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Inference and change detection for high-dimensional time series

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New results about inference and change point analysis of zero mean high dimensional vector time series

$$\mathbf{Y}_{ni} = (Y_{ni}^{(1)}, \ldots, Y_{ni}^{(d)})', \qquad i = 1, \ldots, n,$$

are discussed. Here the dimension, $d = d_n$, of the time series is allowed to grow with the sample size n. The results deal with change-point procedures that can be based on an increasing number of bilinear forms of the sample variance-covariance matrix as arising, for instance, when studying change-in-variance problems for projection statistics and shrinkage covariance matrix estimation.

Contrary to many known results, e.g. from random matrix theory, the results hold true without any constraint on the dimension, the sample size or their ratio, provided the weighting vectors v_n, w_n, are uniformly ℓ_1-bounded. Extensions to ℓ_2-bounded projections are also discussed. The large sample approximations are in terms of (strong resp. weak) approximations by Gaussian processes for partial sum and CUSUM type processes, [1], such as

$$\mathbf{v}_n' \left(\sum_{i=1}^{k} \mathbf{Y}_{ni}\mathbf{Y}_{ni}' - \frac{k}{n} \sum_{j=1}^{n} \mathbf{Y}_{nj}\mathbf{Y}_{nj}' \right) \mathbf{w}_n,$$

which imply (functional) central limit theorems under certain conditions. It turns out that the approximations by Gaussian processes hold not only without any constraint on the dimension, the sample size or their ratios, but even without any such constraint with respect to the number of bilinear form. For the unknown variances and covariances of these bilinear forms nonparametric estimators are proposed and shown to be uniformly consistent.

We present related change-point procedures for the variance of projection statistics as naturally arising in principal component analyses, in financial portfolio management where the convex combination of asset returns matters, in signal processing where data from large sensor arrays has to be analyzed.

Further, we discuss how the theoretical results lead to novel distributional approximations and sequential methods for shrinkage covariance matrix estimators in the spirit of Ledoit and Wolf, given by convex combinations of the form

$$\widehat{\mathbf{\Sigma}}_n^s = (1 - W)\widehat{\Sigma}_n + W i_n \mathbf{I},$$

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where $W \in [0,1]$ is the shrinkage weight, $\mathbf{I}$ the d-dimensional identity matrix and $\widehat{i}_n$ is an estimator of the optimal shrinkage intensity $i_n = \operatorname{tr}(\operatorname{Var}(\mathbf{Y}_n))$. The estimator $\widehat{i}_n$ is obtained. Especially, we discuss an asymptotic confidence interval $[\widehat{i}_n - a_n, \widehat{i}_n + a_n]$ for the shrinkage intensity satisfying

$$\lim_{n \to \infty} P([\widehat{i}_n - a_n, \widehat{i}_n + a_n] \ni i_n) = 1 - \alpha.$$

and related lower and upper bounds for the covariance matrix.

A simulation studies is presented, which investigate the accuracy of the proposed confidence interval. Lastly, we discuss an application to NYSE asset returns over a 22-year period corresponding to $5,651$ trading days.

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References

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