**ON CONDITIONAL OPTIMIZATION OF THE RANDIMIZED PROJECTION AND PROJECTION-MESH FUNCTIONАL ALGORITHMS FOR NUMERICAL SOLUTION OF THE FREDHOLM INTEGRAL EQUATIONS OF THE SECOND KIND**

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 This talk continues the investigations of the paper [1], in which the systematization of numerical (implemented on a computer) randomized functional algorithms for approximation of a solution of Fredholm integral equation of the second kind is carried out. Wherein, three types of such algorithms are distinguished: *the projection, the mesh* and *the projection-mesh methods*.

 In the paper [1], the possibilities for usage of these algorithms for solution of practically important problems are investigated in detail. The disadvantages of the mesh algorithms, related to the necessity of calculation values of the kernels of integral equations in fixed points, are identified. On practice, these kernels have integrated singularities, and calculation of their values is impossible.

 Thus, for applied problems, related to solving Fredholm integral equation of the second kind, it is expedient to use not mesh, but the projection and the projection-mesh randomized algorithms.

 Nevertheless, as opposed to the mesh methods, the usage of *the theory of conditional optimization* (see, for example, [2]) for these algorithms is somewhat complicated.

 In this theory, the question is about the coordinated choice of the parameter $M$ (the number of the nodes or the basic functions) and the parameter $N$ (the number of the trajectories of the applied Markov chain, simulated on a computer) for the implemented functional algorithms. These parameters must provide the given error level (designate it as $γ$) by the minimal computing expenditures $S$.

 Construct the upper boundary $UP(M,N)$ of the algorithm's error $δ(M,N)$, which depends on the parameters $M$ and $N$:

$$δ\left(M,N\right)\leq UP\left(M,N\right) (1)$$

 This function of the two variables is equated to the value $γ$. From the equation of the form

$$UP\left(M,N\right)=γ (2)$$

one of the parameters (for example, $N$) presents in terms of the other parameter: $N=ψ(M)$.

 This ratio is substituted into the expression for the expenditures $S\left(M,N\right)$ (which is also depends on the parameters $M$ and $N$; as a rule, $S\left(M,N\right)=H×M×N;H=const$). As the result, we get the function $\tilde{S}\left(M\right)$ on the one variable $M$ and investigate this function for minimum with the help of the well-known methods of mathematical or numerical analysis.

 The defined values $M\_{min}\left(γ\right), N=ψ(M\_{min}\left(γ\right))$ are named as *the conditionally optimal parameters* of the model (algorithm). The ``conditionally'' of this optimization technique is related to the fact that in the left-hand side of the equation of the form (2) we use not the algorithm's error $δ(M,N)$ itself but it's upper boundary $UP(M,N)$ (may be, this boundary is inexact, rough?!).

 The difficulties for the conducting of such reasoning for the projection methods are related to the existence of the ``tails'' of the infinite orthogonal function systems and the necessity of their estimation for construction of the error upper boundaries of the form (1).

 In this sense, it is rather more simple (and, possibly, effective) to use the projection-mesh methods, for which the search for the conditionally optimal parameters is not too complicated problem [3].

**References**

[1] Voytishek A.V., Shipilov N.M. On randomized algorithms for numerical solution of applied Fredholm integral equations of the second kind // AIP Conference Proceedings 1907, 030015 (2017).

[2] Mikhailov G.A., Voytishek A.V. Numerical Statistical Modelling. Monte Carlo Methods. Moscow, Akademia, 2006 [In Russian].

[3] Voytishek A.V. Functional Estimators of the Monte Carlo Method. Novosibirsk: NSU, 2007 [In Russian].

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