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# Simulation of Branching Random Walks on Multidimensional Lattices

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The processes with generation and transport of particles on the lattice $\mathbf{Z}^d$, $d \geq 1$, are usually called *branching random walks* (BRWs). It is convenient to describe such processes in terms of birth, death, and walks of particles on $\mathbf{Z}^d$. We assume that the structure of an environment is defined by the offspring reproduction law at a finite number of particle generation centers, *branching sources*, situated at points of $\mathbf{Z}^d$. The spatial dynamics of particles is considered under different assumptions about underlying random walks that may be simple symmetric, symmetric or non-symmetric. Such processes are used in numerous applications [1], in particular, in statistical physics, population dynamics, chemical kinetics.

In the last decade, for various models of branching random walks, a series theoretical results were obtained among which the limit theorems about behavior of the process for large times has been obtained, see, e.g. [1]. However, as it is often the case, much more difficult or almost impossible to analyze analytically branching random walks on finite time intervals. So, in this paper we generalize an algorithm for simulating branching random walks introduced in [1] and give new examples of its numerical realization.

In the BRW models with one branching source, the exponential growth of the population may occur when the intensity of the source $\beta$ surpasses the critical value $\beta_c$ [2]. The situation when $\beta > \beta_c$ is close to $\beta_c$ (weakly supercritical BRWs) considered in [3] is of keen interest for the study, for example, of cell evolution [1]. The effect of phase transitions on the asymptotic behavior of a particle population in BRWs was studied analytically in detail by many authors, see, e.g., [4, 5, 6, 7] and the bibliography therein.

In [1], simulation of BRWs with one branching source is applied for numerical estimation of a threshold value of the parameter on finite time intervals. In the present work we propose an approach to simulation of the mean number of particles over the lattice and at every lattice point for BRWs with several sources. Simulation of the process is based on queue data structures, see, e.g., [8], and the Monte Carlo method described, e.g., in [9]. This approach allows to simulate BRWs with sources of different intensities and random walks with jumps not only to neighbor lattice points.

The evolution of particle system in BRWs is carried out in accordance with the rules used for constructing the algorithm for the BRW simulation. Some necessary theoretical results for an interpretation of the simulation are given.

The simulation is illustrated by an example of a BRW with a few sources of equal intencities, which demonstrates how the space configuration of the sources

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affects the behavior of the mean numbers of particle. An analytical investigations of the same BRW can be found in [3, 10]. In conclusion, more general BRWs with a finite number of sources of different intensities and finite variance of jumps considered in [7] are simulated.

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