Effective coefficients in the electromagnetic logging problem with log-normal distribution, multi-scale conductivity and permittivity

Olga N. Soboleva[[1]](#footnote-1), Mikhail I. Epov[[2]](#footnote-2), Ekaterina P. Kurochkina[[3]](#footnote-3)

The quasi-steady condition in a nomagnetic medium are $ωερ\_{0}<0.1$, where $ε=ε^{\*}/ε\_{0}$, $ε\_{0}=8.85∙10^{-12}F/m$ and $ε^{\*}\leq 5-10$ is the relative permittivity, $μ=4π∙10^{-7}$ is the magnetic permeability, $σ=1/ρ\_{0}$, $σ$ is the electric conductivity, $ω$ is the cyclic frequency. For long probes of the well logging, the quasi-steady condition is satisfied with a high accuracy. However, in heterogeneous media, the quasi-steady condition may be violated. The permittivity affects a measured signal and $ε^{\*}$ may be equal to 40. In natural condition as a rule, the spatial geometry of small-scale heterogeneities is not exactly known, and the irregularity of electric conductivity and permittivity abruptly increases as the scale of measurements decreases. The numerical solution of the problem with variations of parameters at all the scales requires high computer costs. So, the small-scale heterogeneities are described by random fields with the joint probability distribution functions and they are taken into account with the help of the effective parameters. Many natural media are "scale regular" in the sense that they can be described by multifractals and hierarchical cascade models by Sahimi[1], Bekele[2]. In this paper, he effective coefficients for Maxwell's equations in the frequency domain for a multi-scale isotropic medium by using a subgrid modelling approach are calculated. The correlated fields of conductivity and permittivity are approximated by the Kolmogorov multiplicative continuous cascades with a lognormal probability distribution. The wavelength is assumed to be large as compared with the scale of heterogeneities of the medium. The equations for effective coefficients are obtained in the first order terms of $ωε(x)/σ(x)$. The obtained effective parameters are frequency-independent, therefore it follows that they are also the effective parameters in the time domain. The theoretical results are compared with the results from the direct 3D numerical simulation. The permittivity also affects the results in a quasi-steady case if the parameters $σ$ and $ε$ are weakly correlated.

**References**

[1] Sahimi M. Flow phenomena in rocks: from continuum models, to fractals, percolation, cellular automata, and simulated annealing. // Rev Modern Phys. 1993, v. 65, p. 1393.

[2] Bekele A., Hudnall H. W., Daigle J.J., Prudente A. and Wolcott M. Scale dependent variability of soil electrical conductivity by indirect measures of soil properties. Journal of Terramechanics, 2005, v. 42, p. 339.

1. Novosibisk State Technical University, Institute of Computational Mathematics and Mathematical Geophysics, Lavrentieva, 6, Novosibirsk, NA 63090, Russia, E-mail: olgasob@gmail.com [↑](#footnote-ref-1)
2. Institute of Petroleum Geology and Geophysics, pr. Akademika Koptyuga, 3, Novosibirsk, NA 63090, Russia, E-mail: Epov@emf.ru; [↑](#footnote-ref-2)
3. Novosibisk State University Baker Hughes Joint Laboratory of The Multi-Scale Geophysics and Mechanics, Pirogova Str, 2, Novosibirsk, NA 63090, Russia, E-mail: e.p.kurochkina@gmail.com [↑](#footnote-ref-3)