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## Identifying and representing clusters of spatial defects in microelectronics planar artefacts

## Riccardo Borgoni<sup>1</sup> Chiara Galimberti<sup>2</sup> Diego Zappa<sup>3</sup>

Statistical process control (SPC) charts are routinely adopted to monitor the stability of different processes over time mainly in manufacturing industries but also in other fields. Control charts detect a process distributional shift when the charting statistic is beyond the control limits. Amongst the different types of control charts, the cumulative sum (CUSUM) charts [2] are widely used in practice. However, practitioners would also be interested in knowing how strong the signal is in order to plan subsequent actions appropriately. For this reason, several authors have suggested control charts based on the p-value [1]. Following this approach the incontrol distribution of the charting statistic is preliminarily computed and the p-value is obtained at any given time point. If the p-value is smaller than a pre-specified significance level  $\alpha$ , the chart points out a process distributional shift. This approach has some benefits if compared to conventional control charts. Firstly, a p-value based control chart always has a vertical axis in the range [0, 1] and, hence, a unique control limit corresponding to  $\alpha$ . Secondly, it provides a measure of how the data suggest a potential out-of-control state even if it is not detected.

Defects that tend to show systematic patterns over the wafer area, can be usually ascribed to specific causes of the production process. A prompt identification of these patterns allows to reduce discards and the need of reworking production items, hence improving the yield. Hereafter, it is assumed that a defect can be represented by a random point occurring in the wafer area, W, and the spatial dynamics of defectivity is thought as governed by a spatial point process over W. The lack of interaction among points is named *complete spatial randomness* (CSR) property. Being N(W) the (random) number of events occurring in W and supposing that N(W) = n, the conditional property of CSR states that these n points are independent and

<sup>&</sup>lt;sup>1</sup> Department: Economia Metodi Quantitativi e Strategie d'Impresa, Università di Milano-Bicocca, Milano, Italy. E-mail: riccardo.borgoni@unimib.it

<sup>&</sup>lt;sup>2</sup> Department: Economia Metodi Quantitativi e Strategie d'Impresa, Università di Milano-Bicocca, Milano, Italy. E-mail: chiara.galimberti19@campus.unimib.it

<sup>&</sup>lt;sup>3</sup> Department: Scienze Statistiche, Università Cattolica delSacro Cuore, Milano, Italy. Email: diego.zappa@unicatt.it

uniformly distributed in W. If the CSR condition holds true, no structures are present in the defectivity process and defects are somehow a physiological result of the fabrication process. Testing the CSR condition consists in identifying a suitable test statistic that summarises the "discrepancy between the data and the CSR hypothesis". Being r<sub>i</sub> the distance of the *i*-th event of the observed point pattern to its nearest event in the sample, we consider the Euclidean norm between the empirical cumulative distribution function (ecdf),  $\hat{G}(r) = n^{-1} \sum_{i=1}^{n} \mathbf{1}(r_i \leq r)$  and the cdf, G(r), of the nearest neighbour distance (NND) expected under the CSR condition denoted by  $D = D(\widehat{G}, G)$  hereafter. A large value of D observed on the data is not favourable to the CSR hypothesis. The p-value is obtained by approximating the distribution of Dunder the null hypothesis via Monte Carlo simulations. A large number B of point patterns under CSR are simulated and for the b-th simulated point pattern the ecdf  $\hat{G}_b^*(r)$  and  $D_b^*$  are calculated. Finally, the p-value is obtained as  $p = (B+1)^{-1} [1+$  $\sum_{b=1}^{B} \mathbf{1}(D_b^* \leq D_{obs})$  where  $D_{obs} = D(\widehat{G}_{oss}, G)$  and  $\widehat{G}_{oss}$  is the ecdf calculated on the actual sample. A p-value control chart for structured spatial defectivity can be naturally based on a statistical test for CSR. Following the CUSUM approach, the defects occurred up to time t are cumulated and the ecdf of the NND is calculated on the cumulated point patterns. Hence, the test statistics,  $D_t$ , and the p-value,  $p_t$ , at time t are worked out as described above. In order to construct the chart,  $p_t$  values are reported on the y axis whereas time values are reported on the x axis as in usual control charts. A horizontal line is also added to the chart at the value of significance level  $\alpha$  one wants to consider in the statistical test. In order to control the variability induced by the simulations that are necessary to estimate the distribution of the test statistics under the null hypothesis, we suggest to calculate a confidence interval of the p-value by bootstrapping the values  $D_{tb}^*$ , b = 1, ..., B simulated at a given time t. The confidence interval is displayed on the chart and the process is considered out of control when this interval is below the  $\alpha$  level adopted for the test.

Once an out-of-control state is detected it is relevant to assess the shape of the clusters occurring on the wafer areas in order to understand the defectivity process and to plan remediation actions. To this end, a clustering algorithm has been implemented to highlight those areas of the wafer more prone to high defectivity. We propose to reconstruct the structure and the shape of the cluster using the alpha-shape and principal curve methodologies.

## References

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