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The Revival of Reduction Principles in the Generation of Optimal Designs for Non-standard Situations

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1 General considerations

It is common sense that experimentation, when properly designed, yields the highest evidence in statistical reasoning. The backbones of experimentation are classical concepts like randomization, blinding and stratification. On top of that the quality of a statistical experiment may be improved by a suitable choice of the experimental settings and a suitable choice of the corresponding numbers of replications. This consideration constitutes essentially the concept of optimal design of statistical experiments.

Based on convex optimization the general theory of optimal design is well developed. However, in practice for every non-standard statistical situation an individual optimal solution still has to be computed which may be challenging in the case of high dimensions and/or nonlinear relationships. While a diversity of algorithmic approaches is available ranging from steepest descent, multiplicative, and quasi-Newton to generic and particle swarm optimization methods involving high computational efforts, there may be still interest in analytical solutions or in reduction of the complexity of the problem to decrease the computational burden or to obtain exact benchmarks on the quality of competing designs.

As reduction principles in the construction of optimal designs we revisit here

- invariance and equivariance,
- majorization, and
- reduction to lower-dimensional problems.

The concept of invariance allows for symmetrization of designs, which typically results in a large number of experimental settings, while the related pure equivariance may provide standardizations, which lead to canonical forms in nonlinear situations (see Radloff and Schwabe, 2016). By majorization the design region, i. e. the number of experimental settings, may be reduced either by the concept

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of Loewner ordering or by model embedding. Finally, in more-dimensional situations the optimal designs may be built from their univariate counterparts as product-type designs under factorization or as some star-type (polypod) designs in certain nonlinear additive models (see Schmidt and Schwabe, 2017). For a general treatment of these concepts in linear settings we refer to Schwabe (1996).

2 Applications

We will exhibit the applicability of these general concepts in a couple of examples.

1. For restricted design regions we characterize optimal designs in a K-factor experiment with binary predictors, when the number of active predictors is bounded. As a by-product we obtain irregular fractions of a 2^K full factorial experiment.
2. In paired comparisons we can derive optimal designs for models with interactions up to second order between binary attributes.
3. For nonlinear (or generalized linear) models, in which the information is based on the value of the linear predictor in K variates, optimal designs can be additively constructed from their counterparts in the corresponding univariate models, if the design region is a (potentially unbounded) hyperrectangle. For a spherical design region similar reductions are possible.
4. For the Gamma model of multiple regression majorization can be used to obtain optimal designs in the case that there is no constant term.

References

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