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Title of abstract

Author's name [1]

It is possible to write an abstract without sections.

1 Introduction

This paper deals with E-optimal designs for polynomial regression on arbitrary segments. In a number of works the designs were found in an explicit form for segments of special types. Kovrigin [4] showed that for $[-1, 1]$ an E-optimal design is located in the extremum points of the Tchebysheff polynomial. It is worth to mention that only short abstract was published. Pukelsheim and Studden [7] rediscovered this result for the more general case of the truncated E-criterion. Heiligers [1] showed that the result remains true for segments $[a, b]$, were a and b have the same sign and for sufficiently small segments $[-r, r]$. He also showed that for every r the multiplicity of eigenvalues of information matrices for any experimental design have multiplicity no more than 2 (in [5] this result was generalized for arbitrary segments). A number of additional particular results was obtained also in Heiligers [2].

2 Formulation of the problem

Let the experimental results $\{y_i\}$ be described by the standard linear regression equation

$$y_j = \Theta^T f(x_j) + \epsilon_j, \quad j = 1, 2, \ldots, n, \tag{1}$$

where $\Theta^T = (\Theta_1, \ldots, \Theta_m)$ is the vector of unknown parameters, $f(x) = (f_1(x), \ldots, f_m(x))^T$ is the vector of known functions assumed to be linearly independent and continuous on a compact set χ. The random errors $\{\epsilon_j\}$ are such that $E\epsilon_i = 0, E\epsilon_i\epsilon_j = \sigma^2\delta_{ij}(i, j = 1, \ldots, n)$, δ_{ij} is the Kroneker symbol and $\sigma^2 > 0$ is unknown constant.

Any discrete probability measure on $\chi : \xi = \{x_1, \ldots, x_n; \mu_1, \ldots, \mu_n\}$, $x_i \in \chi, x_i \neq x_j(i \neq j), \mu_i > 0, \sum \mu_i = 1$ is called an experimental design. Let us consider the matrix

$$M(\xi) = \int_\chi f(x)f^T(x)\xi(dx) = \sum_{i=1}^{n} f(x_i)f^T(x_i)\mu_i$$

to be the information matrix. If all parameters $\Theta_1, \ldots, \Theta_m$ are to be estimated then the E-optimality criterion plays an important role: an experimental design

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is called E-optimal if it maximizes the minimum eigenvalue of the information matrix.

Since the set of all information matrix is compact (see, for example, [3], Ch.10), an E-optimal design exists. In this paper E-optimal designs for the model $f_i(x) = x^{i-1}, i = 1, 2, \ldots, m, \chi = [r_1, r_2]$ where $r_1 < r_2$ are arbitrary real numbers are considered. Model (1) in this case will be called polynomial regression on an arbitrary segment.

References

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