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Approximations of the boundary crossing probabilities for the maximum of moving weighted sums

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1 Introduction

We study approximations of boundary crossing probabilities for the maximum of moving weighted sums of i.i.d. random variables. We demonstrate that the approximations based on classical results of extreme value theory, see [5], [1] and [2], provide some scope for improvement, particularly for a range of values required in practical applications.

2 Formulation of the problem

Let $\varepsilon_1, \varepsilon_2, \ldots$ be a sequence of independent identically distributed random variables with finite mean μ and variance σ^2 and some c.d.f. F. Define the moving weighted sum as

$$\mathcal{S}_{n;L,Q} = \sum_{s=n+1}^{n+L+Q-1} w_{L,Q}(s-n)\varepsilon_t \quad (n = 0, 1, \ldots), \tag{1}$$

where the weight function $w_{L,Q}(\cdot)$ is defined by

$$w_{L,Q}(t) = \begin{cases} t & \text{for } 0 \le t \le Q, \\ Q & \text{for } Q \le t \le L, \\ L+Q-t & \text{for } L \le t \le L+Q-1. \end{cases} \tag{2}$$

where L and Q are positive integers with $Q \le L$.

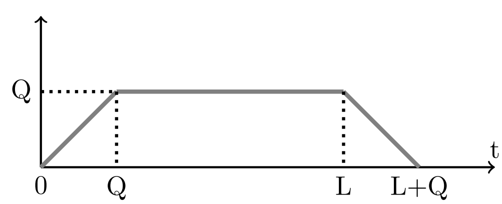

Figure 1: The weight function $w_{L,Q}(\cdot)$, $1 \le Q \le L$.

The weight function $w_{L,Q}(\cdot)$ is depicted in Figure 1. In the special case $Q = 1$, the weighted moving sum (1) becomes an ordinary moving sum.

The main aim of this paper is to study precision of different approximations of boundary crossing probabilities for the maximum of the moving weighted sum; that is,

$$P\left(\max_{n=0,1,\ldots,M} \mathcal{S}_{n;L,Q} > H \right), \tag{3}$$

where H is a given threshold, M is reasonably large and L, Q are fixed parameters.

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3 Application

The particular weight function defined in (2) is directly related to the SSA change point algorithm described in [6] (for an extensive introduction to SSA, we point the reader towards [3, 4]). More precisely, if we let $\varepsilon_j = \xi_j^2$, where $\xi_1, \xi_2, \dots$ are i.i.d. random variables with zero mean, variance δ^2 and finite fourth moment $\mu_4 = E\xi_i^4$, then $\mathcal{S}_{n;L,Q}$ can be seen as a moving weighted sum of squares; we have $\mu = E\varepsilon_j = \delta^2$ and $\sigma^2 = \mathrm{var}(\varepsilon_j) = \mu_4 - \delta^4$. In this particular setting, a good approximation for (3) is needed in the theory of sequential change-point detection because the boundary crossing probability defines the significance levels for the SSA change-point detection statistic.

References

[1] Harald Cramér. A limit theorem for the maximum values of certain stochastic processes. *Theory of Probability & Its Applications*, 10(1):126–128, 1965.

[2] James Durbin. The first-passage density of a continuous gaussian process to a general boundary. *Journal of Applied Probability*, pages 99–122, 1985.

[3] Nina Golyandina, Vladimir Nekrutkin, and Anatoly A Zhigljavsky. *Analysis of Time Series Structure: SSA and related techniques*, volume 90 of *Monographs on Statistics and Applied Probability*. Chapman & Hall/CRC, London, UK, 2001.

[4] Nina Golyandina and Anatoly Zhigljavsky. *Singular Spectrum Analysis for Time Series*. Springer Briefs in Statistics. Springer, 2013.

[5] Malcolm R Leadbetter, Georg Lindgren, and Holger Rootzén. *Extremes and related properties of random sequences and processes*, volume 21. Springer-Verlag New York, 1983.

[6] Valentina Moskvina and Anatoly Zhigljavsky. An algorithm based on singular spectrum analysis for change-point detection. *Communications in Statistics—Simulation and Computation*, 32(2):319–352, 2003.