Abstracts of the Ninth Workshop on Simulation (2018) 1-2

# Computing optimal experimental designs with respect to a compound Bayes risk criterion

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## 1 Introduction

We will consider the problem of computing optimal experimental designs with respect to a compound Bayes risk criterion, which includes various specific criteria, such as a linear criterion for prediction in a random coefficient regression model. We will prove that this problem can be converted into a problem of constrained A-optimality in an artificial model, which allows us to directly use existing theoretical results and software tools. We will demonstrate the application of the proposed method for the optimal design in a random coefficient regression model with respect to an integrated mean squared error criterion. The talk is based on [2].

## 2 Formulation of the problem

Let $\mathcal{X}$ be a finite design space. For an experimental design $\xi$, let $\mathbf{M}(\xi)$ denote the information matrix. The aim is to propose a method for computing optimal approximate and exact designs on $\mathcal{X}$ with respect to the criterion $\Phi$ defined by

$$\Phi(\xi) = \sum_{j=1}^{s} \mathrm{tr}((\mathbf{M}(\xi) + \mathbf{B}_j)^{-1}\mathbf{H}_j) \tag{1}$$

for all designs $\xi$ such that the matrices $\mathbf{M}(\xi)+\mathbf{B}_j$ are non-singular for $j = 1, \ldots, s$, and is defined by $\Phi(\xi) = +\infty$ for all other designs $\xi$. In (1), $\mathbf{B}_1, \ldots, \mathbf{B}_s$ are given non-negative definite $p \times p$ matrices, and $\mathbf{H}_1, \ldots, \mathbf{H}_s$ are given positive definite $p \times p$ matrices.

We will call (1) the compound Bayes risk criterion (CBRC) because for $s = 1$, it is equivalent to the standard Bayes risk criterion; see [1]. The class of CBRC criteria includes the recently proposed criterion for the prediction of individual parameters in random coefficient regression models ([3]) which is our main motivation for studying optimality criteria of this form.

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We will show that it is possible to convert any problem of CBRC-optimality into a problem of linearly constrained $A$-optimality in an auxiliary linear model. This enables computing CBRC-optimal designs by existing methods of mathematical programming, such as second-order cone programming and mixed-integer second-order cone programming ([4]).

## References


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