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# Estimating and Testing for Influence of Specific Structural Factors on Aging of Refrigerated Vehicles

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#### Abstract

This paper introduces a mathematical formulation for the aging curve to study the effects that structural characteristics, as well as operation and maintenance practices, have on the life of refrigerated transport units. From a large database of measurements and real aging data, we fitted the proposed model and we performed inference on parameters using a nonparametric permutation approach. The proposed testing permutation approach, we validated via a Monte Carlo simulation study, appears to be a flexible method, suitable to be implemented for nonlinear models, such as the presented aging curve. The proposed algorithm, although general in its kind, offers a well-rounded approach to make inference on nonlinear model via permutation test. As final results of this, we were able to identify and quantify the most relevant specific structural factors affecting the aging of refrigerated vehicles, which are the presence of meat rails in the roof and the number of leafs for the first and second door.

**Keywords:** aging curve, nonlinear models, permutation tests.

## 1. The aging curve of refrigerated vehicles

The Accord Transport Perishable (ATP), established in 1970 among some European countries and ratified in Italy in 1977, defines the precise structural characteristics of isothermal units at controlled temperature to be used in the transport of perishable products. Over time the insulated capacity of refrigerated transportation vehicles tends to diminish, thus allowing for an increase in the so-called overall coefficient of heat transfer *K* which represents the insulating capacity of the equipment and is defined as

$$K = \frac{W}{S \, \Delta T} \,,$$

where W is the thermal capacity required in a body of mean surface area S to maintain the absolute difference  $\Delta T$  between the mean inside temperature  $T_i$  and the mean outside temperature  $T_e$ , during continuous operation, when the mean outside temperature  $T_e$  is constant. The mean surface area S of the body is the geometric mean of the inside surface area  $S_i$  and the outside surface area  $S_e$  [1].

Several factors contribute to the increase in K: some maybe regarded as structural deformation due to wear and tear while others are related to an increase of water in the polyurethane slab. A mathematical formulation of the aging curve can be taken into consideration to study the effects that structural characteristics, as well as operation and maintenance practices, have on the life of refrigerated transport units. This theoretical

model is derived as a combination of the physical processes involved in the heat transfer within the insulating panel. That model has been calibrated with respect to the data available through the ATP database (for details, see [2]). This model formulation allows to compare the effective age of different type of refrigeration units independently from their manufacturing, structure or use.

Based on this model, the theoretical aging,  $Y_t$  of a refrigeration unit at its time-life t can be computed according to:

$$Y_{t} = 100 \cdot \left( \frac{TC_{t}}{TC_{0}} - 1 \right) \cdot \theta , \qquad (1)$$

where  $TC_0$  and  $TC_t$  are the thermal conductivities respectively computed at time t=0 (i.e. when the refrigeration unit is new) and at time t during the life of the unit (i.e. while the refrigeration unit is in use). These thermal conductivities (see [2]) are obtained using expressions:

$$TC_{0} = \left(\frac{2}{3}\right) \cdot \left(1 - K_{1}\right) \cdot \left(1 - \frac{K_{2}}{2}\right) \cdot K_{3} + K_{1} \cdot \frac{K_{4} \cdot K_{5} + K_{6} \cdot K_{7}}{\left(K_{5} + K_{7}\right)} + \frac{16}{3} \frac{K_{8} \cdot K_{9} \cdot K_{10}}{\sqrt{3.68 \cdot \frac{K_{11}}{K_{12}}}}, \quad (2)$$

$$TC_{t} = \frac{2}{3} \cdot \left(1 - K_{1}\right) \cdot \left(1 - \frac{K_{2}}{2}\right) \cdot K_{3} + K_{1} \cdot \left(K_{13} \cdot t\right) \cdot K_{14} + K_{1} \cdot \left(1 - K_{13} \cdot t\right) \left[\left(\frac{P_{air.t}}{P_{tot.t}}\right) \cdot K_{4} + \left(\frac{P_{R11.t}}{P_{tot.t}}\right) \cdot K_{6}\right] + \frac{16}{3} \frac{K_{8} \cdot K_{9} \cdot K_{10}}{\sqrt{3.68 \cdot \frac{K_{11}}{K_{12}}}},$$
(3)

where (in brackets we report the value of constant  $K_i$ , i=1,...,15)  $K_1$  (0.97) is the porosity of the polyurethane slab, which is given by the ratio of the volume occupied by the air and gas and the total volume of the slab;  $K_2$  (0.85) is the fraction of volume occupied by the solid in the selected geometrical representation of the polyurethane structure. For this particular derivation, the insulating material is represented as in line cubic cells of equal dimensions surrounded by a layer of gas. Constants  $K_3$  (0.29 W/mK),  $K_4$  (0.026 W/mK),  $K_6$  (0.0078 W/mK) and  $K_{14}$  (0.6163 W/mK) are the thermal conductivity coefficients of the solid, the air, the compressed gas (R11) and the water in the polyurethane slab, respectively;  $K_5$  (1000 Pa) and  $K_7$  (90000 Pa) are the initial partial pressures of the air and of the compressed gas, respectively,  $P_{tot,t}$  is the total pressure in the polyurethane slab at time t [Pa], and is given by the sum of  $P_{air,t}$  and  $P_{R11,t}$ , the partial pressures of the air and gas (R11) at time t, respectively;  $P_{air,t}$  and  $P_{R11,t}$  are computed according to the following expressions:

$$P_{air,t} = K_5 + (K_{15} - K_5) \cdot (1 - P(t, A_{air})), \tag{4}$$

$$P_{R11,t} = K_7 \cdot P(t, A_{R11}), \tag{5}$$

where  $K_{13}$  (3.0e-12 mc/mc-s) is the flux of condensed water in the slab;  $K_8$  (5.6704E-8 W/mK<sup>3</sup>) is the Stephan-Boltzmann constant;  $K_9$  (298 K) is the mean temperature of two faces of polyurethane slab;  $K_{10}$  (0.0005 m) is the mean equivalent diameter of the cells and  $K_{11}$  (35 Kg/mc) and  $K_{12}$  (1200 Kg/mc) are the densities of the foam and the solid, respectively.

In (4),  $K_{15}$  (101325 Pa) is the partial pressure of the air outside the refrigeration unit and  $P(t,A_i)$ , where i=2,3 are referred to air and gas respectively, is a function of (6) that computes the partial pressure of a gas at time t based on the parameter  $A_i$  which is in turn given by (7):

$$P(t,\beta_i) = \frac{8\sqrt{2}}{1.01895 \cdot \pi^2} \left[ e^{-\beta_i \tau} - \frac{e^{-9 \cdot \beta_i \tau}}{9} \right], i=2,3,$$
 (6)

$$\beta_i = \left(\frac{\pi}{2K_{16}}\right)^2 \cdot \tilde{\beta}_i, i=2,3, \tag{7}$$

where  $K_{16}$  (0.1 m) is the mean thickness of the polyurethane layer and  $\beta_i$  is a calibrated unknown parameter corresponding to the coefficient of diffusivity of the air or of the compressed gas respectively for  $\tilde{\beta}_2$  and  $\tilde{\beta}_3$ . Equation (6) represents the mean pressure over time and across the polyurethane layer and was derived as a simplification of the diffusivity processes of the gases present in the polyurethane layer, based on Fick's Law [3]. For a more detailed introduction on the mathematical formulation of the aging curve we refer the reader to [2].

From an engineering point of view, the parameter  $\theta$  in (1) can be interpreted as the aging velocity of the refrigeration unit. To account for structural characteristics and specifications that might contribute to the overall aging of the isothermal unit, we can represent the expected value of  $\theta$  as an additional linear model such as in (8). This allows evaluating the aging results of refrigerated transportation systems that might differ by structural factors or by method of employment.

$$E(\theta) = \beta_1 + \beta_4 X_1 + \sum_{j=1}^{6} \beta_{5j} X_{2j} + \beta_6 X_3 + \beta_7 X_4 + \beta_8 X_5 + \beta_9 X_6 + \beta_{10} X_7 + \beta_{11} X_8 + \beta_{12} X_9,$$
(8)

where  $\beta_1$  is a constant,  $\beta_i$ , i = 4,6,...,12, and  $\beta_{2i}$ , j = 1,...,6 are parameters related to the possibly relevant factors potentially affecting the aging velocity. The variables under study (some binary or nominal categorical, some other numerical) are:  $X_1$ : Type of use;  $X_{2j}$ , j = 1,...,6: Type of transported perishables (j = 1 Dairy, Fish, Fruit and vegetables, General Perishable, Meat, Poultry);  $X_3$ : Number of leafs for the second door;  $X_4$ : Number of leafs for the first door;  $X_5$ : Total perimeter doors;  $X_6$ : Presence of refrigerating unit in the vehicle;  $X_7$ : Presence of meat rails in the roof of the vehicle;  $X_8$ : Average thermal thickness,; $X_9$ : Average geometrical thickness.

# 2. Descriptive data analysis

For this study, a database of nearly 3,300 records of measurements and real aging data, was available from the Laboratories of Chill Techniques (LCT) within the Italian National Research Council, Construction Technologies Institute, Padova, Italy, for the decade 1998 to 2007. The LCT is one of the centres certified to measure the overall coefficient of heat transfer in transported refrigeration systems.

In fact, the ATP prescribes specific rules to employ and to maintain refrigerated transportation units. Among these are quality compliance standards and quality control tests and their frequency. In particular, checks for conformity with the standards prescribed in the accord are to be made: (a) before the equipment is put into service; (b) at least once every six years and (c) whenever required by the competent authority. Equipment checks are performed in insulated chambers by measuring the overall coefficient of heat transfer in specific operating conditions and in selected environment settings. In particular, insulating capacity is measured in continuous operation either by the internal heating method (i.e. a heating source is placed inside the empty isothermal unit and the heat transferred to the outside is measured) or by the internal cooling method (i.e. a cooling source is placed inside the refrigeration system and the heat exchange between inside and outside is measured).

At the LCT, tests are conducted either on road or railroad refrigerated truck in a 28 mlong insulated chamber. Measurements and environment conditions within the insulated chamber are computer controlled. The aging process in insulated units is measured as the increase in the heat transferred between the inside and the outside of the unit also known as increase in the thermal conductivity.

As first basic statistics on the available dataset, a classification of these data based on age of the vehicles and year in which the observation was taken is presented in Table 1. The last column in Table 1 "All G." represents the number of vehicles of a specific age over the course of the ten-year monitoring period (from 1998 to 2007) for which data are available. It is evident from this table that for several "Age" groups there are no sufficient data to calibrate the model. Therefore, only the subset from Age = 6 to Age = 20 will be used to fit the aging model in this study.

Table 1. Counting of vehicle age in each year

Age	1998	1999	2000	2001	2002	2003	2004	2005	2006	2007	All G.
1-5	1	1	0	1	1	1	3	1	1	1	11
6-10	89	50	30	57	27	36	41	47	38	42	457
11-15	194	207	155	247	260	255	255	268	274	215	2330
16-20	25	29	33	23	25	44	41	62	86	71	439
21-25	5	3	4	6	7	6	4	5	15	13	68
26-30	2	0	0	1	0	2	0	0	2	3	10
Total	316	290	222	335	320	344	344	383	416	345	3315

Figure 1 displays a graphical summary of the observed measurements of aging for the 3315 vehicles. Note that the shape of the frequencies histogram is not symmetric (skewness = 1.57).

#### Descriptive Statistics

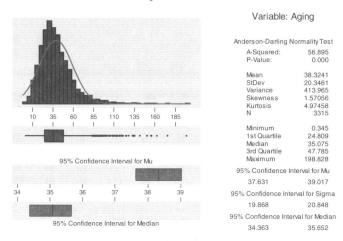


Figure 1. Graphical summary of Ageing distribution.

Figure 2 represents the scatter plot of observed ageing versus vehicle's age, for the subset of vehicles aged 6-20 years. Red dots, connected by a solid line, represent the sample mean of ageing by vehicle's age.

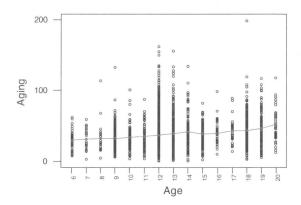


Figure 2. Scatter plot of Ageing versus Age (subset Age 6-20).

Tables 2 and 3 display descriptive statistics of the specific structural factors of refrigerated vehicles. More specifically, Table 2 summarises the counting of vehicles by specific structural factors of categorical-type, while Table 3 shows counting, sample mean and standard deviation by specific numerical-type structural factors.

Table 2. Counting of vehicles by specific structural factors (categorical type) and by year.

Factor / Category	1998	1999	2000	2001	2002	2003	2004	2005	2006	2007	All G.
Type of use											
Private activity								240	242	199	681
Transportation activity								142	174	146	462
missing	316	290	222	335	320	344	344	1			2172
Type of perishables											
Dairy	41	30	26	42	40	41	41	48	62	46	417
Fish	46	38	20	39	35	55	42	49	53	33	410
Fruit and vegetables	2	2	4	15	32	25	44	39	55	58	276
General Perishable	22	20	37	41	16	31	44	56	41	29	337
Meat	66	63	45	57	46	55	65	72	64	58	591
Poultry	19	15	10	12	9	26	12	24	29	23	179
missing	120	122	80	129	142	111	96	95	112	98	1105
Refrigerating unit											
No	20	21	11	17	22	32	34	29	27	37	250
Yes	296	269	211	318	298	312	310	354	389	308	3065
Meat rails in the roof											
No	230	191	163	225	251	279	285	320	333	277	2554
Yes	86	99	59	110	69	65	59	63	83	68	761

**Table 3.** Basic statistics of vehicles by specific structural factors (numerical type) and by year.

es or ,	CILICIC	D U J	Decilia	Derere		ore laser		71-7	J ,	/
1998	1999	2000	2001	2002	2003	2004	2005	2006	2007	All G.
316	290	222	335	320	344	342	381	415	343	3308
						2	2	1	2	7
0.98	0.98	1.00	1.02	1.06	1.06	1.13	1.11	1.11	1.12	1.06
0.40	0.46	0.51	0.39	0.44	0.40	0.58	0.58	0.56	0.54	0.50
316	289	221	332	316	343	344	382	415	342	3300
	1	1	3	4	1		1	1	3	15
0.85	0.88	0.79	0.83	0.75	0.84	0.78	0.71	0.66	0.63	0.77
0.72	0.71	0.68	0.67	0.74	0.93	0.71	0.83	0.68	0.69	0.75
				320	344	344	383	411	345	2147
316	290	222	335						5	1168
				16223	16744	17507	16881	16382	16573	16716
				5193	5206	7041	8346	6349	6841	6636
316	290	222	335	320	344	344	383	416	345	3315
85.3	84.9	83.2	83.0	78.7	79.5	78.5	78.2	77.7	77.0	80.3
10.8	9.5	10.2	9.3	9.6	9.5	8.7	8.7	8.0	7.8	9.6
275	269	208	304				383	416	345	2200
41	21	14	31	320	344	344				1115
85.6	85.7	84.7	84.5				82.0	81.7	80.8	83.3
10.5	9.3	13.5	9.0				8.9	8.2	8.7	9.7
	316 0.98 0.40 316 0.85 0.72 316 85.3 10.8 275 41 85.6	1998 1999  316 290  0.98 0.98 0.40 0.46  316 289 1 0.85 0.88 0.72 0.71  316 290  85.3 84.9 10.8 9.5  275 269 41 21 85.6 85.7	1998         1999         2000           316         290         222           0.98         0.98         1.00           0.40         0.46         0.51           316         289         221           1         1         1           0.85         0.88         0.79           0.72         0.71         0.68           316         290         222           85.3         84.9         83.2           10.8         9.5         10.2           275         269         208           41         21         14           85.6         85.7         84.7	1998         1999         2000         2001           316         290         222         335           0.98         0.98         1.00         1.02           0.40         0.46         0.51         0.39           316         289         221         332           1         1         3         0.85         0.88         0.79         0.83           0.72         0.71         0.68         0.67           316         290         222         335           85.3         84.9         83.2         83.0           10.8         9.5         10.2         9.3           275         269         208         304           41         21         14         31           85.6         85.7         84.7         84.5	1998         1999         2000         2001         2002           316         290         222         335         320           0.98         0.98         1.00         1.02         1.06           0.40         0.46         0.51         0.39         0.44           316         289         221         332         316           0.85         0.88         0.79         0.83         0.75           0.72         0.71         0.68         0.67         0.74           316         290         222         335         16223           5193           316         290         222         335         320           85.3         84.9         83.2         83.0         78.7           10.8         9.5         10.2         9.3         9.6           275         269         208         304         4         4         31         320           85.6         85.7         84.7         84.5         84.5         84.5         84.5         84.5	1998         1999         2000         2001         2002         2003           316         290         222         335         320         344           0.98         0.98         1.00         1.02         1.06         1.06           0.40         0.46         0.51         0.39         0.44         0.40           316         289         221         332         316         343           0.85         0.88         0.79         0.83         0.75         0.84           0.72         0.71         0.68         0.67         0.74         0.93           316         290         222         335         16223         16744           5193         5206           316         290         222         335         320         344           85.3         84.9         83.2         83.0         78.7         79.5           10.8         9.5         10.2         9.3         9.6         9.5           275         269         208         304         44         121         14         31         320         344           85.6         85.7         84.7         84.5         85.6	1998         1999         2000         2001         2002         2003         2004           316         290         222         335         320         344         342           0.98         0.98         1.00         1.02         1.06         1.06         1.13           0.40         0.46         0.51         0.39         0.44         0.40         0.58           316         289         221         332         316         343         344           0.85         0.88         0.79         0.83         0.75         0.84         0.78           0.72         0.71         0.68         0.67         0.74         0.93         0.71           316         290         222         335         16223         16744         17507           316         290         222         335         320         344         344           316         290         222         335         320         344         344           85.3         84.9         83.2         83.0         78.7         79.5         78.5           10.8         9.5         10.2         9.3         9.6         9.5         8.7	1998         1999         2000         2001         2002         2003         2004         2005           316         290         222         335         320         344         342         381           0.98         0.98         1.00         1.02         1.06         1.06         1.13         1.11           0.40         0.46         0.51         0.39         0.44         0.40         0.58         0.58           316         289         221         332         316         343         344         382           0.85         0.88         0.79         0.83         0.75         0.84         0.78         0.71           0.72         0.71         0.68         0.67         0.74         0.93         0.71         0.83           316         290         222         335         16223         16744         17507         16881           5193         5206         7041         8346           316         290         222         335         320         344         344         383           85.3         84.9         83.2         83.0         78.7         79.5         78.5         78.2	1998         1999         2000         2001         2002         2003         2004         2005         2006           316         290         222         335         320         344         342         381         415           0.98         0.98         1.00         1.02         1.06         1.06         1.13         1.11         1.11           0.40         0.46         0.51         0.39         0.44         0.40         0.58         0.58         0.56           316         289         221         332         316         343         344         382         415           0.85         0.88         0.79         0.83         0.75         0.84         0.78         0.71         0.66           0.72         0.71         0.68         0.67         0.74         0.93         0.71         0.83         0.68           316         290         222         335         16223         16744         17507         16881         16382           5193         5206         7041         8346         6349           316         290         222         335         320         344         344         383         416 <td>316         290         222         335         320         344         342         381         415         343           0.98         0.98         1.00         1.02         1.06         1.06         1.13         1.11         1.11         1.11         1.12           0.40         0.46         0.51         0.39         0.44         0.40         0.58         0.58         0.56         0.54           316         289         221         332         316         343         344         382         415         342           1         1         3         4         1         1         1         3         342         343         344         382         415         342           0.85         0.88         0.79         0.83         0.75         0.84         0.78         0.71         0.66         0.63           0.72         0.71         0.68         0.67         0.74         0.93         0.71         0.83         0.69           316         290         222         335         320         344         344         383         411         345           85.3         84.9         83.2         83.0</td>	316         290         222         335         320         344         342         381         415         343           0.98         0.98         1.00         1.02         1.06         1.06         1.13         1.11         1.11         1.11         1.12           0.40         0.46         0.51         0.39         0.44         0.40         0.58         0.58         0.56         0.54           316         289         221         332         316         343         344         382         415         342           1         1         3         4         1         1         1         3         342         343         344         382         415         342           0.85         0.88         0.79         0.83         0.75         0.84         0.78         0.71         0.66         0.63           0.72         0.71         0.68         0.67         0.74         0.93         0.71         0.83         0.69           316         290         222         335         320         344         344         383         411         345           85.3         84.9         83.2         83.0

Linear correlation between observed aging and other specific structural factors of refrigerated vehicles are presented in Table 4 (missing values have been handled by

using pairwise deletion). As highlighted in the table (significant correlations in bold, at  $\alpha$ -level=0.05), and as we can reasonably expecting there are several factors significantly correlated with aging.

**Table 4.** Correlations between aging and other specific structural factors.

Factor	All cases	subset Age=12
Type of use	0.08	0.08
No. of leafs 2-nd door	-0.06	-0.02
No. of leafs 1-st door	0.03	0.01
Perimeter doors	0.22	0.25
Refrigerating unit	0.25	0.25
Meat rails in the roof	0.04	0.05
Thermal thickness	0.14	0.18
Geometrical thickness	0.19	0.22

#### 3. Parameter Estimation

Since the distribution of aging is far to be symmetric (Figure 1), as confirmed by the fact that mean and median of the aging distribution are respectively 38.3 and 35.1, we prefer to perform the parameter estimation procedure via minimizing the sum of the absolute values of the residuals. For this goal, suitable MatLab routines were implemented in order to numerically estimate, using the Nelder-Mead algorithm [4], the three parameters of the aging nonlinear model presented in the first section of the present paper. Note that, as first estimation attempt, we hold fixed the velocity parameter  $\theta$ . Table 5 displays the estimated values of the three aging model parameters, by several periods of increasing length.

**Table 5.** Estimate of model parameters for several periods via minimizing of sum of abs of residuals.

Period	$\beta_2$ (air diffusivity)	$\beta_3$ (gas diffusivity)	$\theta$ (velocity)	N	Predicted value for age=12
1998-1999	5.3E-12	1.0E-18	1.27	606	38.0
1998-2000	6.6E-12	8.0E-19	1.13	828	38.6
1998-2001	6.3E-12	9.0E-19	1.15	1163	38.4
1998-2002	6.3E-12	8.1E-19	1.10	1483	36.7
1998-2003	8.3E-12	5.7E-19	0.90	1827	34.9
1998-2004	1.0E-11	7.5E-19	0.81	2171	34.2
1998-2005	1.1E-11	7.8E-19	0.79	2554	34.2
1998-2006	1.1E-11	8.3E-19	0.79	2970	34.4
1998-2007	1.2E-11	6.9E-19	0.78	3315	34.7

In order to measure a goodness of fit of estimated model, we can calculate the sum of LAD - least absolute deviation of residuals which is equal to 14.4.

Hence, we fixed the estimated values of air and gas diffusivity ( $\hat{\beta}_2 = 1.2\text{E-}11$ ,  $\hat{\beta}_3 = 6.9\text{E-}19$ ), then we proceeded to analyzing in details the relation between velocity parameter and the effect of specific structural factors of refrigerated vehicle.

In Table 6 and 7 the velocity parameter  $\theta$  has been separately estimated by subgroup of specific categorical-type structural factors and as a linear combination of each other numerical-type specific structural factors respectively.

**Table 6.** Estimate of velocity in subgroups of specific categorical-type structural factors.

Structural factor	$\theta$ (velocity)	N	Predicted value for age=12
Type of use			
Transportation activity	0.78	462	34.8
Private activity	0.80	681	36.0
Type of perishables			
Dairy	0.82	417	36.9
Fish	0.80	410	35.6
Fruit and vegetables	0.75	276	33.3
General Perishable	0.73	337	32.6
Meat	0.84	591	37.5
Poultry	0.69	179	31.0
Refrigerating unit			
No	0.67	250	29.8
Yes	0.78	3065	35.1
Meat rails in the roof			
No	0.74	2554	33.2
Yes	0.90	761	40.1

**Table 7.** Dependence of velocity from numerical-type specific structural factors.

Structural factor	a	b	N	mean	std. dev.
	I	model: $\beta_1 = a + b * f$	actor		
No. of leafs 2-nd door	0.72	0.05197	3315	1.071	0.530
No. of leafs 1-st door	0.68	0.13131	3315	0.784	0.798
	model:	$\beta_1 = a + b * standard$	lized(factor)		
Perimeter doors	0.82	0.05	2134	16716.3	6635.62
Thermal thickness	0.74	0.13	1752	80.32	9.60
Geometrical thickness	0.73	0.15	2168	83.25	9.70

It is possible to consider a more realistic aging curve model where we take into account simultaneously for all available specific structural factors of refrigerated vehicle, according to the linear model presented in Equation (8).

Table 8 summarises the estimates of each parameter for specific structural factors, along with their own bootstrap standard error [5].

As a measure of goodness of fit of estimated model, we can calculate the sum of least absolute deviation (LAD) of residuals which is 13.2. Hence, when comparing this value with those of the three-parameter aging curve model, we can note that the reduction of LAD of residuals is 8.5%.

**Table 8.** Estimate of parameters for specific structural factors.

Factor	Parameter	Estimate	Bootstrap SE of estimate
Constant	$\beta_1$	0.483	0.0078
Type of use	$eta_4$	0.071	0.0024
Type of perishables			
Dairy	$\beta_{51}$	0.027	0.0045
Fish	$\beta_{52}$	0.032	0.0060
Fruit and vegetables	$\beta_{53}$	0.007	0.0030
General Perishable	$\beta_{54}$	0.012	0.0022
Meat	$\beta_{55}$	-0.056	0.0041
Poultry	$\beta_{56}$	-0.039	0.0032
No. of leafs 2-nd door	$\beta_6$	0.117	0.0029
No. of leafs 1-st door	$\beta_7$	0.140	0.0019
Perimeter doors	$eta_8$	0.032	0.0014
Refrigeration unit	$\beta_9$	0.083	0.0033
Meat rails in the roof	$\beta_{10}$	0.158	0.0025
Thermal thickness	$\beta_{11}$	0.051	0.0020
Geometrical thickness	$\beta_{12}$	-0.016	0.0026

# 4. Inference on parameters of aging model via permutation tests

Significance testing on nonlinear model parameters within the parametric approach is traditionally a difficult topic due to the complexity of studying the null distribution of test statistics. The main concern to determine the accuracy and the significance of the estimates for the nonlinear regression coefficients has been addressed by literature to classical asymptotic procedures, such as Wald test and likelihood test, or to resampling methods, such us bootstrap and jackknife [6,7] or, more recently, to Bayesian inference [8]. The validity of asymptotic testing approach is obviously connected with the assumption on random error (usually normal). Moreover, when the sample size is not particularly large asymptotic assumption may be suspected. With reference to bootstrap and jackknife approaches, we highlight that they are basically heuristic procedures and, more specifically, these resampling techniques are neither conditional nor unconditional inferential method. In contrast, permutation methods are nonparametric conditional procedures where conditioning is performed with respect to the sub-space associated with the set of sufficient statistics under the null hypothesis for all nuisance entities, including the underlying, known or unknown, distribution [9,10].

The importance of the permutation approach in resolving a large number of inferential problems is well-documented in the literature, where the relevant theoretical aspects emerge, as well as the extreme effectiveness and flexibility from an application point of view. Provided that data are sampled from only one underlying distribution, the permutation tests are nonparametric procedures conditional on the observations, which are always a set of sufficient statistics under the null hypothesis, for any underlying distribution. Supposing that the null hypothesis implies exchangeability of data with respect to the levels of factors, the permutation tests are not only independent of the likelihood model relative to population distribution, but also enjoy some important properties: they are exact tests, enjoy the property of similarity and are conditionally unbiased and consistent procedures. Furthermore, in rather general conditions, it is possible to weakly extend the conclusions of the permutation tests to the reference population [10].

Hence, permutation approach is a general and flexible method, suitable to be implemented for both linear and nonlinear models. Several authors propose the application of permutation test for testing on single coefficients of multiple linear regression model [11]. Mainly, literature proposals suggest to permute residuals according some specific permutation strategy [12,13,14,15]. Note that, since the null hypothesis actually allows for exchangeability of random errors, a permutation test based on residuals is an approximated conditional inferential procedure. Anderson and Legendre [16] and Anderson and Robinson [17] performed a simulation study on several permutation test proposals for linear model showing that all approaches are generally asymptotically equivalent. However, the reduced model permutation approach by Freedman and Lane [12] showed the most coherent and reliable results.

Although recently Chiou and Mueller [18] addressed the idea of using residuals of functional regression to develop a randomization test for functional regression, nowadays permutation tests did not have give raise to notable interest in the field of nonlinear model. Permutation tests for significance testing on single coefficients of a nonlinear model have been only occasionally proposed for hypothesis testing on parameters of piecewise regression model [19], canonical correlation analysis [20], shape analysis [21] and Pharmacokinetics [22].

With the aim of performing inference on parameters affecting the aging of refrigerated vehicles, we decided to fit to our aging nonlinear model the approach proposed by [12]. For this purpose, let the null hypothesis of interest be  $H_0$ :  $\beta_k = 0$  vs.  $H_1$ :  $\beta_k \neq 0$ , where  $\beta_k$  is a single individual parameter (a scalar) from  $\beta$ , the whole set of parameters in the nonlinear model at hand. The steps of the proposed algorithm are the followings:

- o estimate the parameter vector  $\boldsymbol{\beta}$  under  $H_1$ , i.e.  $\hat{\boldsymbol{\beta}}_1$ , and store the value of interest:  $\hat{\boldsymbol{\beta}}_k$ ;
- o estimate the parameter vector  $\boldsymbol{\beta}$  under  $H_0$ , i.e.  $\hat{\boldsymbol{\beta}}_0$  (it means that the k-th independent variable has been excluded from the model);
- o calculate the vector of estimated response values (under  $H_0$ ):  $\hat{\mathbf{Y}}_0 = f(X; \hat{\boldsymbol{\beta}}_0)$ ;
- o calculate the vector of residuals:  $\mathbf{R}_0 = \mathbf{Y} \hat{\mathbf{Y}}_0$  (under  $H_0$ );
- o randomly permute the elements of  $\mathbf{R}_0$  into  $\mathbf{R}_0^*$ ;
- o calculate  $\mathbf{Y}_0^* = \hat{\mathbf{Y}}_0 + \mathbf{R}_0^*$ ;
- o from  $\mathbf{Y}_0^*$ , re-estimate  $\hat{\boldsymbol{\beta}}_1^*$ , i.e. the parameter vector of  $\boldsymbol{\beta}$  and store the value  $\hat{\boldsymbol{\beta}}_k^*$ ;
- o carry out B-1 independent repetitions of the last three steps, so that we have  $\hat{\beta}_{kj}^*$ , j=1,...,B, (i.e. a random sampling from the permutation distribution of  $\hat{\beta}_k$ );
- o the permutation estimated p-value  $\hat{p}$  for  $\beta_k$  is given by

$$\hat{p} = \# (|\hat{\beta}_k *| \ge |\hat{\beta}_k|) / B;$$

o if  $\hat{p} < \alpha$ , the null hypothesis  $H_0$  is rejected at significance level  $\alpha$ .

In case the interest is on simultaneously testing for a subset of parameters, the permutation test can be carried out with the same rationale by using an appropriate test statistic, for example  $S = (SSE_0 - SSE_1)/SSE_1$  where  $SSE_0$  and  $SSE_1$  are respectively the sum of square of residuals from models under  $H_0$  and under  $H_1$ .

## 5. Simulation study

In order to validate the proposed approach, we carried out a suitable Monte Carlo simulation study. For this goal let us consider the following nonlinear model:

$$Y_i = \frac{\theta}{1 + \beta_2 e^{-\beta_3 X_{ii}}} + \varepsilon_i, i=1,...,n.$$
(9)

Moreover, we assume that

$$\theta = \beta_1 + \beta_4 X_{2i} + \beta_5 X_{3i} + \beta_6 X_{4i} + \beta_7 X_{5i}$$
 (10)

where  $X_2$  and  $X_3$  are dummy variables representing some sort of possible fixed effects towards Y while  $X_4$  and  $X_5$  are numerical covariates. Note that expression (9) represents the well-known three-parameter logistic model while equation (10) allow us to change the increasing s-shape form of (9) in relation of several factors. In practice, the model for simulation study mimics in a more simple way the aging model presented in the first part of the work.

We set the value of parameters as follows:  $\beta_1 = \beta_2 = \beta_3 = 1$ ,  $\beta_4 = \beta_6 = 0$ ,  $\beta_5 = 0.05$ ,  $\beta_7 = 0.03$  and we generate the value of numerical covariates and random errors as follows:  $X_{1i}$  are i.i.d. from Uniform[0,4],  $X_{2i}$  and  $X_{3i}$  are i.i.d. from Bernuolli[1/2],  $X_{4i}$  and  $X_{5i}$  are i.i.d. from a N(0,1) and  $\varepsilon_i$  are i.i.d. from a N(0,0.1), since such distributions seem appropriate to represent real data configurations.

Note that, in our simulation parameters  $\beta_5$  and  $\beta_7$  represent a possible different model which we would like to identify using the proposed permutation testing procedure.

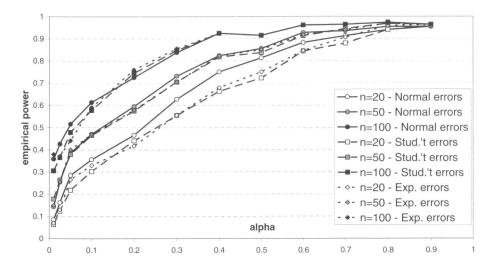
Suitable MatLab routines were implemented in order to numerically estimate the parameters of the nonlinear model using the Nelder-Mead algorithm [4] and to execute the proposed permutation test. These programs are available upon request by authors.

The considered simulation setting consists of 1000 Monte Carlo simulations for the generation of 100 observations (n=50), where the true values are added to standard normally distributed random errors. For each one of the 1000 simulated data we separately estimated the permutation p-values (with 1000 random permutations) following the proposed algorithm for each hypothesis of interest. The rejection rates under each specific alternative are displayed in Table 9.

**Table 9.** Permutation test rejection rates.

Hypothesis		1 6610		01111010		Al	pha					
to be tested	.01	.025	.05	.1	.2	.3	.4	.5	.6	.7	.8	.9
empirical size												
$\beta_4 = 0$	.013	.033	.052	.117	.196	.307	.399	.458	.588	.719	.804	.928
$\beta_6 = 0$	.013	.020	.065	.111	.203	.333	.444	.516	.601	.706	.791	.915
empirical power												
$\beta_4 = \beta_5 = \beta_6 = \beta_7 = 0$	.109	.139	.179	.348	.512	.612	.711	.786	.836	.896	.945	.970
$\beta_5 = 0$	.105	.170	.255	.340	.549	.634	.699	.765	.824	.869	.922	.954
$\beta_7 = 0$	.144	.255	.386	.471	.595	.732	.824	.856	.928	.935	.954	.954

Note that the proposed permutation tests show in general an appropriate behavior either for maintaining the proper nominal level under the null hypothesis and for identifying the true alternatives. For example, when setting the significance level  $\alpha$  at 0.05, we rejected the false null hypothesis 25.5% times for  $\beta_5 = 0$  and 38.6% times for  $\beta_7 = 0$ . We performed also additional simulations (not reported here in details) with different sample sizes (n=20, n=100) and different random distributions (Exponential and Student's t with 2 d.f. random errors). This additional results essentially confirmed findings on Table 9. A summary of these additional simulations (for parameter  $\beta_7$ ) are displayed in Figure 3.



**Fig. 3.** Freedman and Lane empirical power for nonlinear testing on  $\beta_7$  by different sample size and by different random distribution.

# 6. Application of permutation tests for hypothesis testing on parameters of aging model

Hence, we can carry on to appropriately apply the Freedman and Lane [12] approach for inference on parameters of aging nonlinear model. Presented in Table 10 are the permutation *p*-values (with 1000 independent random permutations), obtained for each factor affecting the aging curve of refrigerated vehicles. Permutation *p*-values has been calculated by stratifying the dataset into one-year periods. The reason is because within a shorter period data are more comparable and we could highlight possible time effects, hence inference is more reliable.

Results confirm that the most relevant structural factors, affecting the aging curve of refrigerated vehicles are

- 1. Meat rails in the roof;
- 2. Number of leafs for the first and second door:
- 3. Perimeters doors;
- 4. Thermal thickness:

### 5. Refrigeration unit.

**Table 10.** Permutation *p*-values associate with the analyzed structural factors.

Г	Year										
Factor	1998	1999	2000	2001	2002	2003	2004	2005	2006	2007	all
Type of use	-	-	-0	-	-	-	-	0.029	0.006	0.347	0.024
Type of perishables	.072	.006	.868	.484	.138	.216	.840	.196	.802	.341	.030
# of leafs 2-nd door	.156	.054	.138	.019	.018	.617	.144	.043	.000	.000	.000
# of leafs 1-st door	.000	.000	.000	.000	.000	.108	.000	.000	.000	.000	.000
Perimeter doors	-	-			.263	.012	.011	.058	.078	.461	.000
Refrigeration unit	.096	.150	.054	.396	.000	.335	.160	.000	.311	.048	.006
Meat rails in the roof	.000	.323	.120	.283	.000	.000	.011	.000	.000	.012	.000
Thermal thickness	.000	.090	.030	.164	.563	.000	.305	.072	.521	.030	.000
Geom. thickness	.012	.431	.287	.352	-	-	-	.326	.299	.024	.144

Note: the symbol "-" means that no data were available (see Table 2 and 3).

Results in Table 10 may suggest several practical conclusions. In fact, the more relevant factors affecting the aging curve of refrigerated vehicles are those with a smaller permutation *p*-value: *Number of leafs for the first door* and *Meat rails in the roof*. When these structural characteristics are introduced in the refrigerated vehicle we can expect a great changing in the related aging curve.

As illustration of partial results, presented in Figure 4 we have four estimated aging curves considering the contribution of two significant detected factors. More precisely:

- A: Meat rails in the roof = NO, Number of leafs for the first door = 0
- B: Meat rails in the roof = YES, Number of leafs for the first door = 0
- C: Meat rails in the roof = NO. Number of leafs for the first door = 2
- D: Meat rails in the roof = YES, Number of leafs for the first door = 2

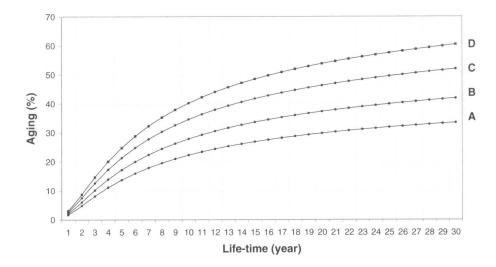


Fig. 4. Four estimated aging curve by considering the contribution of few significant detected factors

### 7. Conclusions

In this work we presented a theoretical aging curve of refrigerated vehicles as a combination of the physical processes involved in the heat transfer within the insulating panel. Several statistical analyses have been performed from a large database of measurements of real aging data, which is available from the Laboratories of Chill Techniques (LCT) within the Italian National Research Council, Construction Technologies Institute, Padova, Italy. In our study the parameters of aging nonlinear model have been estimated, by taking into account for several specific structural factors of refrigerated vehicle. In order to make inference on parameters of structural factors, we applied a proper algorithm within the nonparametric permutation framework. Hence, by applying permutation tests on the problem of the aging of refrigerated vehicles for transport of perishable products we can state that the most relevant specific structural factors affecting the aging are the presence of meat rails in the roof and the number of leafs for the first and second door.

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