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Testing for Structural Breaks in Factor Copula Models

Hans Manner [1], Florian Stark[2], and Dominik Wied[2]

1 Introduction

We propose new fluctuation tests for detecting structural breaks in factor copula models and analyze the behavior under the null hypothesis of no change. In the model, the joint copula is given by the copula of random variables, which arise from a factor model. Parameters are estimated with the simulated method of moments (SMM). Due to the discontinuity of the SMM objective function, it is not trivial to derive a functional limit theorem for the parameters.

2 Test and Behavior under the Null Hypothesis

The dynamic dependence implied by the copula $C(.,\theta_t^0)$ is determined by the unknown parameters θ_t^0 for $t = 1,\dots,T$. We are interested in estimating the $p \times 1$ vectors $\theta_t^0 \in \Theta$ of the copula, by information from the data and information generated by simulations from the factor copula model $C(.,\theta_t)$ for all t, implied by the following factor structure

$$X_{it} = \sum_{k=1}^{K} \beta_{ik}^t Z_{kt} + q_{it}, \quad i = 1,\dots,N,$$

where $q_{it} \overset{iid}{\sim} F_{\boldsymbol{q}_t}(\alpha_{\boldsymbol{q}_t})$ and $Z_{kt} \overset{init}{\sim} F_{\boldsymbol{Z}_{kt}}(\gamma_{kt})$ for $t = 1,\dots,T$. Note that Z_{kt} and $\boldsymbol{q}_{it}$ are independent $\forall i,k,t$ and the Copula for $\boldsymbol{X}_t = [X_{1t},\dots,X_{Nt}]'$ is given by $\boldsymbol{X}_t \sim \boldsymbol{F}_{\boldsymbol{X}_t} = C(G_{1t}(x_{1t};\theta_t),\dots,G_{Nt}(x_{Nt};\theta_t);\theta_t)$, with marginal distributions $G_{it}(.,\theta_t)$ and
$\theta_t = \left[\{\{\beta_{ik}^t\}_{i=1}^N\}_{k=1}^K, \alpha_{\boldsymbol{q}_t}', \gamma_{1t}',\dots,\gamma_{Kt}'\right]'$. For the estimation, we use the simulated method of moments (SMM) to receive estimators $\theta_{sT,S}$ of $\theta_{\lfloor sT \rfloor} = \theta_t$. The estimators are defined as $\theta_{sT,S} := \underset{\theta\in\Theta}{\arg\min}\, Q_{sT,S}(\theta)$, where $Q_{sT,S}(\theta) := g_{sT,S}(\theta)'\hat{W}_{sT} g_{sT,S}(\theta)$, $g_{sT,S}(\theta) := \hat{m}_{sT} - \tilde{m}_S(\theta)$ and $\hat{W}_{sT}$ a positive definite weight matrix. $\hat{m}_{sT}$ are $k \times 1$ vectors of averaged pairwise dependence measures computed from the data and $\tilde{m}_S(\theta)$ is the corresponding vector of dependence measures using simulations from $\boldsymbol{F}_{\boldsymbol{X}_t}$. We are interested in testing

$$H_0 : \theta_1 = \theta_2 = \dots = \theta_T \qquad H_1 : \theta_t \neq \theta_{t+1} \text{ for some } t = \{1,\dots,T-1\}.$$

[1]Department of Economics, University of Graz, Graz, Austria, E-mail: hans.manner@uni-graz.at

[2]Institute of Econometrics and Statistics, University of Cologne, Cologne, Germany

with the test statistic S, defined as

$$S := \max_{1 \le t \le T} \frac{\left(\frac{t}{T}\right)^2}{1/T + 1/S} (\theta_{t,S} - \theta_{T,S})'(\theta_{t,S} - \theta_{T,S}),$$

where $\theta_{sT,S}$ is the SMM estimator up to the information at time point $t = \lfloor sT \rfloor$ and $\varepsilon > 0$. Under suitable assumptions we receive the following limit distribution result for our test statistic

Theorem 1. Under the null hypothesis $\theta_1 = \theta_2 = \cdots = \theta_T$ and suitable assumptions, we have for $\varepsilon > 0$

$$\frac{s}{\sqrt{1/T + 1/S}} (\theta_{sT,S} - \theta_0) \xrightarrow{d} A^*(s), \qquad T, S \to \infty, \quad \forall s \in [\varepsilon, 1], \varepsilon > 0,$$

with $A^*(s) = (G'WG)^{-1} G'WA(s)$ and $A(s)$ a certain Gaussian process, where G is the approximated derivative of $g_{sT,S}(\theta_0)$ and W a weighting matrix.

Corollary 1. Under the null hypothesis $\theta_1 = \theta_2 = \cdots = \theta_T$ and suitable Assumptions, we receive for our test statistic

$$\sup_{s \in [\varepsilon,1]} \frac{s^2}{1/T + 1/S} (\theta_{sT,S} - \theta_{T,S})'(\theta_{sT,S} - \theta_{T,S}) \xrightarrow{d} \sup_{s \in [\varepsilon,1]} (A^*(s) - sA^*(1))'(A^*(s) - sA^*(1)).$$

as $T, S \to \infty$

For the computation of the p-values we use a multiplier resampling scheme that takes the serial dependence into account. We then test the finite-sample performance of the procedure under the null using simulations and provide an empirical application.

References

[1] Oh D., Patton A.J. *Simulated Method of Moments Estimation for Copula-Based Multivariate Models* // Journal of the American Statistical Association, 2013, v. 108, p. 689–700.

[2] Oh D., Patton A.J. *Modelling Dependence in High Dimensions with Factor Copulas* // Journal of Business and Economic Statistics, 2017, v. 35, p. 139–154.