Analysis of Multicast Queuing Systems   
with Random Requirements   
of Overlapping Resources

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In this paper we consider queueing system designed for the performance analysis of multicast telecommunication systems [1]. During each busy period system transmits a stream of media data. Busy periods are separated by idle periods. Busy period starts when the first customer connects to the stream. Later more customers may connect to the stream. When the stream transmission ends all customers get disconnected and the idle period starts.

Let the sequence of idle and busy periods of the system form the alternating renewal process [2] with the cumulative distribution functions of the length of idle and busy periods given by  and  respectively. We assume that during transmission of the stream customers arrive according Poisson arrival process with the rate λ and no more than *N* customers may be connected to the stream. Arriving customers switch up to the highest available quality within their network bandwidth, which are independent random variables with the cumulative distribution function (CDF) . A newcomer never decreases the amount of resources occupied in the system but it can increase this amount. So the total amount of occupied resources is equal to the maximum of these random variables each with CDF . In such a system blocking may occur if and only if at the instant of a new customer arrival there are *N* customers in the system.

System can be described by a stochastic process , where  if the system is idle at time *t* and otherwise  is equal to the number of customers connected to the stream. We denote  the stationary probability of the empty state and  the stationary joint probability that there are *i* customers in the system and the maximum bandwidth required for the transmission is less than *x*.

Let *a* and *b* be the mean length of idle and busy periods respectively,

, ,

then the stationary probability of the idle state can be calculated as [2]



Probability  that at the moment *t* after the start of a busy period it is steel propagating and there was exactly *i* arrivals by time *t* is given by

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Therefore, the mean length of the time interval inside each busy period when there are *i* customers in the system and the maximum amount of occupied resources is less then *x* can be calculated as





Since  we have





In telecommunication the main performance characteristics of the system are the blocking probability, the mean number of customers in the system and the mean amount of resources occupied in the system, which are given by the following formulae

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**References**

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