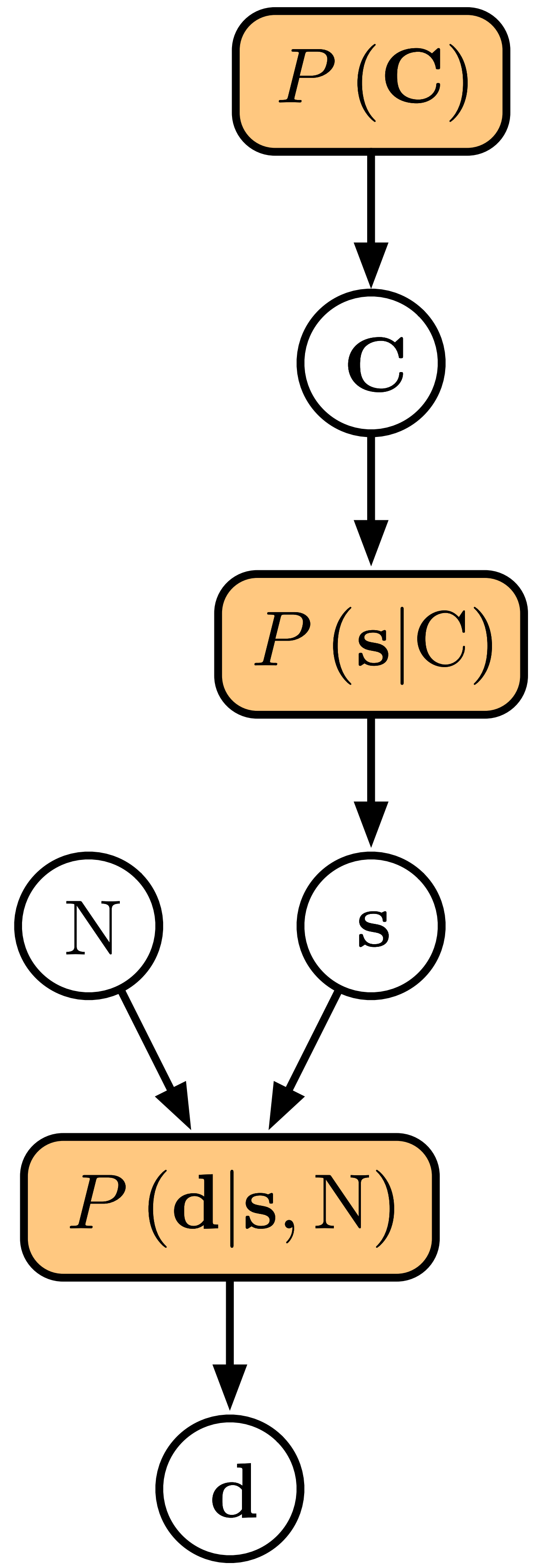
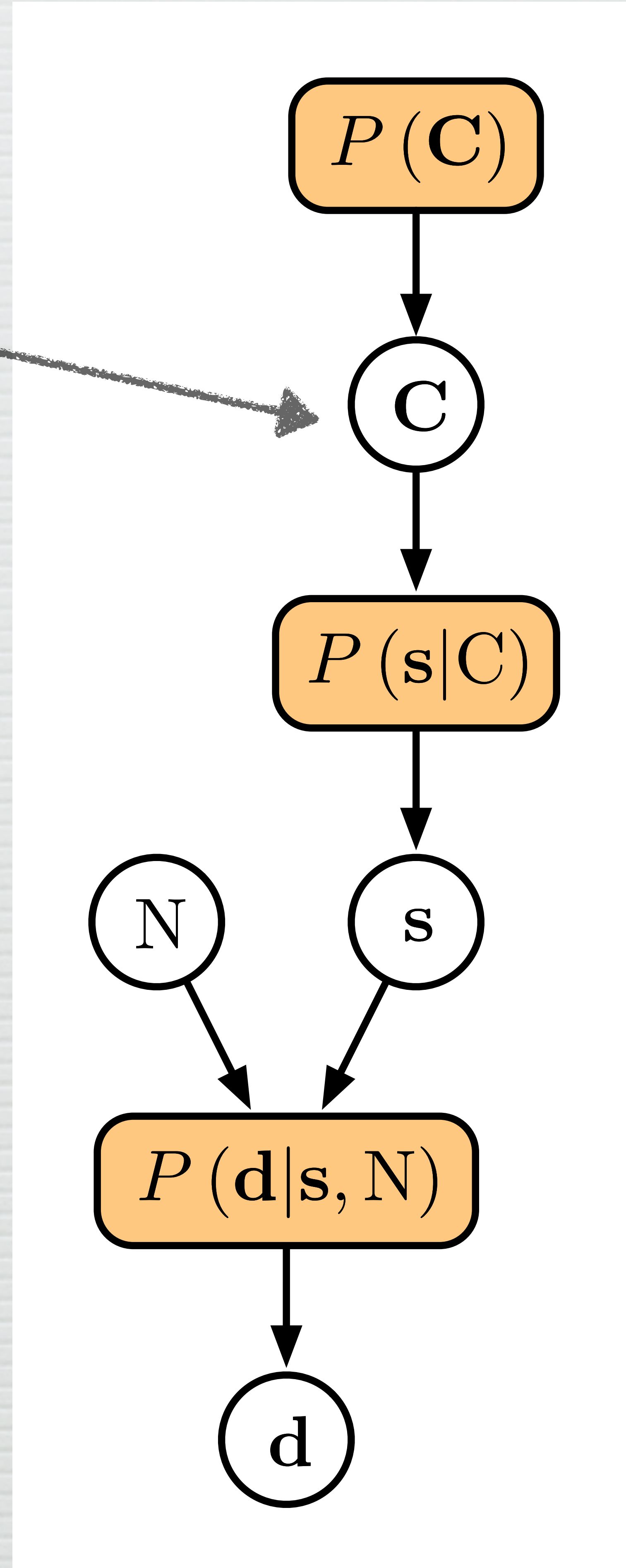



no mask issues
no explicit E/B separation

SHEAR POWER SPECTRA

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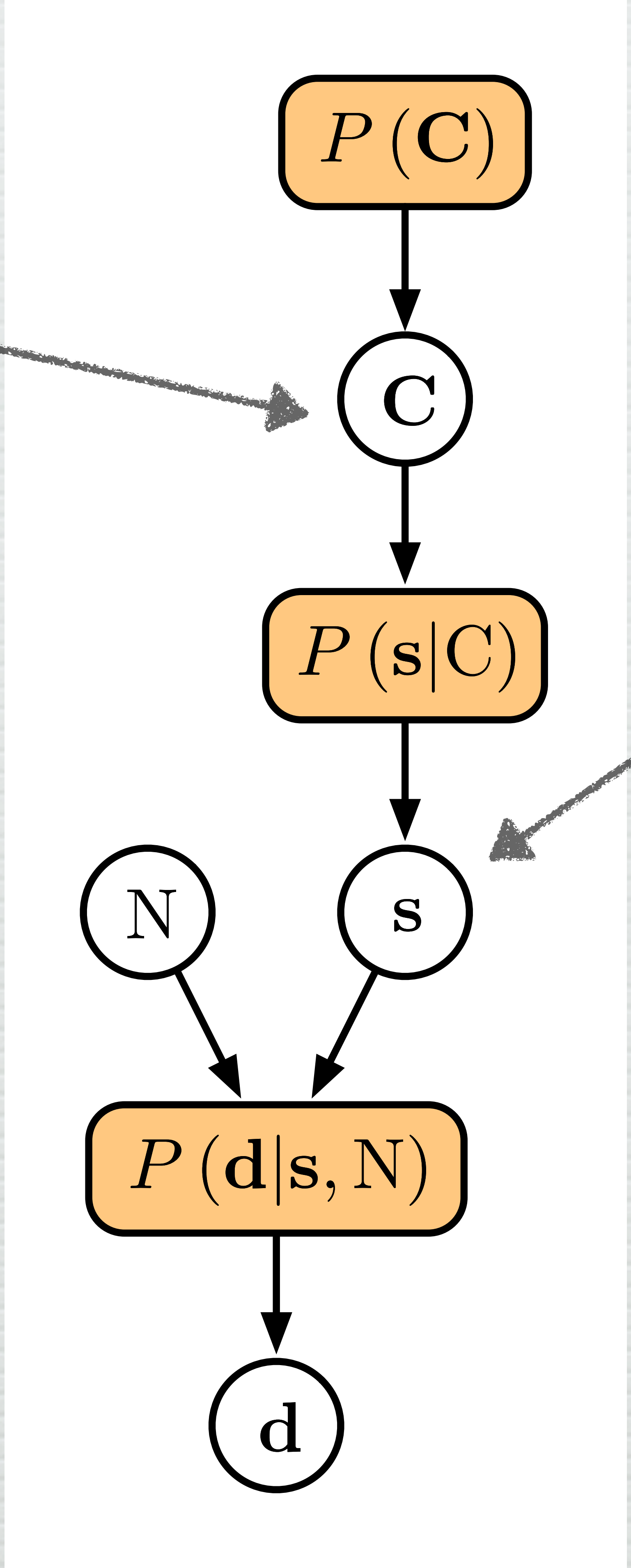


**TOMOGRAPHIC SHEAR FIELDS**

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SHEAR POWER SPECTRA

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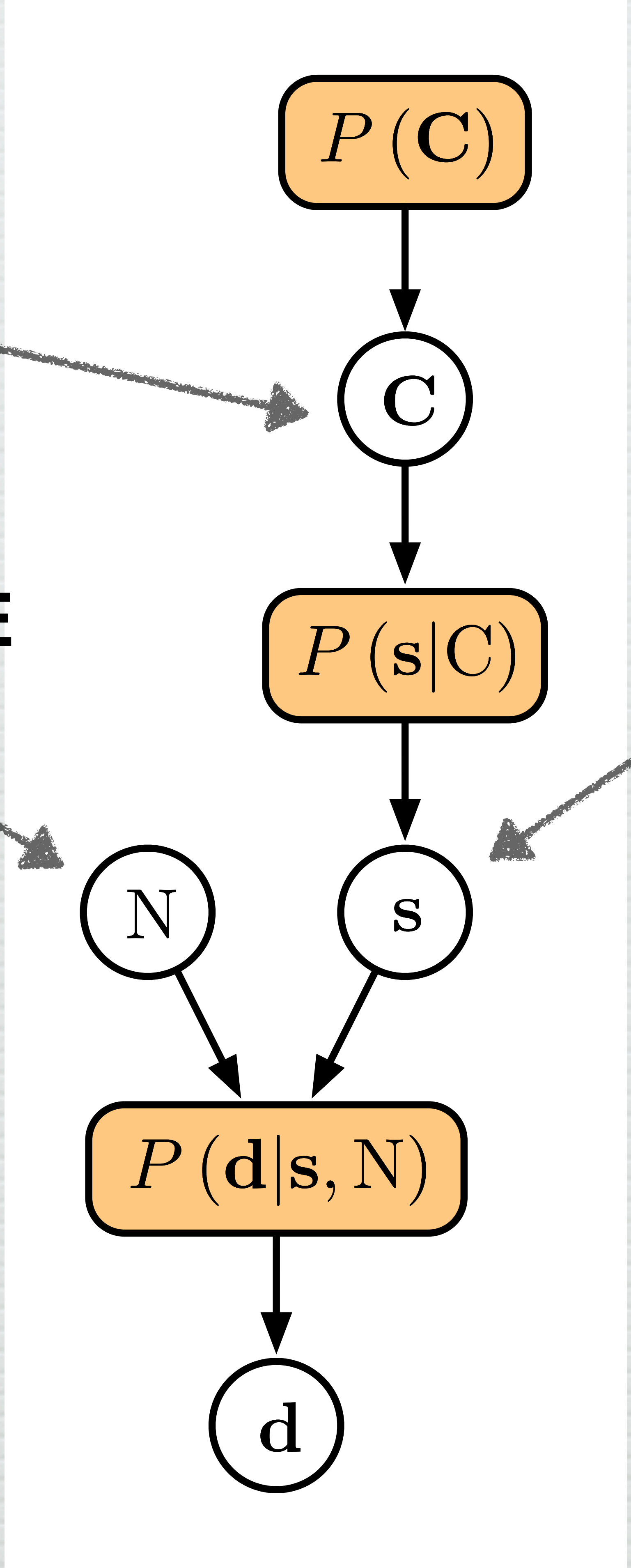
**ELLIPTICITY NOISE**

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SHEAR POWER SPECTRA

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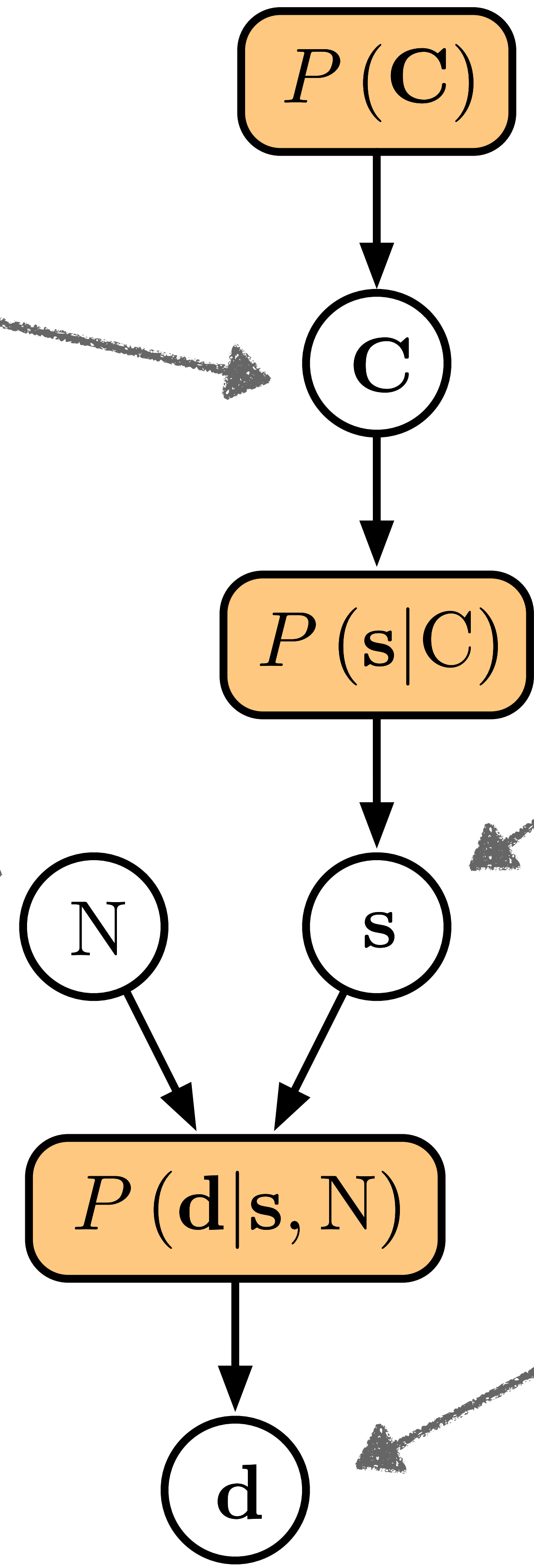
ELLIPTICITY NOISE

NOISY SHEAR MAPS

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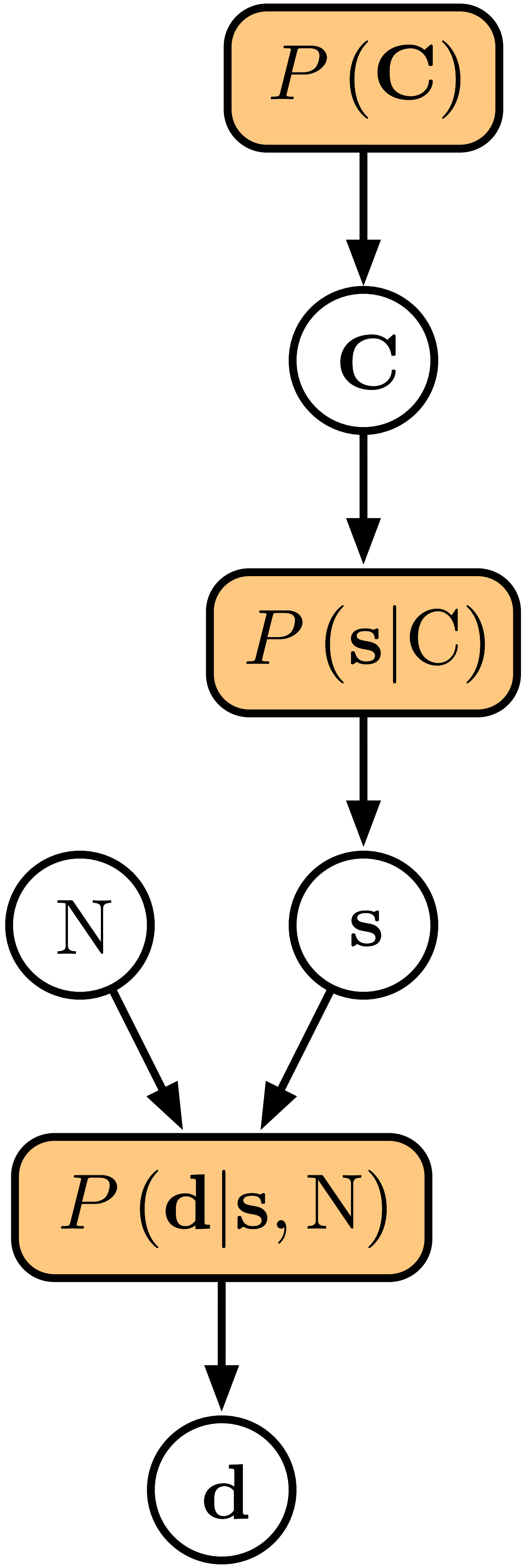


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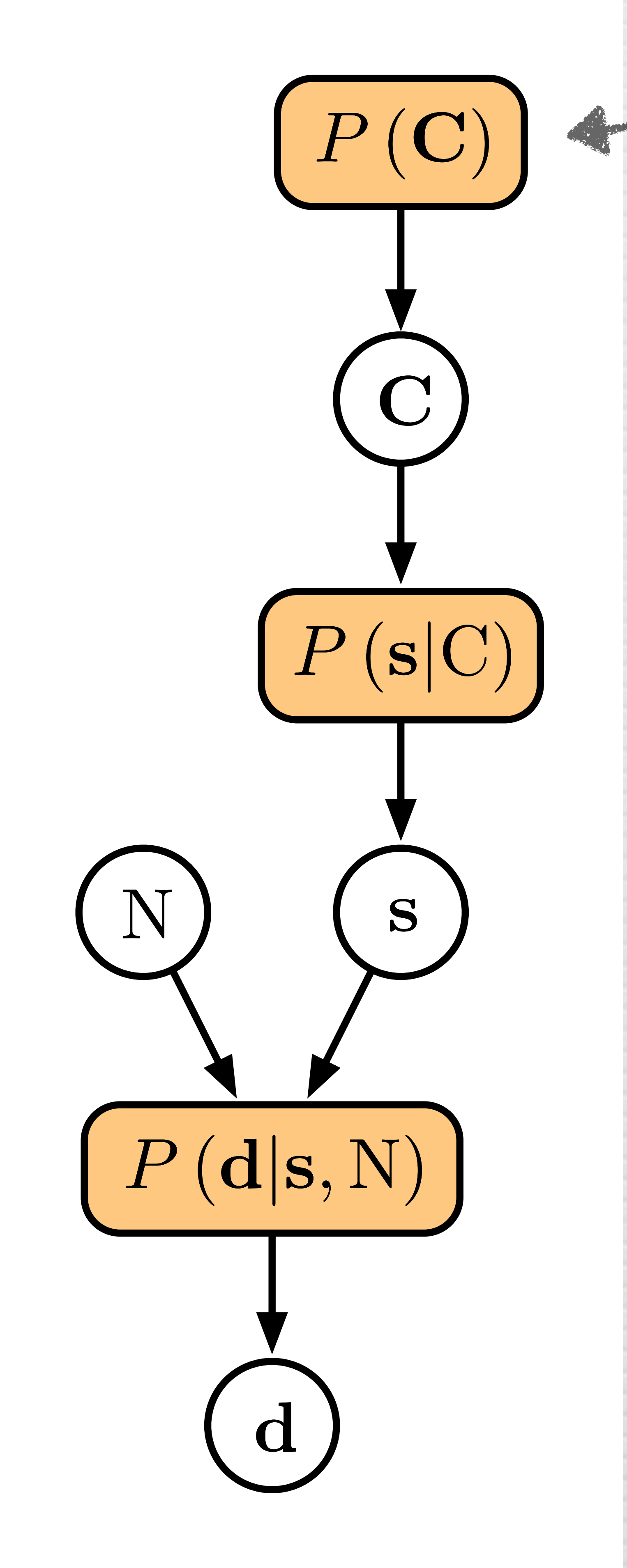
PRIOR
(for “display” or cosmology)

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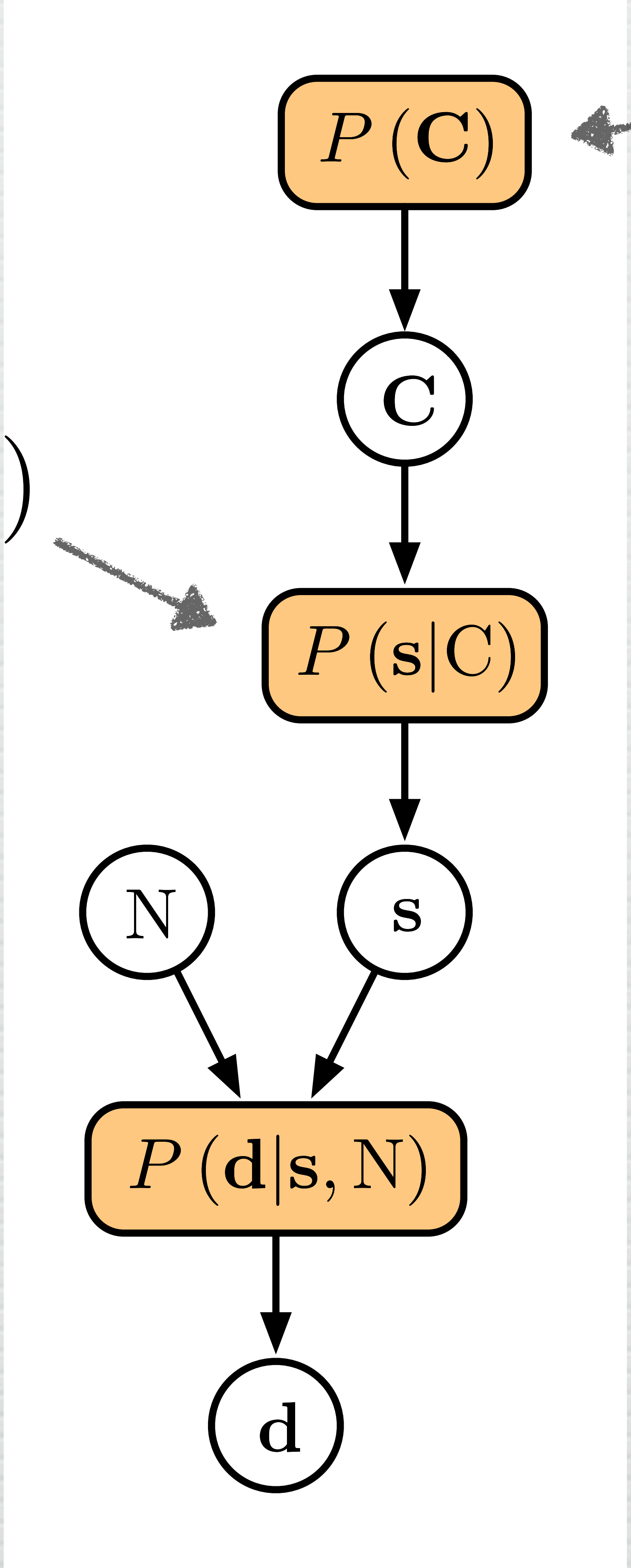


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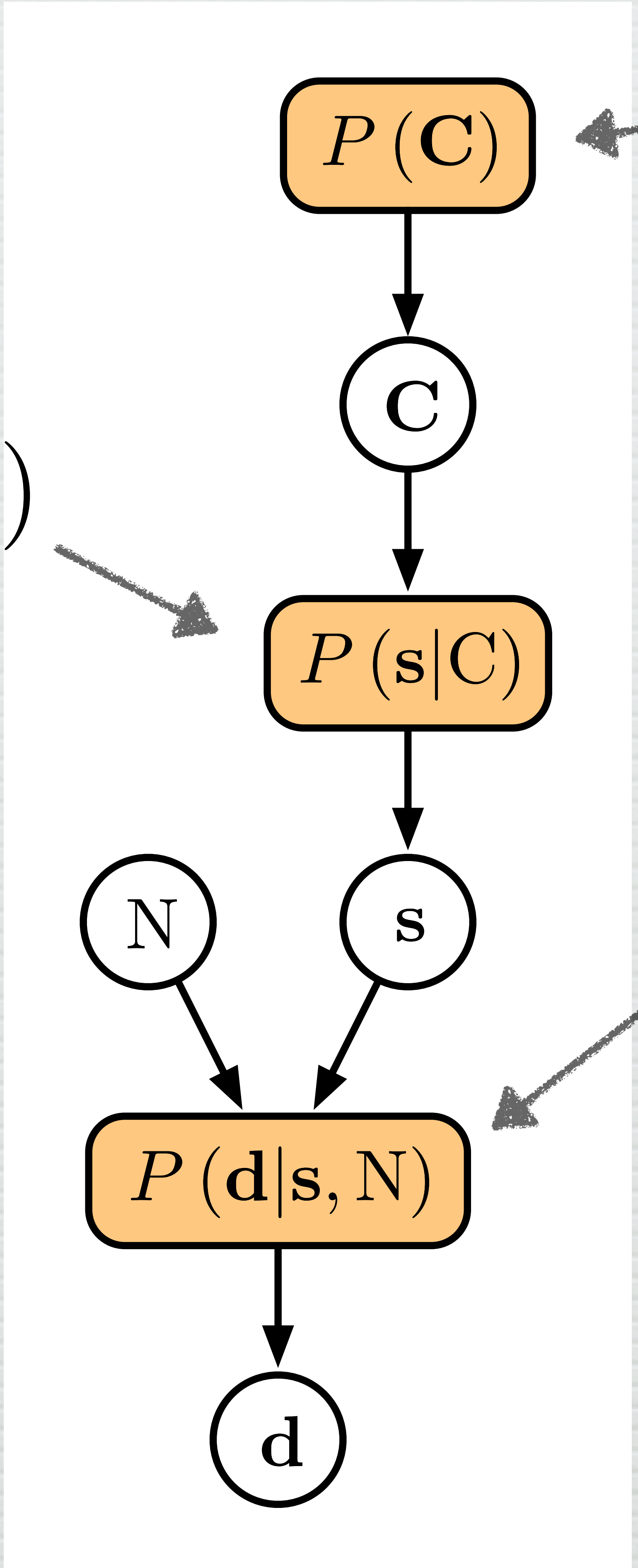


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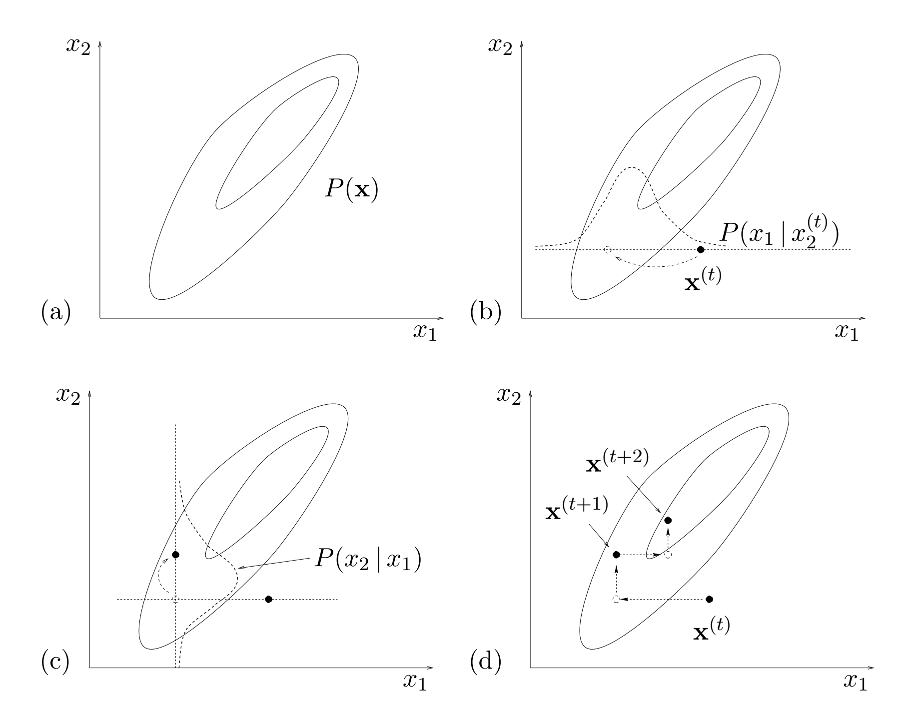
# Gibbs Sampling

- Algorithm:
  - $x_1^{(n+1)} \sim \mathrm{P}(x_2^{(n)}, x_3^{(n)},\ldots)$
    $x_2^{(n+1)} \sim \mathrm{P}(x_1^{(n+1)}, x_3^{(n)},\ldots)$
    $x_3^{(n+1)} \sim \mathrm{P}(x_1^{(n+1)}, x_2^{(n+1)},\ldots)$
- Note that conditionals are just the full distribution with the other parameters held fixed
  (up to normalization).
- In a hierarchical model, get the full posterior by multiplying out all the distributions that appear
  - See Alan Heavens' talk tomorrow…




McKay, *Information Theory…*

# Wiener Filters (Wiener realization/prediction)

- Wiener filter (in the language of BBKS 86; cf. Adler 81)

$$\langle s|d\rangle = \left\langle sd^\dagger\right\rangle \left\langle dd^\dagger\right\rangle^{-1} d$$

- For realizations, also need fluctuations about the mean

$$\left\langle \delta s\, \delta s^\dagger | d\right\rangle = \left\langle ss^\dagger\right\rangle - \left\langle sd^\dagger\right\rangle \left\langle dd^\dagger\right\rangle^{-1} \left\langle ds^\dagger\right\rangle$$

- E.g., $d = s+n$ = signal + noise (zero-mean Gaussians)

$$\left\langle sd^\dagger\right\rangle = \left\langle s(s+n)^\dagger\right\rangle = \left\langle ss^\dagger\right\rangle + \left\langle sn^\dagger\right\rangle = \left\langle ss^\dagger\right\rangle$$

- Even reduces to optimal/unbiased CMB mapmaking in $N\rightarrow\infty$ limit

# Gibbs Sampling for shear

$$\mathbf{C}^{i+1} \leftarrow P(\mathbf{C}|\mathbf{s}^i) = \mathcal{W}^{-1}(\cdot)$$

$W^{-1}$ = Inverse Wishart distribution

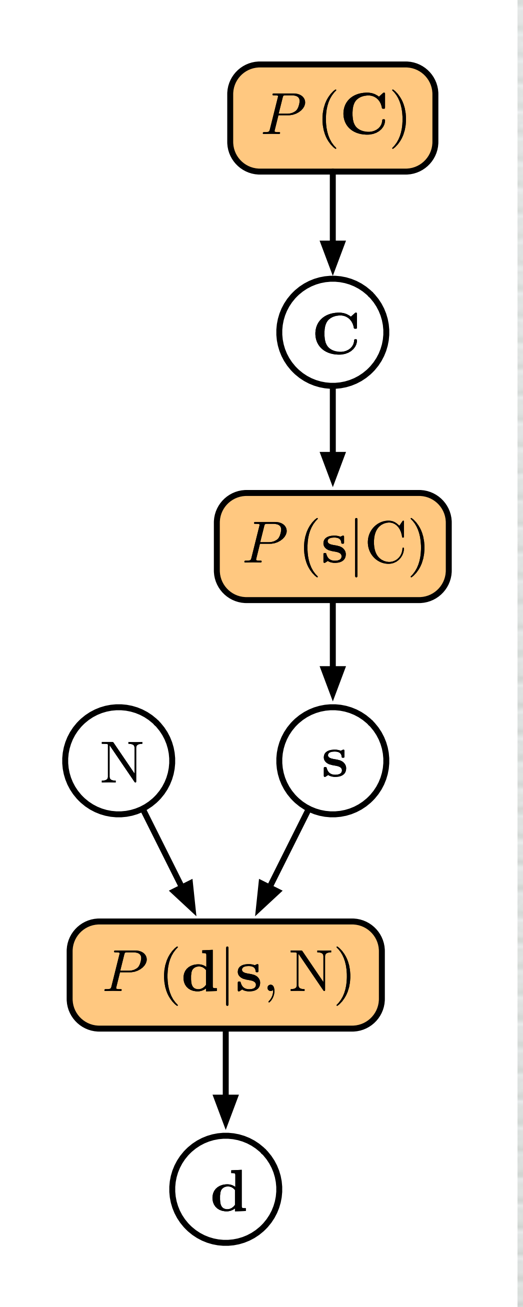
$$\mathbf{s}^{i+1} \leftarrow P(\mathbf{s}|\mathbf{C}^{i+1}, \mathbf{N}, \mathbf{d})$$

$$= \mathcal{N}\left(\mathbf{d}_{\mathrm{WF}}, \mathbf{C}_{\mathrm{WF}}\right)$$

WF = Wiener Filter:

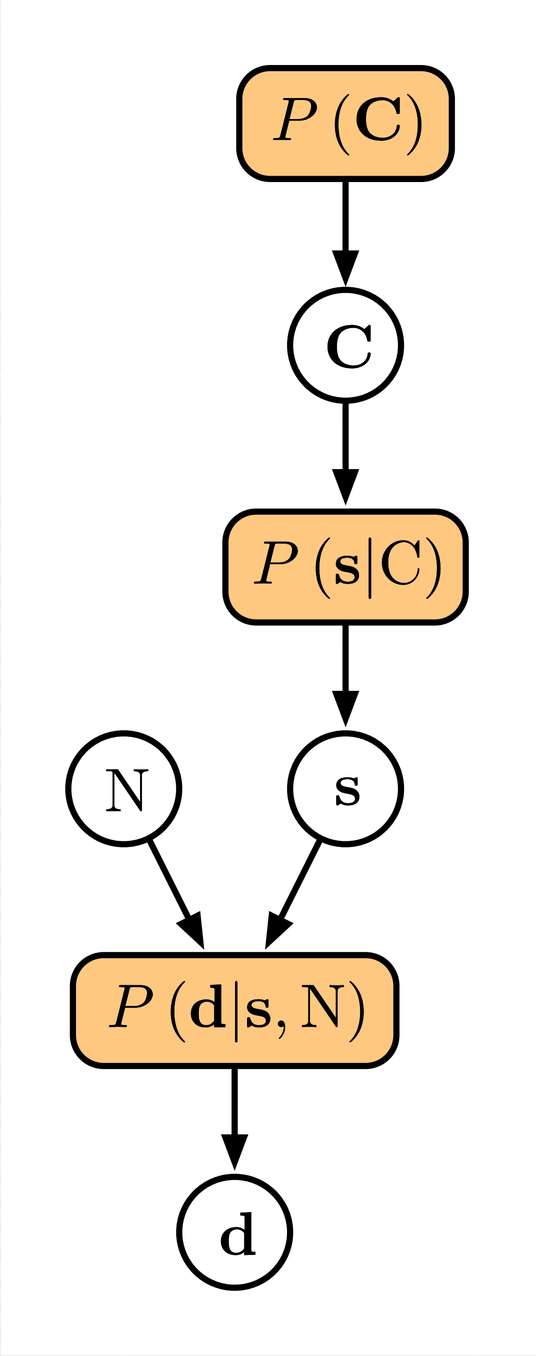
$$\mathbf{d}_{\mathrm{WF}} = (\mathbf{C}^{-1} + \mathbf{N}^{-1})^{-1}\mathbf{N}^{-1}\mathbf{d}$$

$$\mathbf{C}_{\mathrm{WF}} = (\mathbf{C}^{-1} + \mathbf{N}^{-1})^{-1}$$

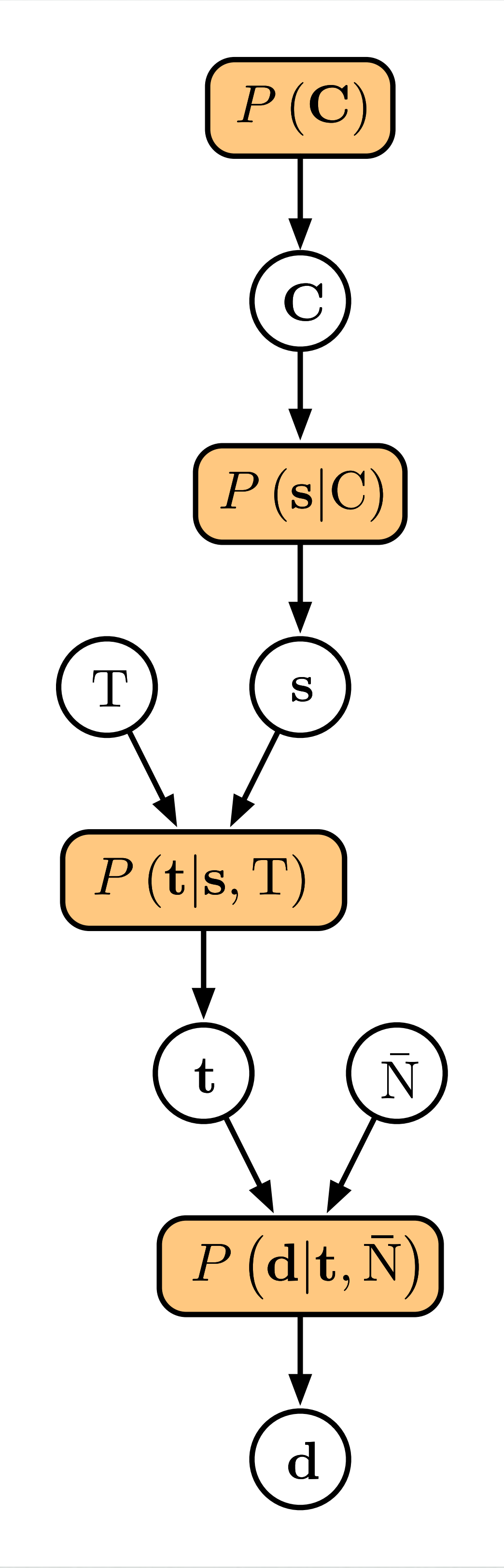


# Messenger fields

**Elsner & Wandelt 2012, 2013**
**Jasche & Lavaux 2015**




Avoid $O(N^3)$ operations by flipping between harmonic & pixel bases



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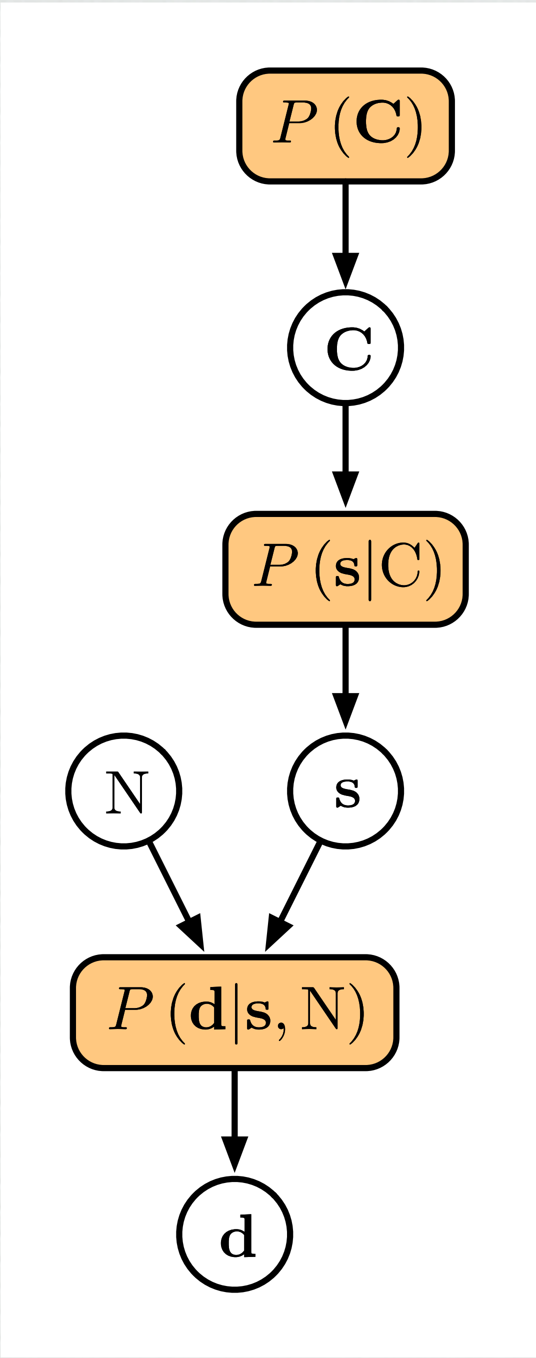
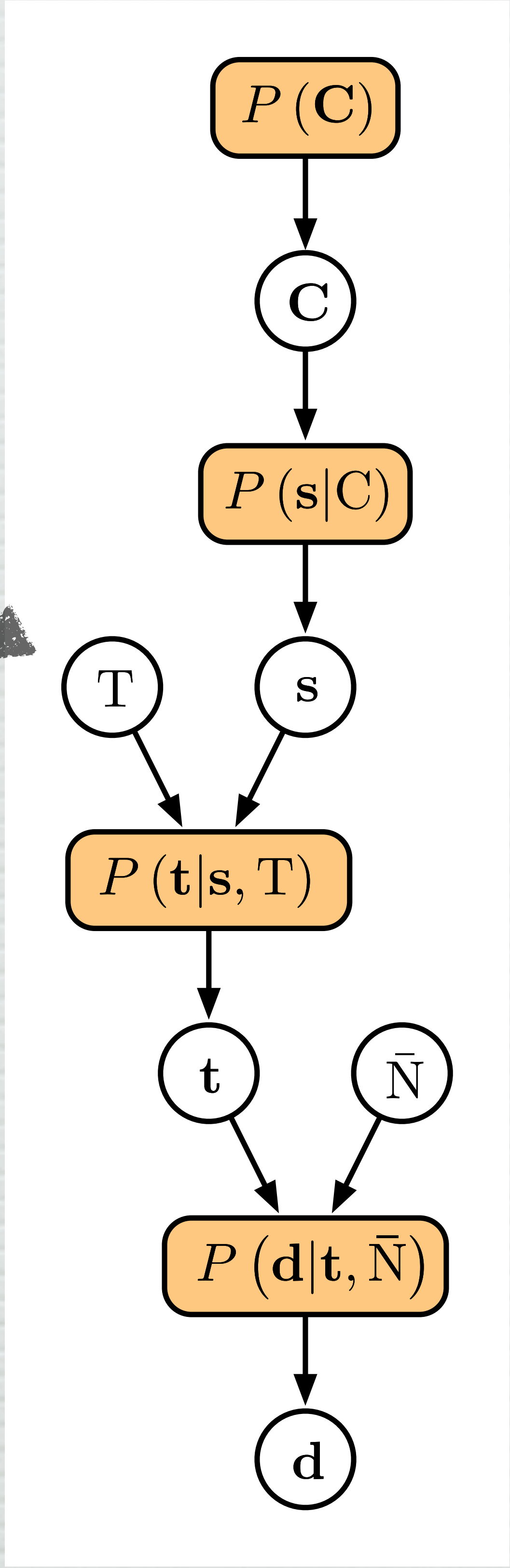



Avoid $O(N^3)$ operations by flipping between harmonic & pixel bases

**ISOTROPIC NOISE**

$$\mathbf{T} = \tau \mathbf{I}$$



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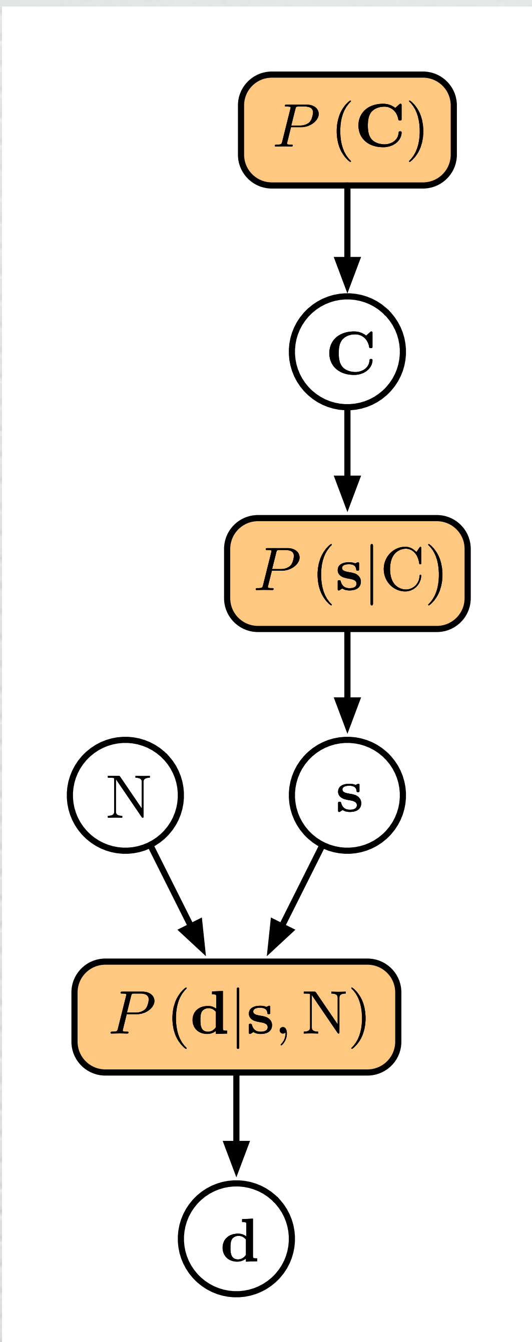


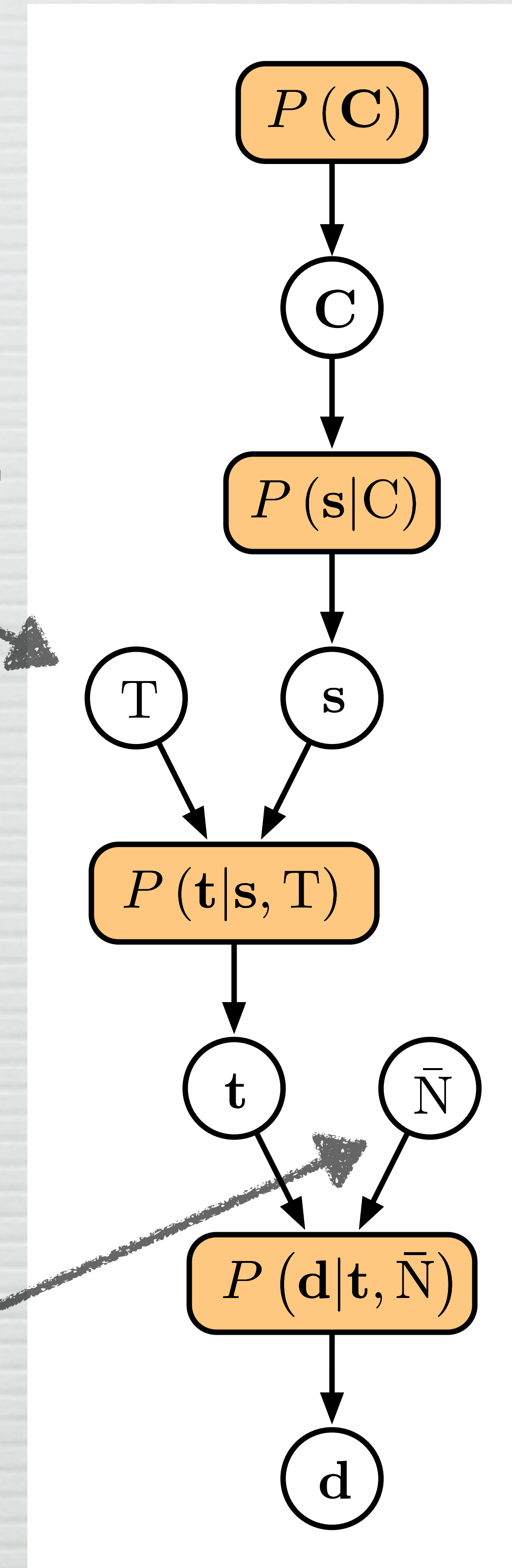

Avoid $O(N^3)$ operations by flipping between harmonic & pixel bases

**ISOTROPIC NOISE**

$$\mathbf{T} = \tau \mathbf{I}$$

**ANISOTROPIC NOISE**

$$\bar{\mathbf{N}} = \mathbf{N} - \mathbf{T}$$



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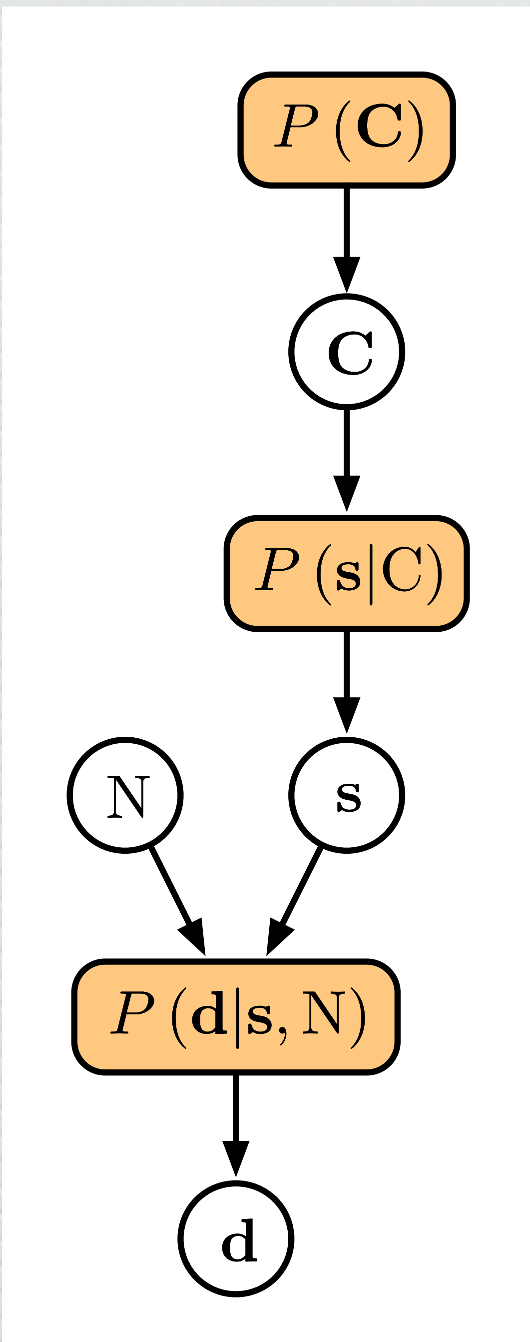



Avoid $O(N^3)$ operations by flipping between harmonic & pixel bases

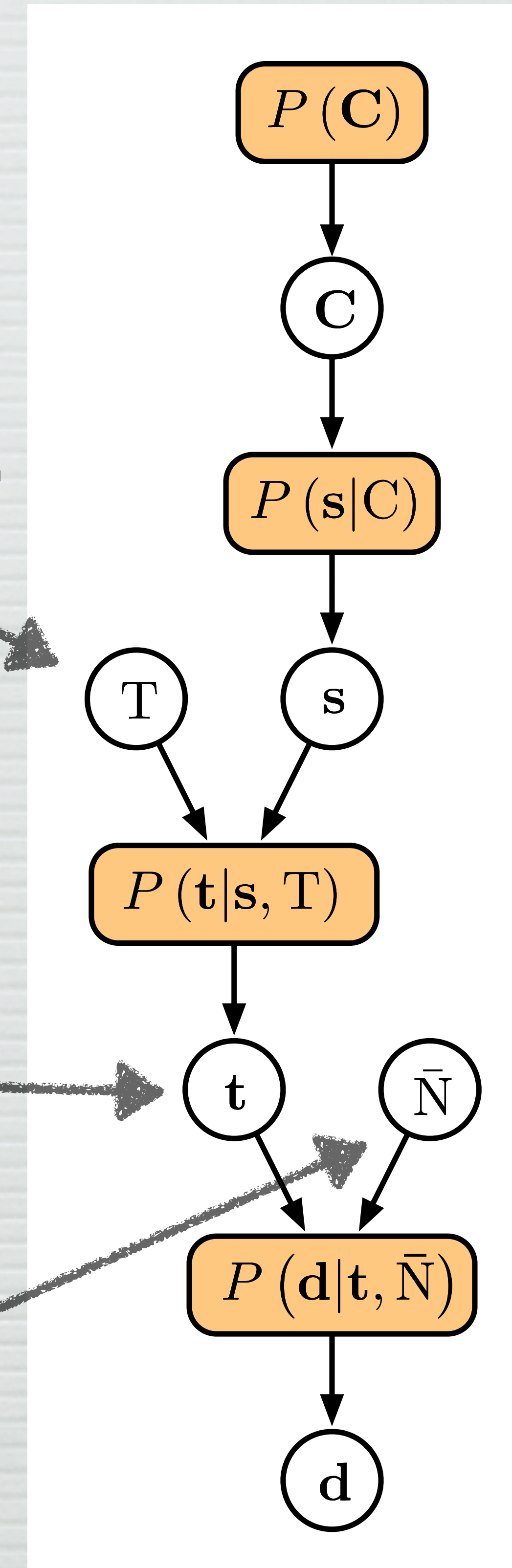
**ISOTROPIC NOISE**

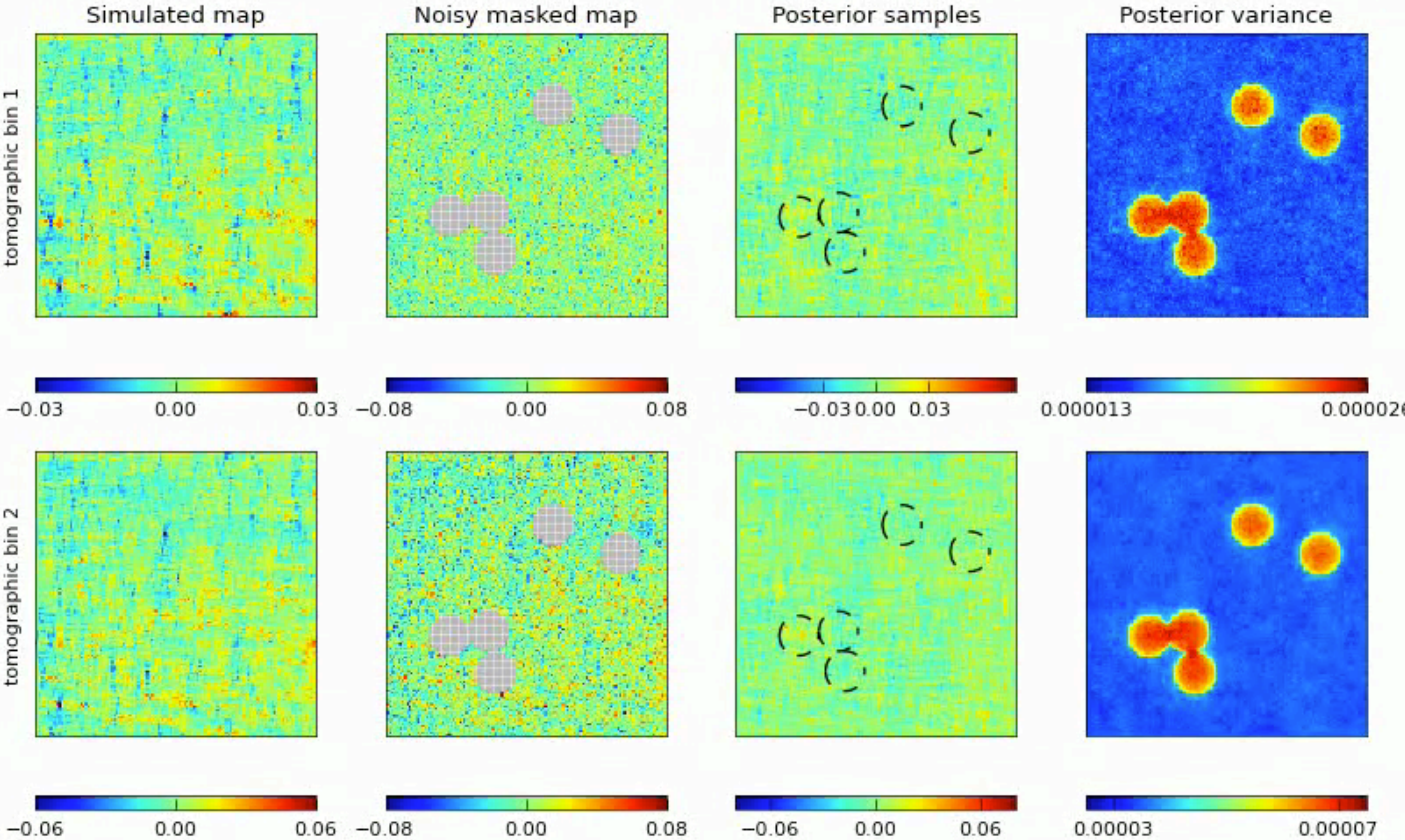
$$\mathbf{T} = \tau \mathbf{I}$$

**MESSENGER FIELD**

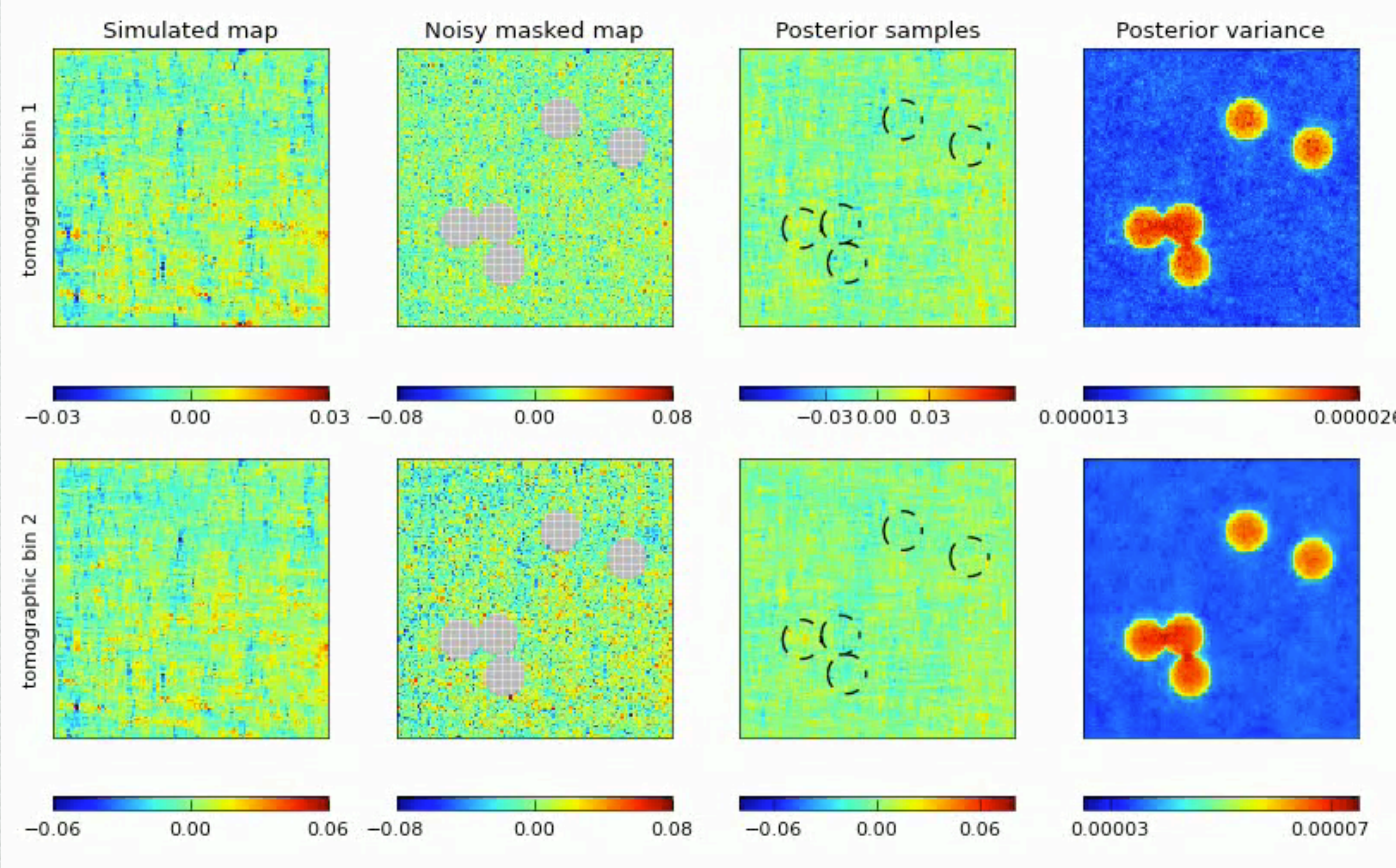
**ANISOTROPIC NOISE**

$$\bar{\mathbf{N}} = \mathbf{N} - \mathbf{T}$$





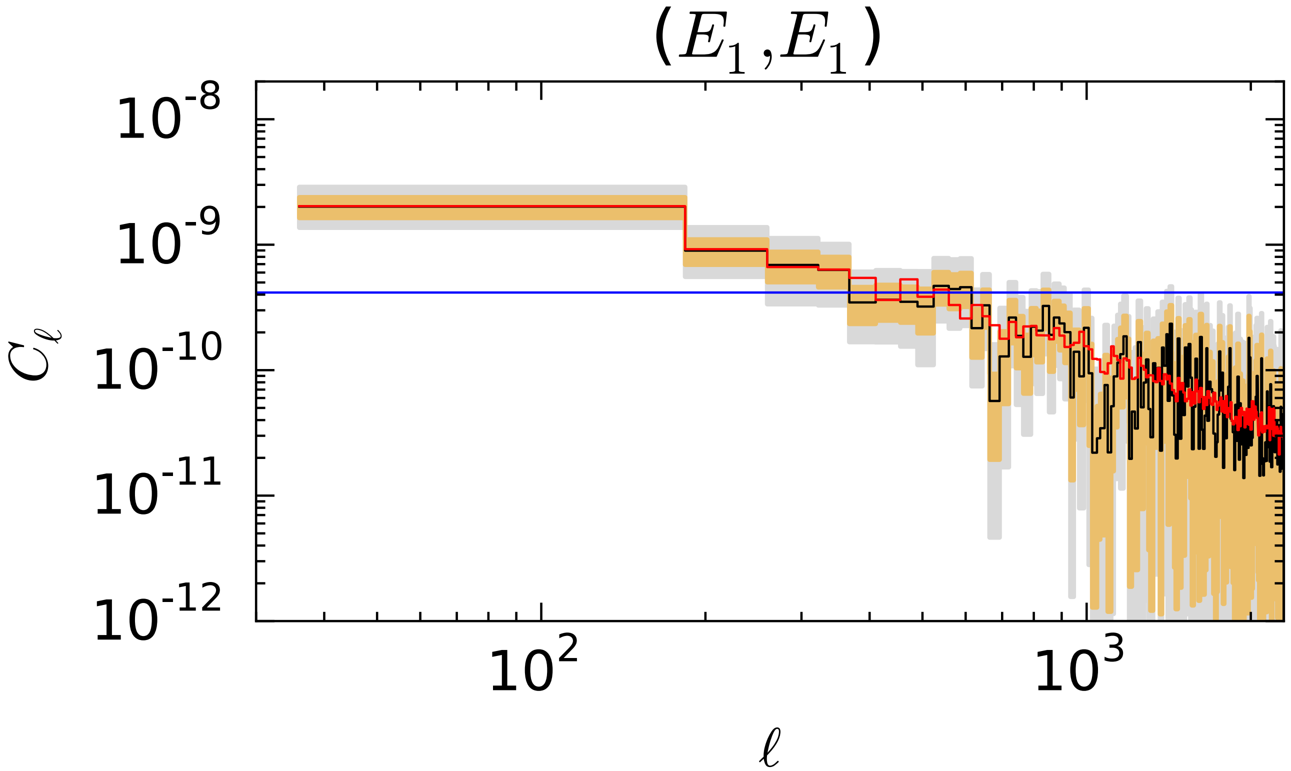

SUNGLASS simulations (Kiessling et al 2011)

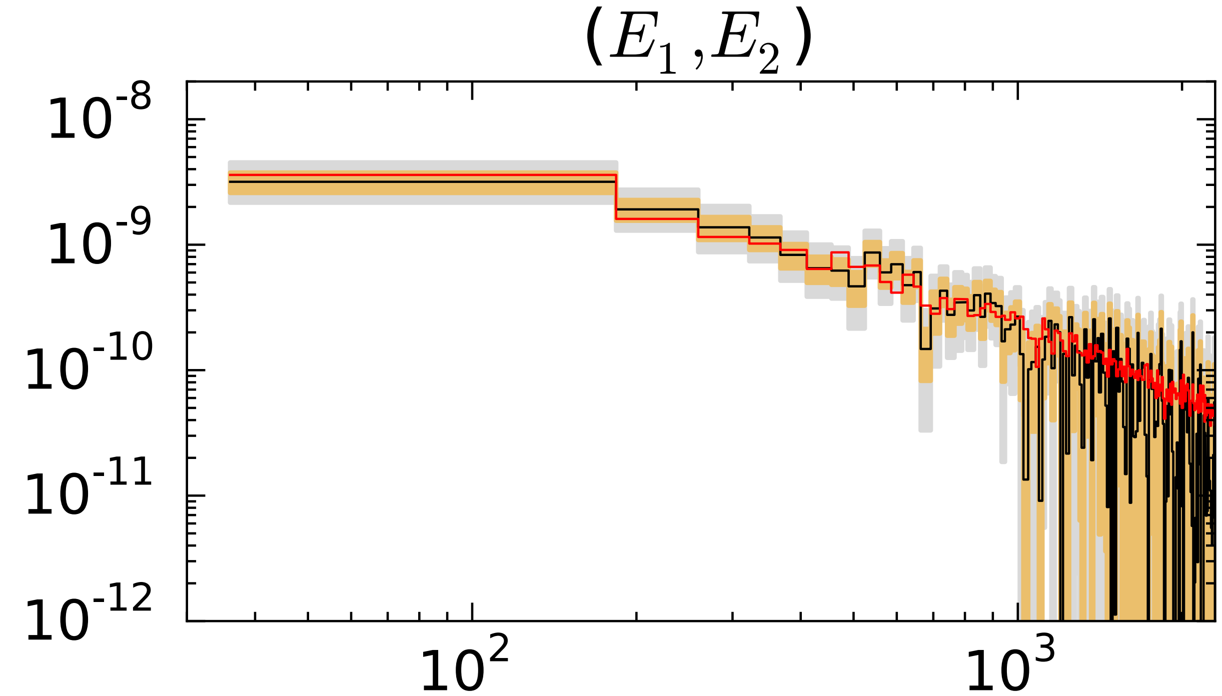



SUNGLASS simulations (Kiessling et al 2011)

# Spectra from simulations

**SUNGLASS (Kiessling et al. 2011), 67,467 model params**




**68% Credible region**

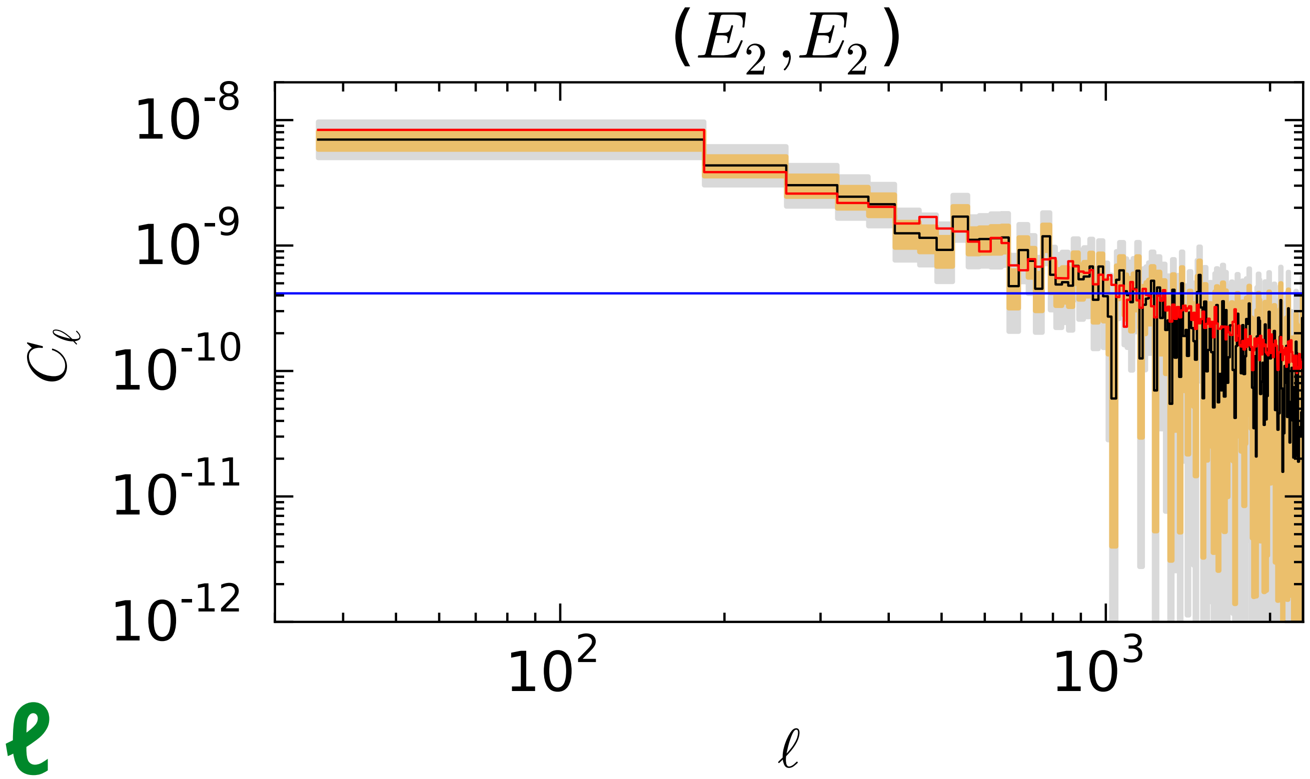
**95% Credible region**

**Posterior mean**

**Simulation**

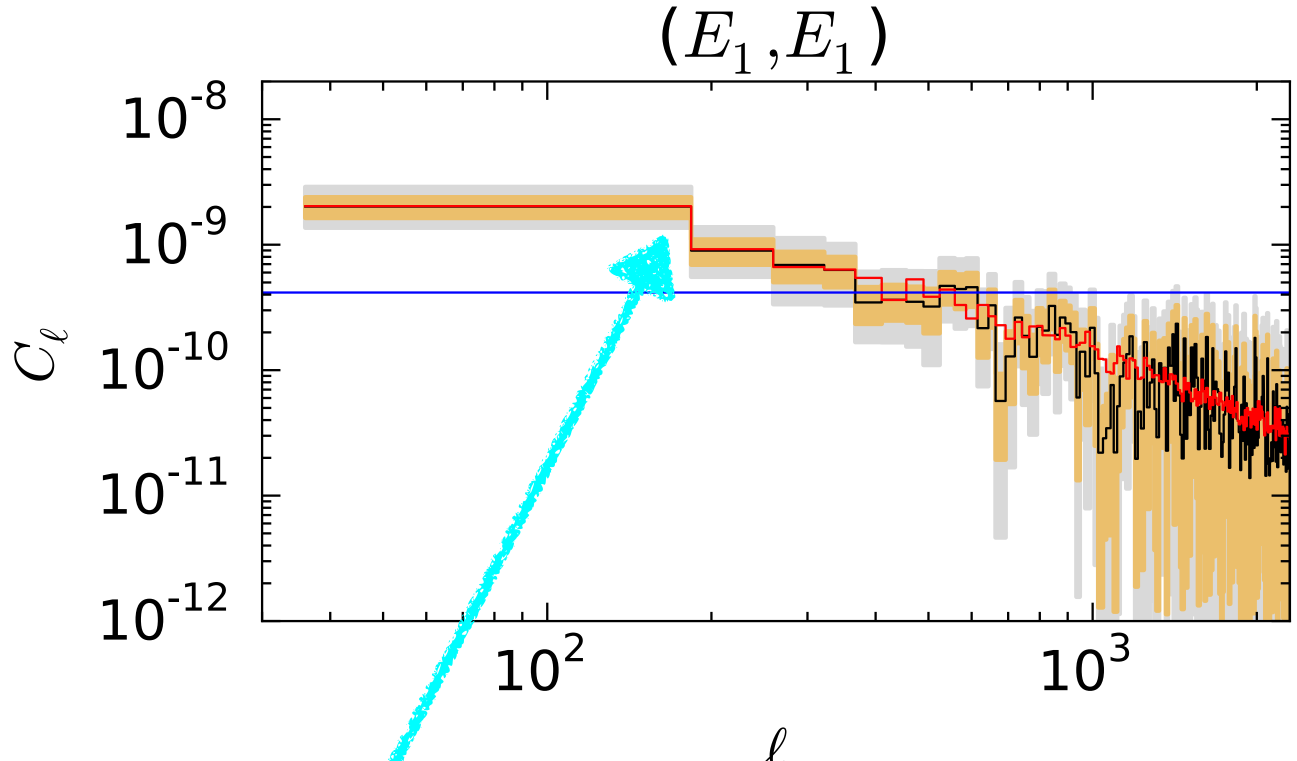
**Noise**

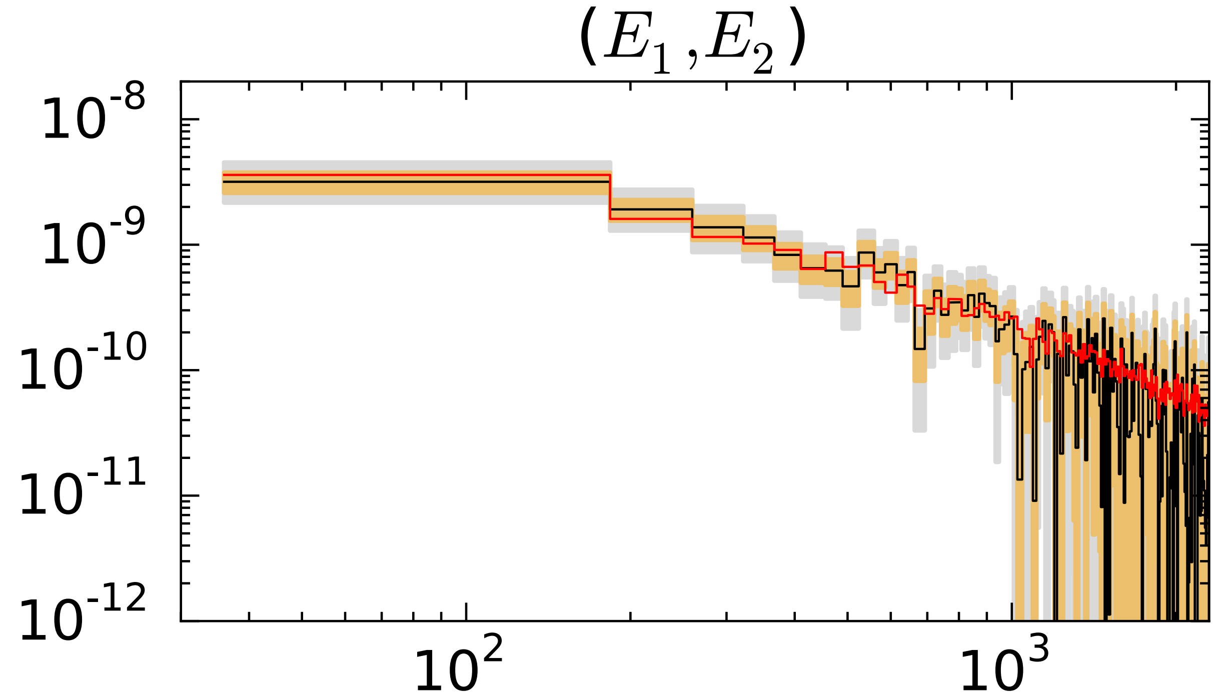
**E modes recovered well below shot noise at high ℓ**

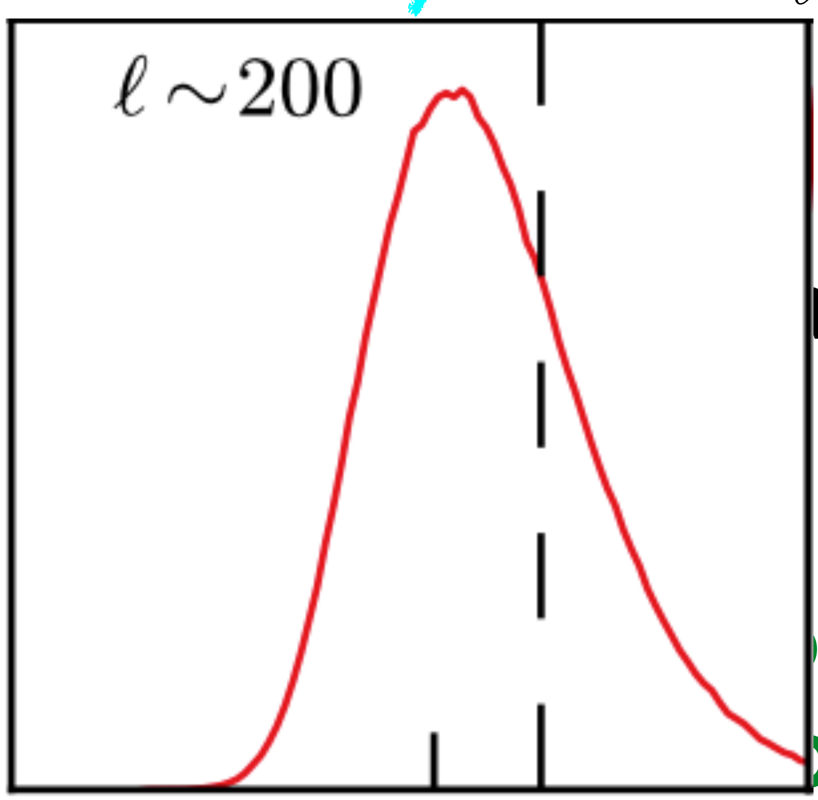


# Spectra from simulations

SUNGLASS (Kiessling et al. 2011), 67,467 model params




region

region

n

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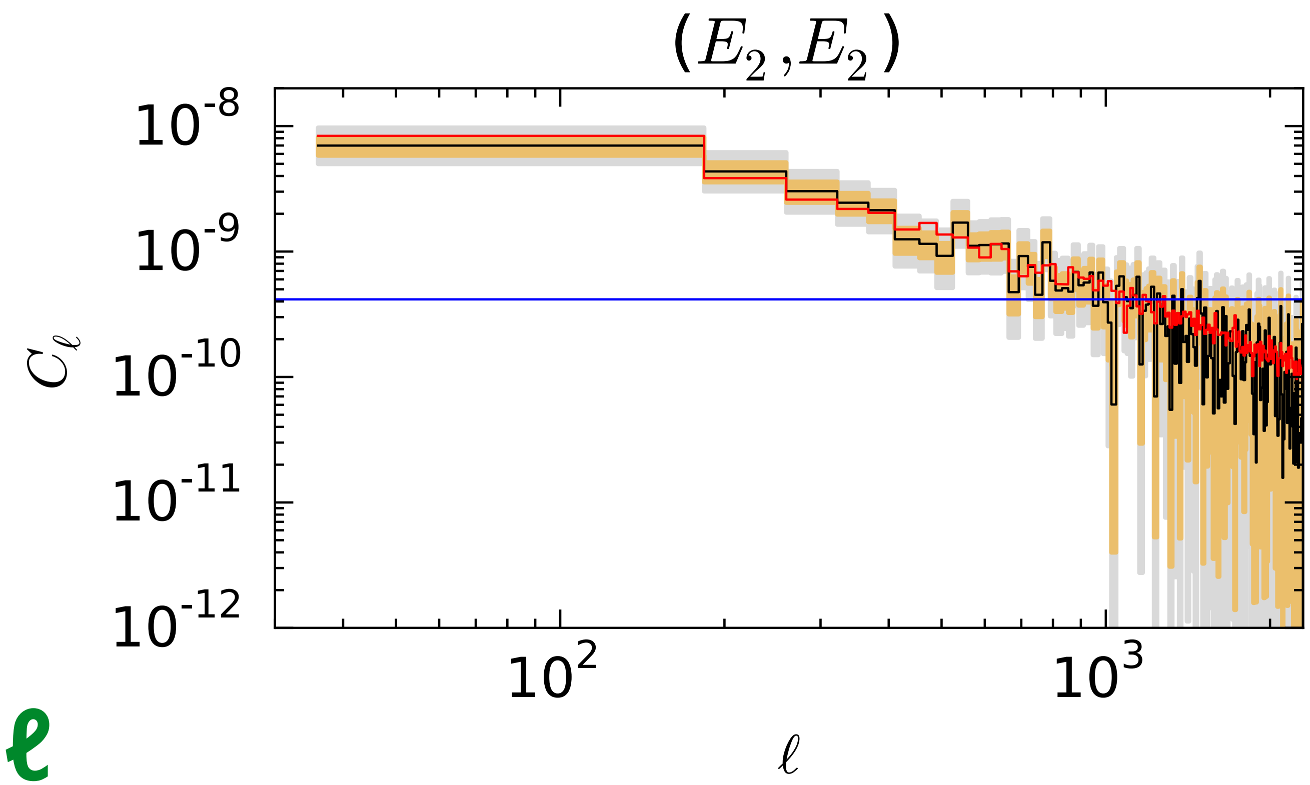
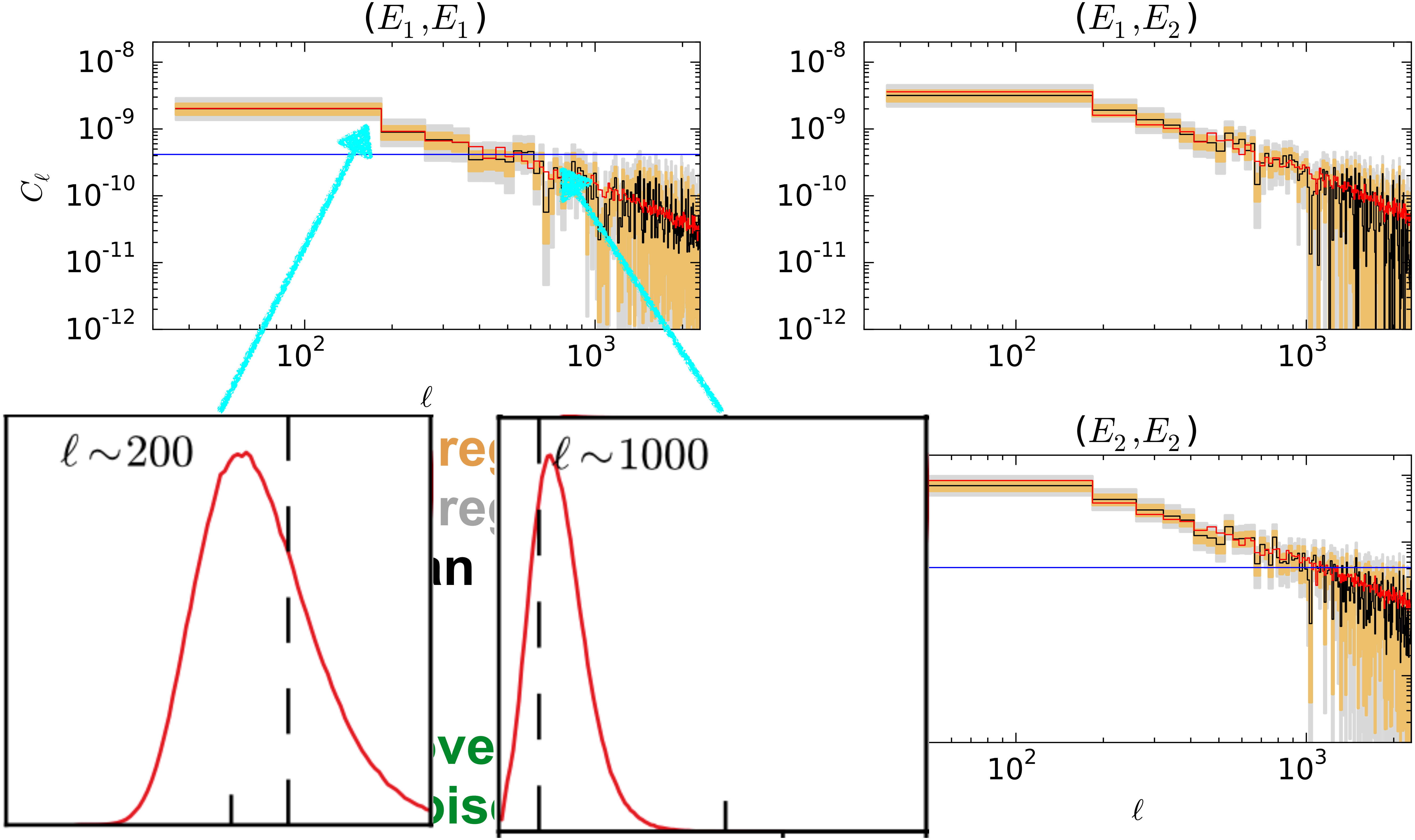


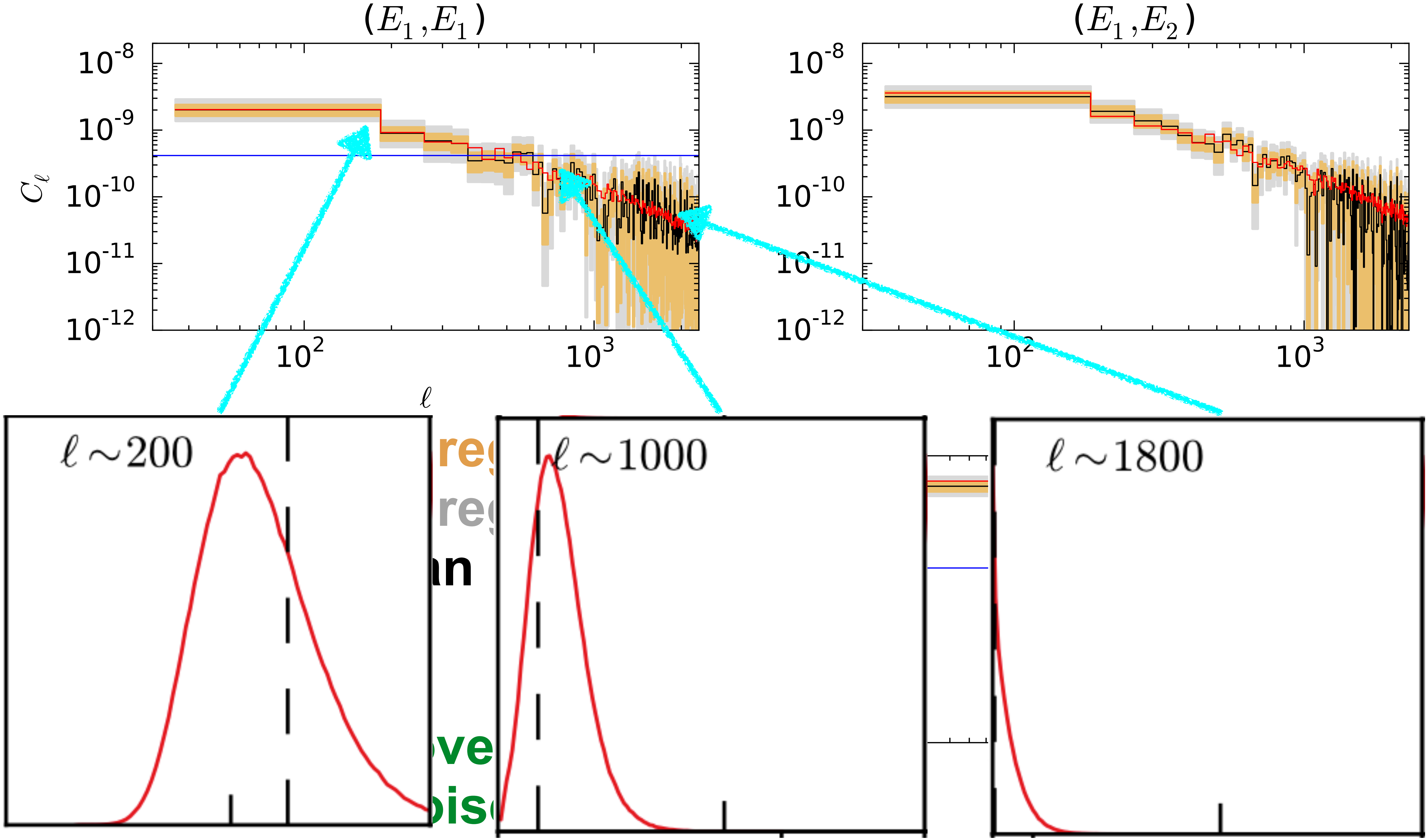
# Spectra from simulations

SUNGLASS (Kiessling et al. 2011), 67,467 model params



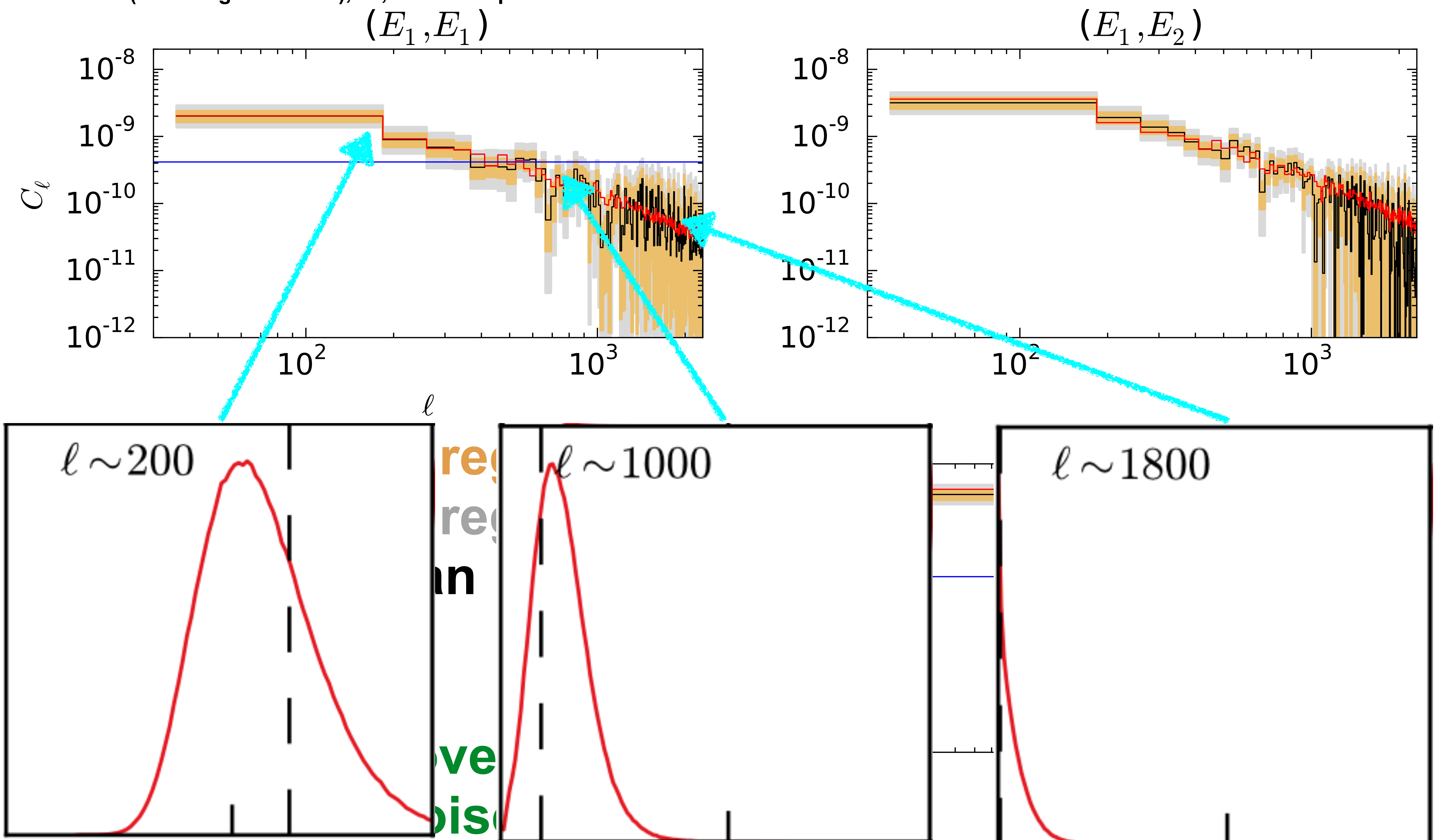
# Spectra from simulations

SUNGLASS (Kiessling et al. 2011), 67,467 model params



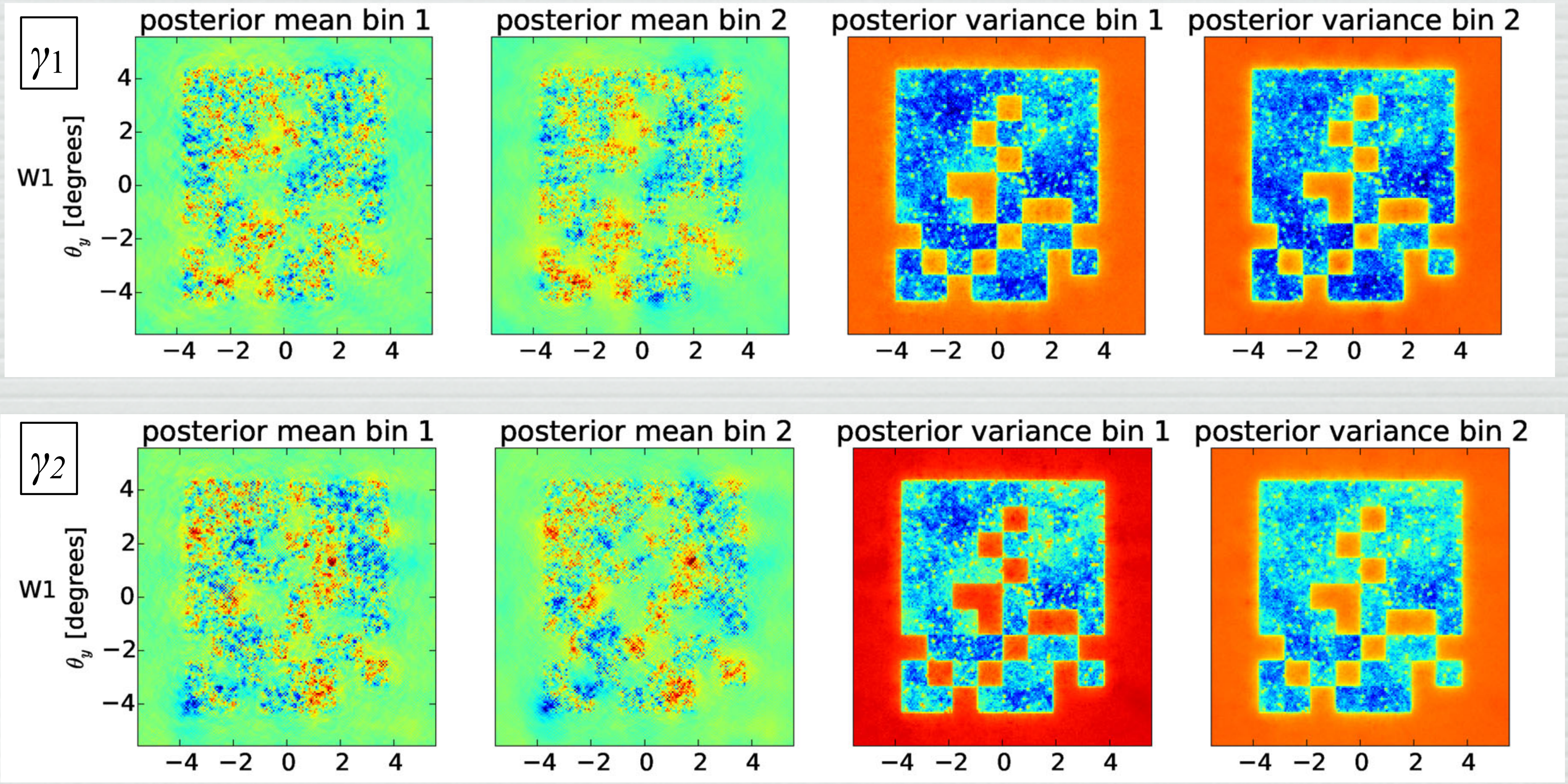
# Spectra from simulations

SUNGLASS (Kiessling et al. 2011), 67,467 model params

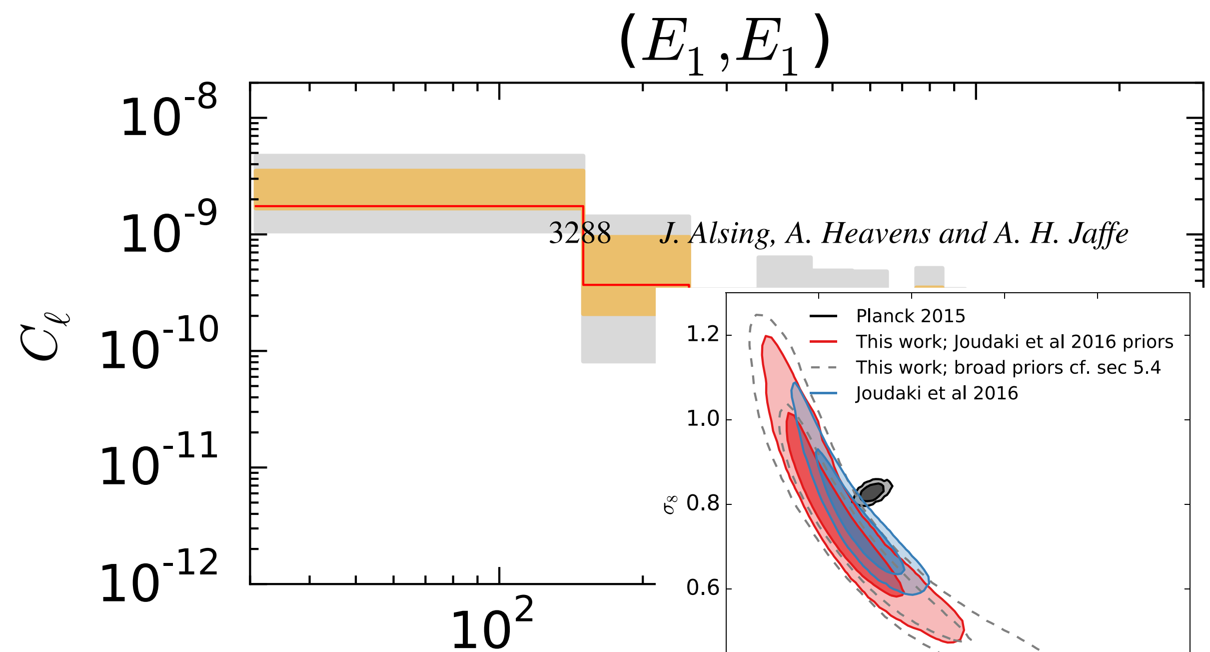



EE, EB and BB consistent with sims

# Application to CFHTLens data: maps (fields)



# Application to CFHTLens data: spectra




3288 *J. Alsing, A. Heavens and A. H. Jaffe*

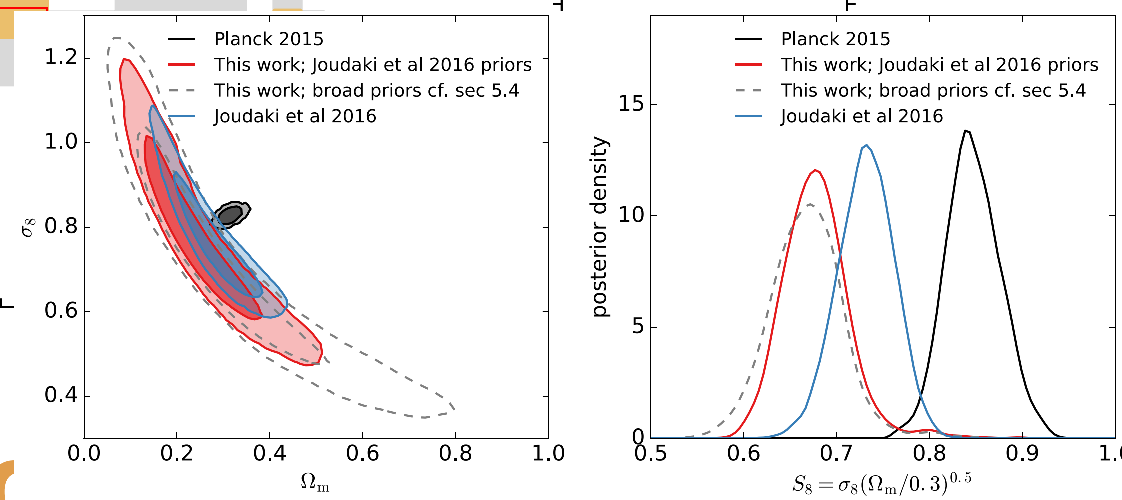
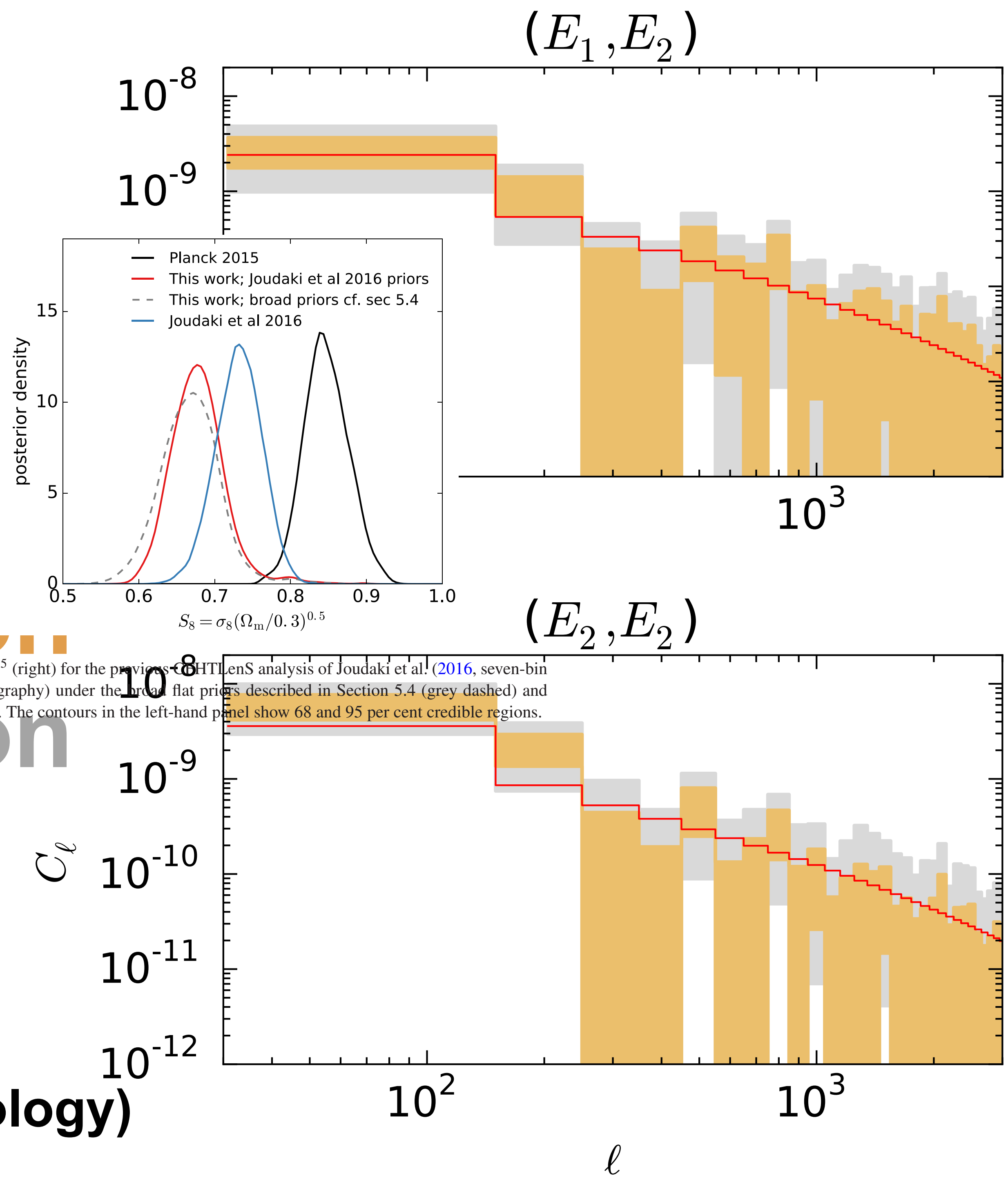



**Figure 12.** Comparison of constraints in $\sigma_8$–$\Omega_m$ (left) and $S_8 = \sigma_8(\Omega_m/0.3)^{0.5}$ (right) for the previous CFHTLenS analysis of Joudaki et al. (2016, seven-bin tomography; blue), Planck 2015 (black) and the present work (two-bin tomography) under the broad flat priors described in Section 5.4 (grey dashed) and under the same prior assumptions (see footnote 9) as Joudaki et al. (2016, red). The contours in the left-hand panel show 68 and 95 per cent credible regions.




**68% Credible region**

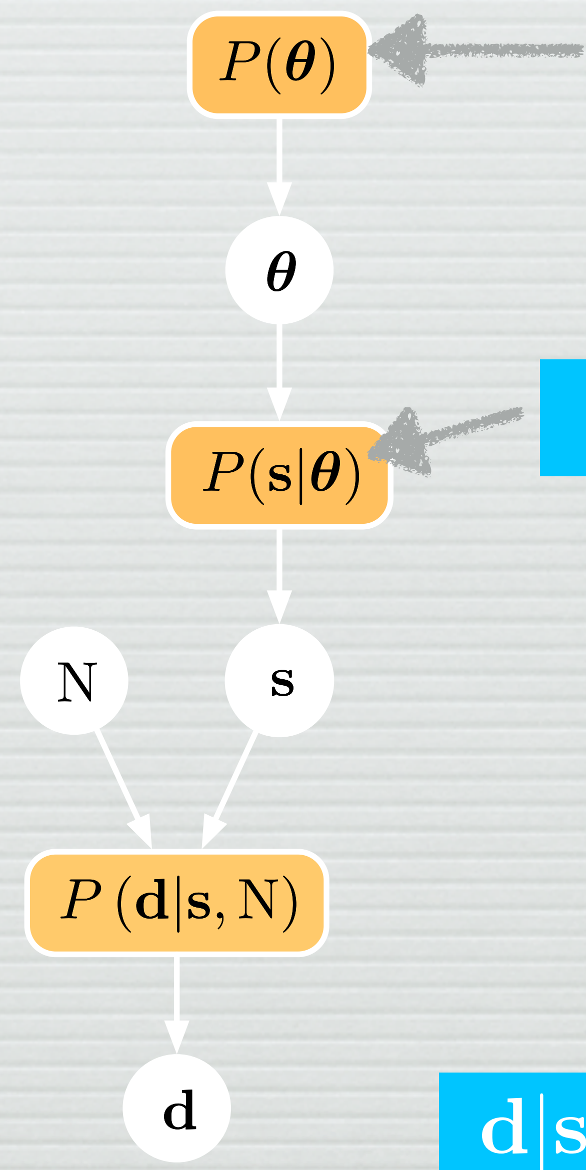
**95% Credible region**

**Planck 2015**

**Note that regions depend on priors (but go away when inferring cosmology)**

# Application to CFHTLens data: parameters

- Pros:
  - no P(C|data) density estimation
  - no $\ell$ binning
  - good at low $\ell$
  - few parameters
- Cons:
  - likelihood function much more complicated fn of parameters
  - no independent estimate of spectra (but cheap enough to run both)
- Could also use similar techniques to indirectly estimate correlation fn




**Cosmology prior**

$$\mathrm{s}|\boldsymbol{\theta} \sim \mathcal{N}(\mathbf{0}, \mathbf{C}(\boldsymbol{\theta}))$$

**(Can be extended to non-Gaussian shear)**

$$\mathbf{d}|\mathrm{s} \sim \mathcal{N}(\mathrm{s}, \mathbf{N})$$

# Application to CFHTLens data: parameters

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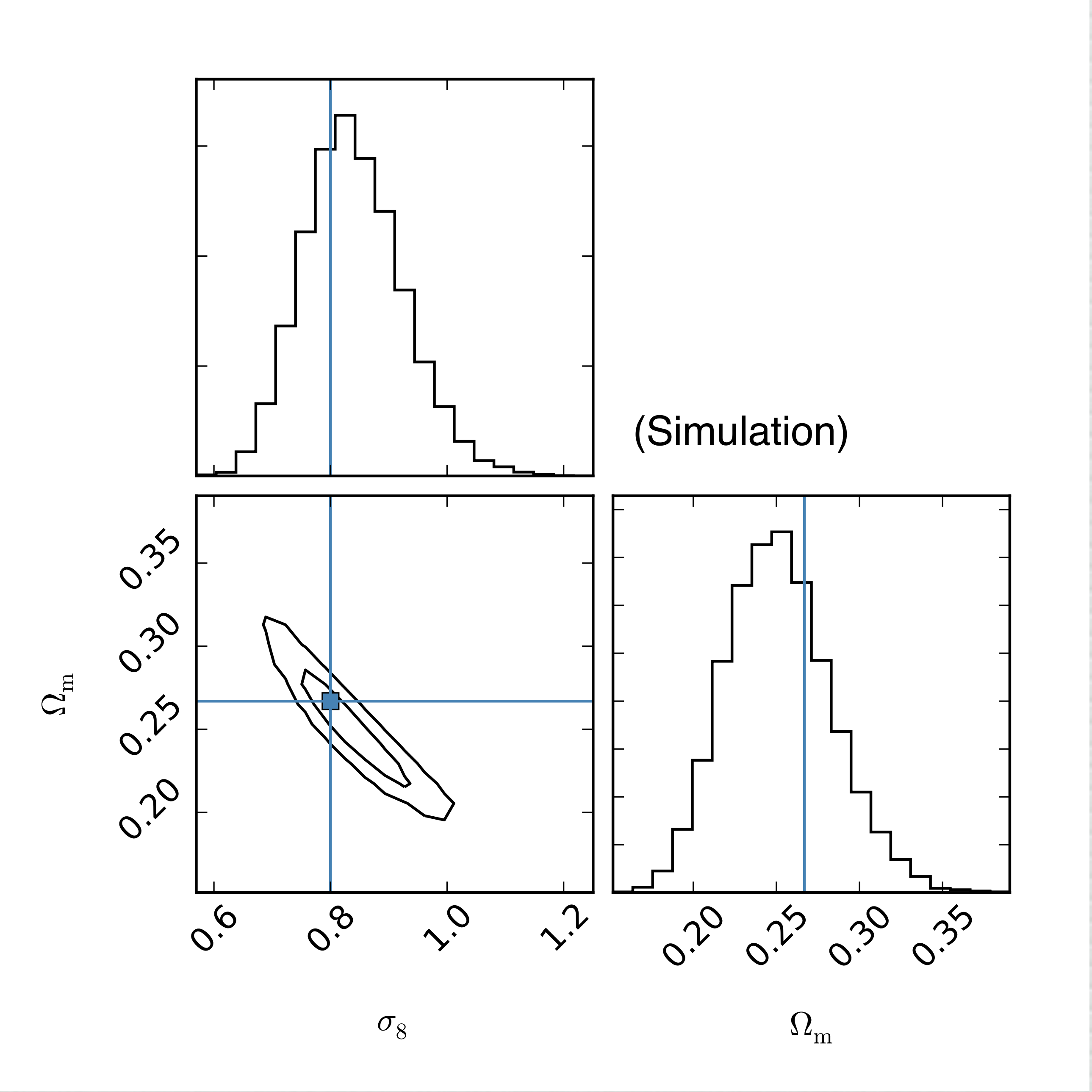

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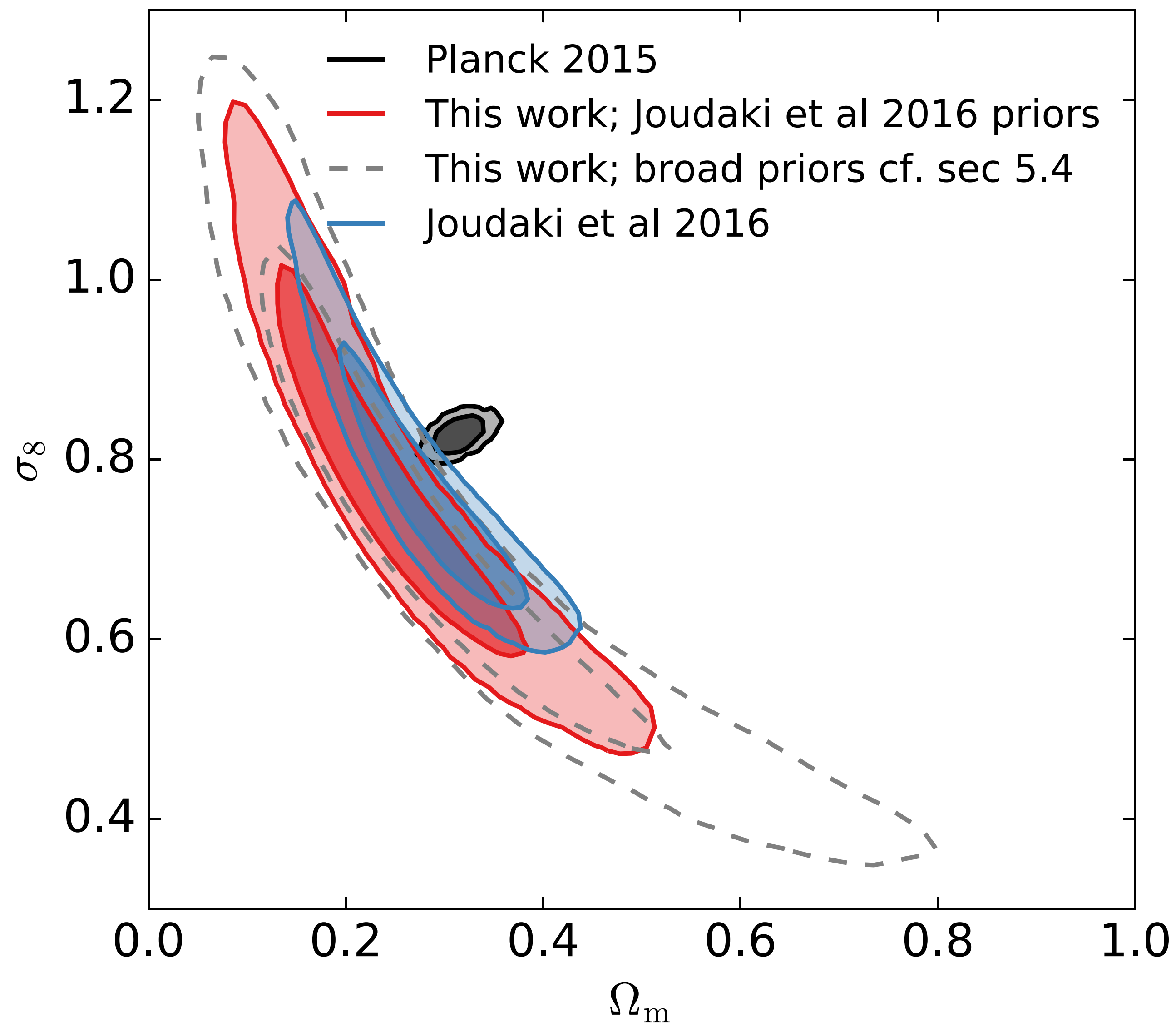


# Hamiltonian Monte Carlo (HMC)

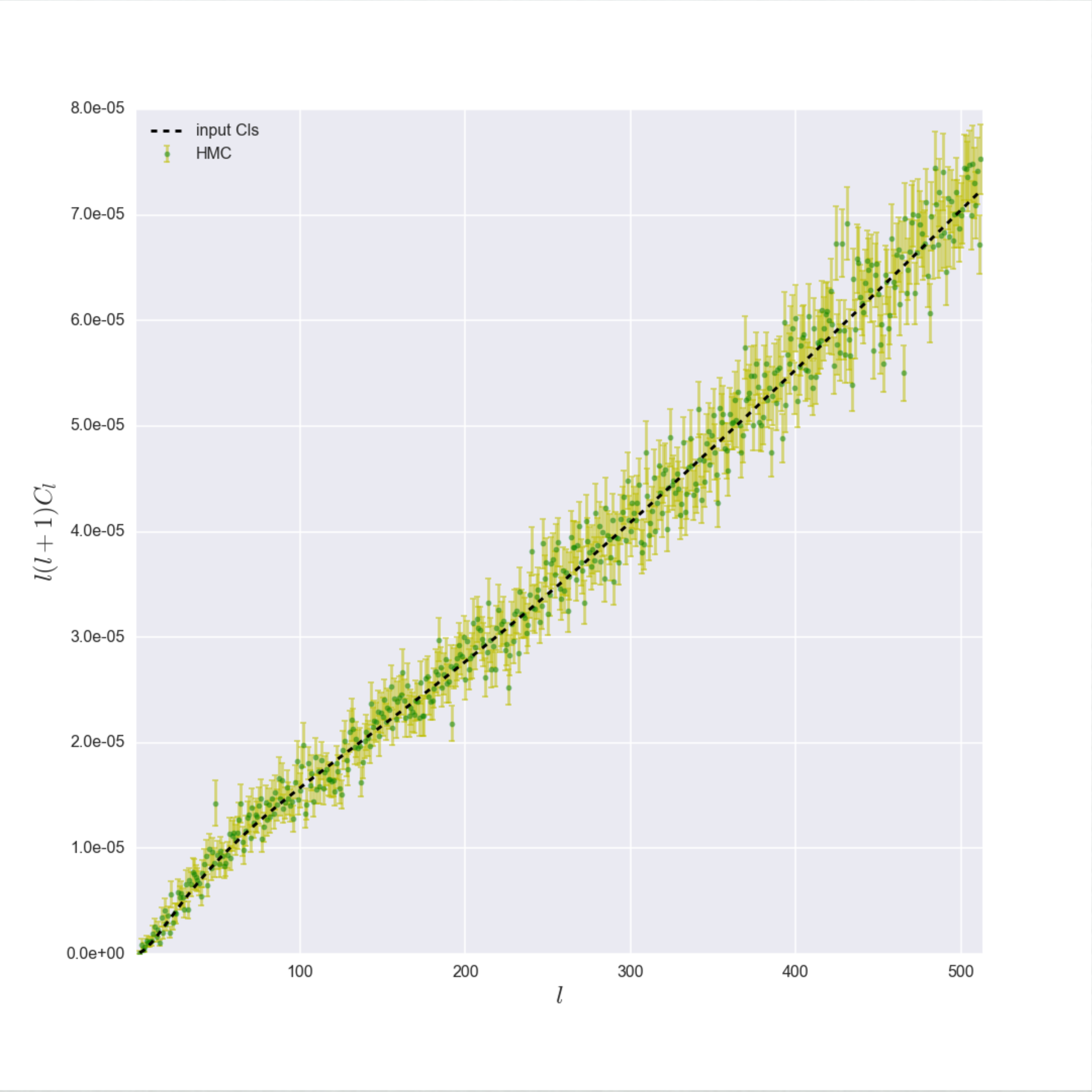
- *(aka Hybrid Monte Carlo; Duane et al 1987)*
- Analogy with dynamical systems, which explore (***position, momentum***) phase space over time
  - Potential $U(\theta_i) = -\ln \mathrm{P}(\theta_i)$ w/ "positions" $\theta_i$
  - KE $K(u_i) = \frac{1}{2}\boldsymbol{u}\cdot\boldsymbol{u}$ w/ "momenta" $u_i \sim \mathrm{N}(0,\sigma^2)$
  - Hamiltonian $H(\theta_i, u_i) = U(\theta_i) + K(u_i)$
  - Density $\mathrm{P}(\theta_i, u_i) = e^{-H(\theta, u)}$
    - 2N parameters!
  - Evolve as dynamical system
    - ignore (marginalize over) momenta

$$\dot{\theta}_i = \frac{\partial H}{\partial u_i} = u_i$$

$$\dot{u}_i = -\frac{\partial H}{\partial \theta_i} = \frac{\partial \ln P}{\partial \theta_i}$$

# HMC for shear

- Based on `BlackPearl` (Balan et al)
- Better behaviour than Gibbs
  - over wide S/N range
  - with strong degeneracy
  - (but see Racine et al 2016)
- Euclid level 0 sims
  - full sky, uniform noise
  - Recovers input



# Beyond Gaussian Random Fields for shear on sphere(s)

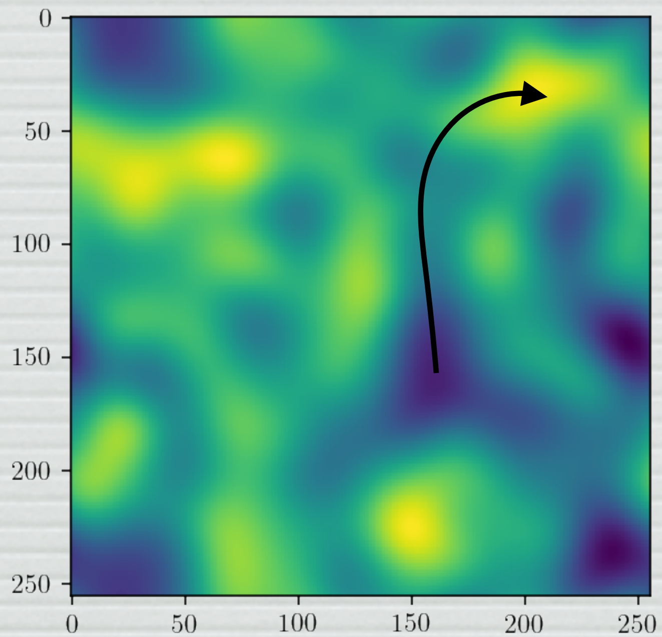
- (Non-) Gaussianity & non-linearity
  - tests w/ lognormal indicate only small effect (CFHTLS)
  - Ideally would propagate full nonlinear physics (e.g., 2LPT a la Leclercq, Jasche & Wandelt)
- Radial information
  - Self-consistently including photo-z
  - From tomography to 3D?
    - Lots of modes, very low S/N per model
    - Related to discussion optimal (?) modes to describe the ball?
- Mass mapping: the shear field is not fundamental

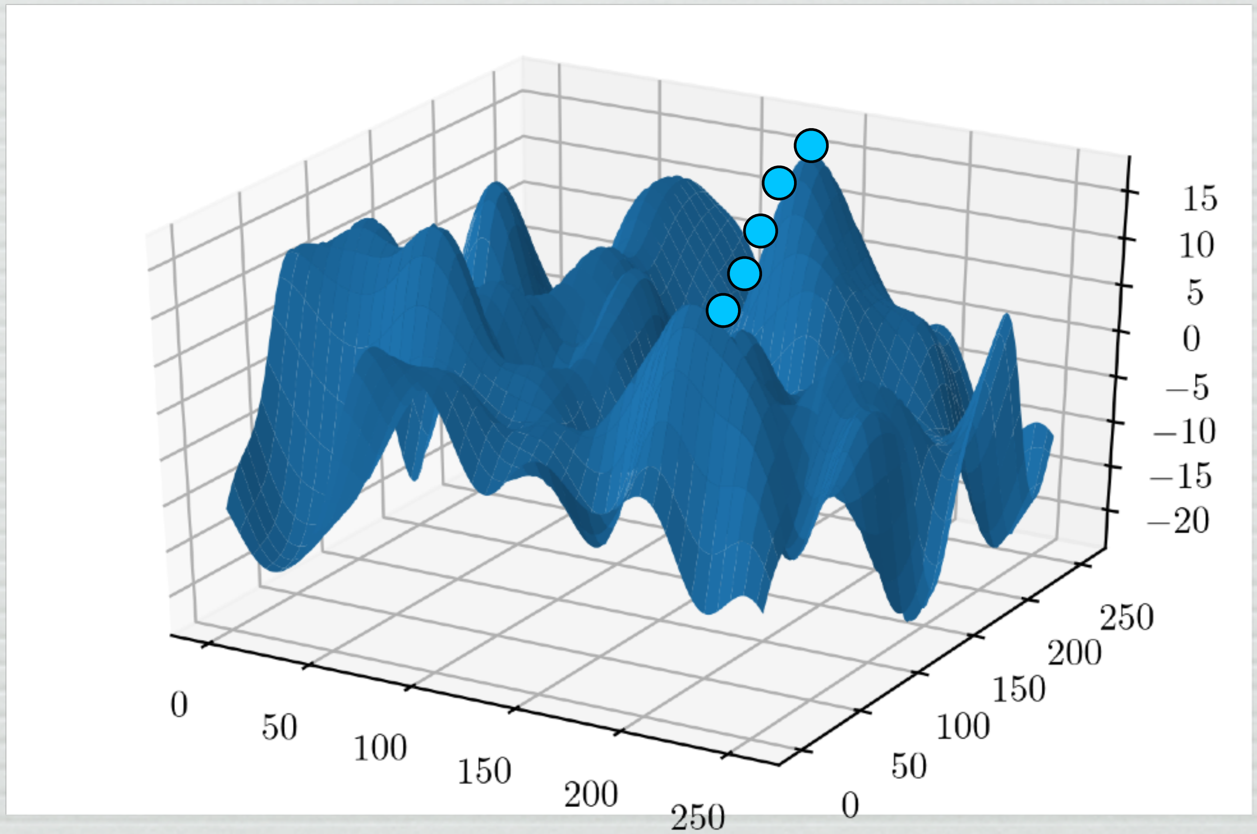
# Conclusions (BHMs)

- (Mostly) Bayesian methods can [optimally] extract cosmological information from astronomical data
- As always, can incorporate prior information on measurements
- More importantly, hierarchical models incorporate dependences of parameters at different levels
  - only need true priors on external parameters
    - i.e., not intermediate maps, power spectra, &c., except for display purposes
- In practice, some steps may be limited by computing power…

# Field Trajectories in a Gaussian Random Potential

- Where did the initial random field come from?
- Assumed to be the result of inflationary dynamics of *one or more* scalar fields in the early Universe
  - There may be many scalar fields at high energy.
  - The physical processes that effect them may be "complex"
    - Model the potential $V(\vec{\varphi})$ as a Gaussian Random Field, isotropic in field space (Euclidean norm on $\vec{\varphi}$)

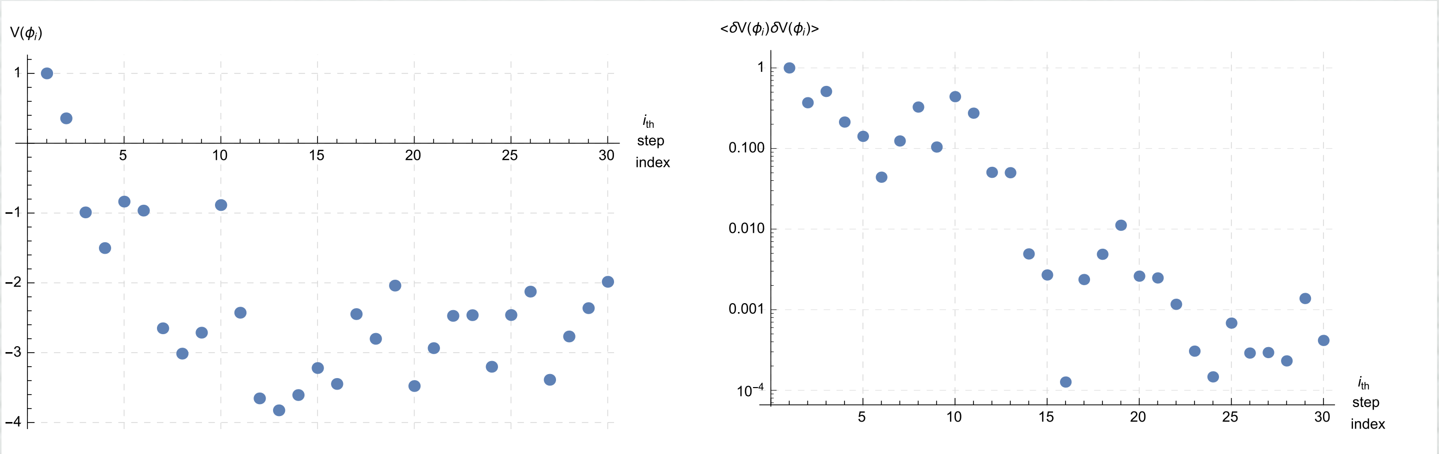


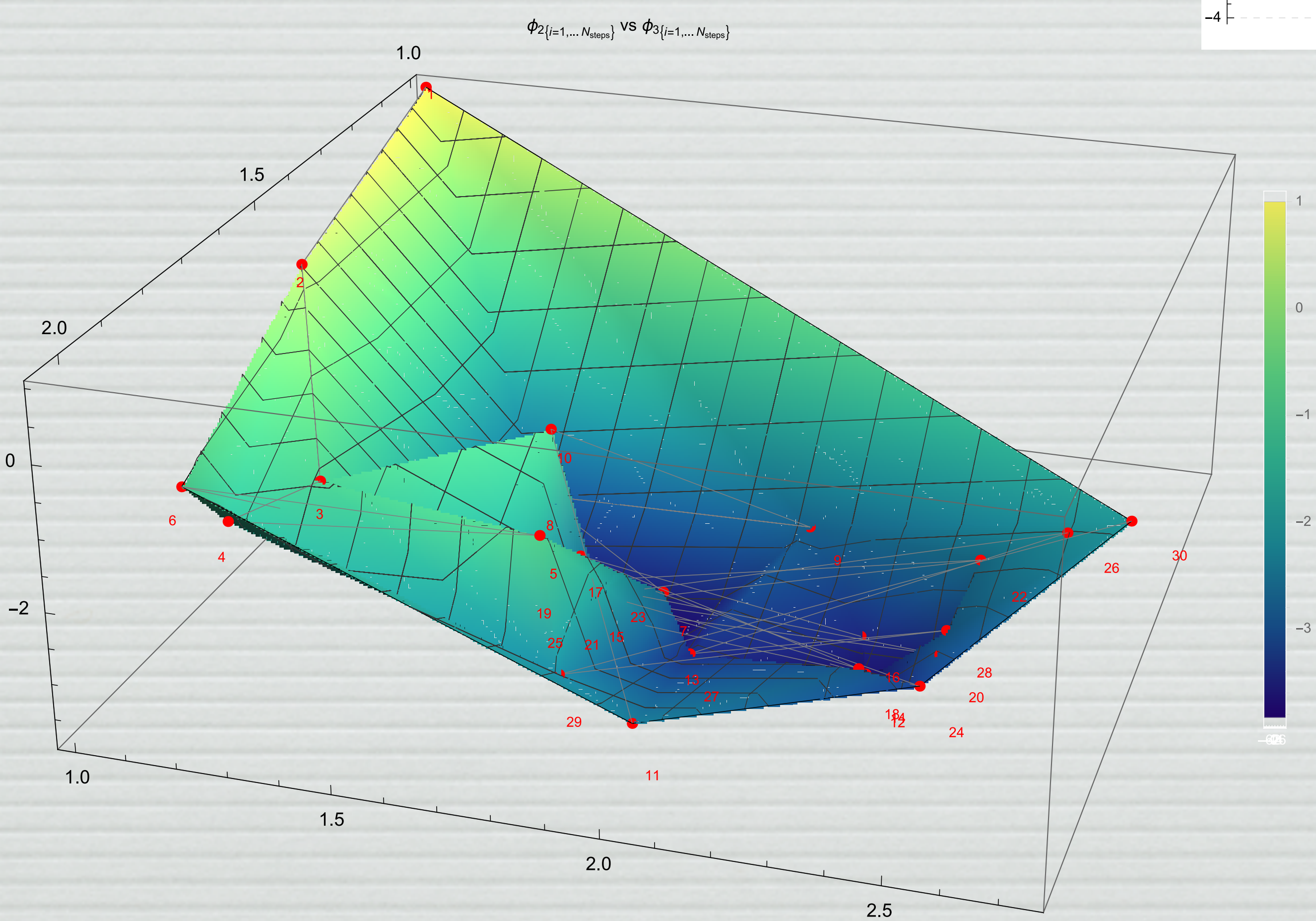



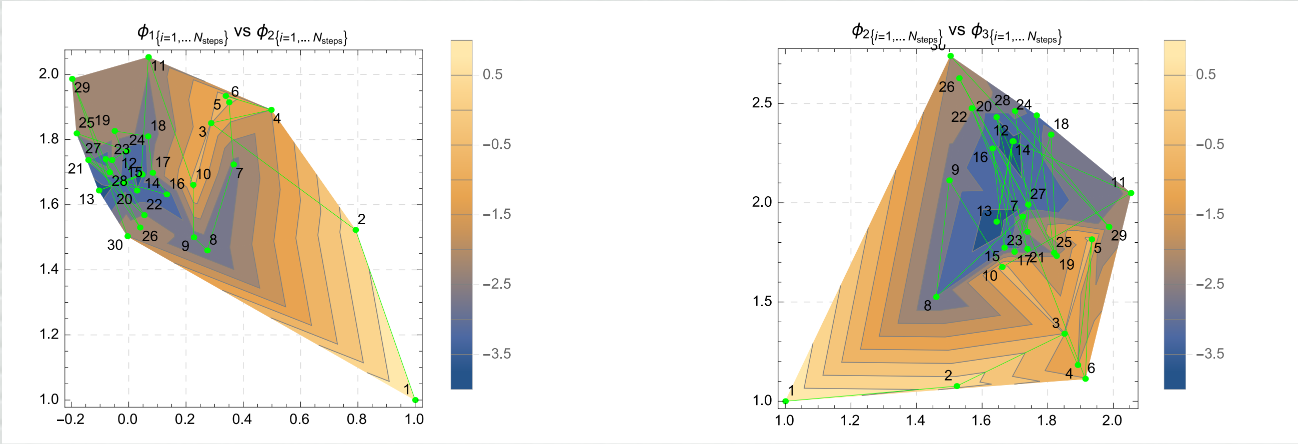
# The scalar potential as a random field

- Model potential $V(\vec{\varphi})$ as a Gaussian Random Field, *isotropic in field space* (Euclidean norm on $\vec{\varphi}$)
  - Search for (e.g.) inflationary trajectories
  - Even with FFTs, expensive in high dimensions, esp. if we need to condition on properties of the potential (e.g., saddle-point inflation)
  - Lots of wasted volume in field space.
- Solution: only realise the potential along the trajectory — constrained realisation/Wiener filter
  - Scales as $O(\#\text{ of points on trajectory})^p$ — naively $p \simeq 4$
  - Wiener formulae for $\langle V_{i+1} | V_{\{1\ldots i\}} \rangle, \langle (\delta V_{i+1})^2 | V_{\{1\ldots i\}} \rangle$
  - Also add derivatives $\nabla_{\varphi_u} V$ & $\nabla_{\varphi_u} \nabla_{\varphi_v} V$ to "signal" and "data"
    - needed for trajectories and predictions
  - related work: Bachlechner 2017, Masoumi, Vilenkin, Yamada 2017

- Add Hamiltonian dynamics of field trajectory
- here, d=8 dimensions, conditioning on $V, \nabla_{\varphi_u} V$



# “Typical” trajectories

- With many fields, we may be able to use the tools of complexity theory to ignore the detailed dynamics of many fields
  - e.g., Dias, Frazer, Marsh 2017 — random matrix theory
    - differs in detail from Gaussian Random fields, but similar in spirit
- work in progress: no conclusions… yet