Optimal Design for a Causal Structure

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1. Introduction

Linear models and linear mixed models are important statistical tools. But in many natural phenomena, there is more than one dependent (endogenous) variable involved and these variables are related in a sophisticated way. Structural Equation Modeling (SEM) is often used to model the relationship between the endogenous (dependent) variables and the exogenous (independent) variables. SEM allows us to estimate the parameters that explain the relationships among the variables [1]. It was first implemented in research to not only separate, but also demonstrate the direction of direct and indirect effects [5], and measure the relative magnitude of each causal factor [3].

Traditional optimal design theory and literature focuses on optimal design for univariate linear and univariate mixed models. There are four objectives for our research. First, we will use the modified Federov search algorithm to produce a D-optimal design for a causal structure for both the 3SLS and FIML estimators. Next, we will obtain a D-optimal design for the estimate of the endogenous and exogenous parameters of a mixed random causal structure. Then, we will use Composite criteria [6] to produce an optimal design for the estimate of the endogenous, exogenous, and random parameters (variance components) of a mixed random causal structure. Finally, we will develop a search algorithm that produces a D-optimal design for the estimates of the endogenous and exogenous parameters of a causal structure with fixed blocks. We then compare the efficiency of each of the optimal designs for causal structures with the optimal design for the univariate case.

In each of the four cases above, the causal relationship changed the optimal designs dramatically. The new optimal designs were much more efficient. Even orthogonal

designs, which are universally optimal in the univariate case [2][4], are not optimal with causal structural models. The new optimal designs for causal structures were much more efficient than orthogonal designs.

**Keywords:** Optimal design, D-optimality criteria, causal structure modeling, three-stage least squares, orthogonal design, endogenous variables, exogenous variables

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