# Forecasting European Industrial Production with Singular Spectrum Analysis

Hossein Hassani[a,b*], Saeed Heravi[c] and Anatoly Zhigljavsky[a]

[a] *Statistics Group, School of Mathematics, Cardiff University, CF24 4AG, UK.*

[b] *Statistical Research and Training Center (SRTC), Tehran, Iran*

[c] *Cardiff Business School, Cardiff University, CF10 3EU, UK.*

**Abstract**

In this paper, the performance of Singular Spectrum Analysis (SSA) technique is assessed by applying it to 24 series measuring the monthly seasonally unadjusted industrial production for important sectors of the German, French and UK economies. The results are compared with those obtained using Holt-Winter and ARIMA models. All three methods perform similarly in the short-term forecasting and in predicting the direction of change (DC). However, at longer horizons, SSA significantly outperforms ARIMA and Holt-Winter methods.

**Keywords:** Singular Spectrum Analysis, ARIMA, Holt-Winter method, Forecasting, European Industrial Production series.

## 1 Introduction

The Singular Spectrum Analysis (SSA) is a powerful technique for nonparametric time series analysis and forecasting. SSA decomposes the original time series into a sum of small number of independent and interpretable components such as slowly varying trend, oscillatory components and noise. Theoretical and practical foundations of the SSA technique can be found in Golyandina et al. (2001) and an introduction to the subject is given in Elsner and Tsonis (1996).

SSA has a wide range of applications; from meteorology and physics to economics and financial mathematics. SSA was first applied to extract tendencies and harmonic components in meteorological and geophysical time series (Vautard et al., 1992). In recent years SSA has been developed and applied to many practical problems (see, for example Ghil et al., 2002, and Moskvina & Zhigljavsky, 2003).

SSA is especially useful for analyzing and forecasting series with complex seasonal components and non-stationarity. Thus, unlike ARIMA models, choosing an appropriate degree of differencing is not an important issue in SSA. The data considered in this study has a complex structure of this kind; as a consequence, we found superiority of SSA over classical techniques.

* *E-mail addresses*: HassaniH@cf.ac.uk (H. Hassani), HeraviS@cf.ac.uk (S. Heravi), ZhigljavskyAA@cf.ac.uk (A. A. Zhigljavsky).

Although some probabilistic and statistical concepts are employed in the SSA-based methods, no statistical assumptions such as stationarity of the series or normality of the residuals are required and SSA uses the bootstrapping to obtain the confidence intervals for the forecasts. Another important aspect of the SSA (which can be very useful in economics) is that, unlike many other methods, it works well even for small sample size (Vautard et al., 1992, Hassani, 2007).

This study uses eight monthly industrial production indices for Germany, France and the UK, previously analysed in linear and nonlinear contexts by Osborn et al. (1999) and Heravi et al. (2004). The eight series examined for the three countries, Germany, France and the UK, are interesting and important since they cover production in the major industrial sectors. They also reflect diverse types of industries.

Osborn et al. (1999) have considered the extent and nature of seasonality in these series. Their findings show that seasonality accounts for over 90% of the variation in almost all French series. The strong seasonal pattern for the traditional industrial sector in France is associated with declines in production during the summer. Seasonality also accounts for at least 80% of variation in all series in Germany and in all series (except vehicles) in the UK. Osborn et al. (1999) demonstrated that seasonalities for these series are much larger than those reported for monthly output in the United States at the two-digit level (Miron, 1996, Table 3.3). The difference in pattern of seasonality between the European countries and the United States is associated to differences in traditions and institutions. Based on seasonal unit root tests, Osborn et al. (1999) found that most of the series should be modelled using conventional first difference. However annual difference specification often produced the most accurate out-of-sample forecasts.

In contrast to Moody et al. (1993) and Swanson & White (1997a,b), Heravi et al. (2004) found relatively little evidence of non-linearity in most series. Comparing linear and neural network forecasts, they found that linear models generally produce more accurate post-sample forecasts than neural network models at horizons of up to a year in terms of root mean square error.

Here we examine the out-of-sample forecast accuracy of the SSA technique and compare it with ARIMA models and the Holt-Winter method. The structure of the paper is as follows. The next section briefly describes the SSA technique and provides some general rules for selecting its parameters. Section 3 outlines the data for the study. Our forecast results are then presented and described in Section 4 and some conclusions are given in Section 5. Appendix A briefly describes the data and appendix B provides an example of the SSA analysis.

# 2 Singular Spectrum Analysis (SSA)

The main purpose of SSA is to decompose the original series into a sum of a small number of time series, so that each subseries can be identified as either a trend, periodic or quasi-periodic component (perhaps, amplitude-modulated), or noise. This is followed by a reconstruction of the original series.

The SSA technique consists of two complementary stages: decomposition and reconstruction. At the first stage we decompose the time series and at the second stage we reconstruct the original time series and use the reconstructed time series for forecasting. Here we provide a brief discussion on the methodology of the Basic SSA method; see

Golyandina et al. (2001) for more information and many variations of the Basic SSA.

### Short description of the Basic SSA

The main idea of the Basic SSA is as follows. Consider the real-valued nonzero time series $Y_T = (y_1, \ldots, y_T)$ of sufficient length $T$. Let $K = T - L + 1$, where $L$ is some integer called the window length (we can assume $L \leq T/2$). Define the so-called 'trajectory matrix'

$$\mathbf{X} = [X_1, \ldots, X_K] = (x_{ij})_{i,j=1}^{L,K} = \begin{pmatrix} y_1 & y_2 & y_3 & \cdots & y_K \\ y_2 & y_3 & y_4 & \cdots & y_{K+1} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ y_L & y_{L+1} & y_{L+2} & \cdots & y_T \end{pmatrix}.$$

Note that $\mathbf{X}$ is a Hankel matrix (by the definition, these are the matrices such that their $(i,j)$-th entries depend only on the sum $i+j$). We then consider $\mathbf{X}$ as a multivariate data with $L$ characteristics and $K = T - L + 1$ observations. The columns $X_j$ of $\mathbf{X}$, considered as vectors, lie in an $L$-dimensional space $\mathbb{R}^L$. Define the matrix $\mathbf{X}\mathbf{X}^T$. Singular value decomposition (SVD) of $\mathbf{X}\mathbf{X}^T$ provides us with the collections of $L$ eigenvalues $\lambda_1 \geq \lambda_2 \geq \ldots \geq \lambda_L \geq 0$ and the corresponding eigenvectors $U_1, U_2, \ldots, U_L$, where $U_i$ is the normalised eigenvector corresponding to the eigenvalue $\lambda_i$ $(i = 1, \ldots, L)$.

The SVD of the trajectory matrix can be written as:

$$\mathbf{X} = \mathbf{E}_1 + \cdots + \mathbf{E}_d, \tag{1}$$

where $\mathbf{E}_i = \sqrt{\lambda_i} U_i V_i^T$ $(i = 1, \ldots, d)$, $d$ is the number of non-zero eigenvalues of $\mathbf{X}\mathbf{X}^T$, and $V_1, \ldots, V_d$ are the principal components defined as $V_i = \mathbf{X}^T U_i / \sqrt{\lambda_i}$. The collection $(\sqrt{\lambda_i}, U_i, V_i)$ is referred to as the $i$-th eigentriple of the matrix $\mathbf{X}$.

A group of $r$ (with $1 \leq r \leq d$) eigenvectors determines an $r$-dimensional hyperplane in the $L$-dimensional space $\mathbb{R}^L$ of vectors $X_j$. The $L_2$-distance between vectors $X_j \in \mathbb{R}^L$ and this $r$-dimensional hyperplane is equal to $\sum_{j \notin I} \lambda_j$ and can be rather small which would mean that $\tilde{\mathbf{X}}$, the projection of $\mathbf{X}$ into this hyperplane, approximates well the original matrix $\mathbf{X}$. Subsequent averaging over the diagonals of $\tilde{\mathbf{X}}$ allows us to obtain a series that can be considered as an approximation to the original series.

### Selection of parameters

Here we consider a version of SSA where we split the set of indices $\{1, 2, \ldots, d\}$ into two groups only: $I = \{1, \ldots, r\}$ and $\bar{I} = \{r+1, \ldots, d\}$. We associate the group $I$ (and the related matrix $\mathbf{E}_I = \mathbf{E}_1 + \ldots + \mathbf{E}_r$) with signal and the group $\bar{I}$ with noise. The SSA method requires then the selection of two parameters, the window length $L$ and the number of elementary matrices $r$. There are specific rules for selecting these parameters; their choice depends on structure of the data and the analysis we want to perform. Detailed description of parameter selection procedures is given in Golyandina et al. (2001). Here we summarize a few general rules.

The window length $L$ is the single parameter that should be selected at the decomposition stage. Larger values of $L$ (we can always assume $L \leq T/2$) lead to a more detailed decomposition; in selecting $L$ we should try to achieve sufficient separability of the components. The following quantity (called the weighted correlation or *w-correlation*) is a natural measure of dependence between two time series $Y_T^{(1)}$ and $Y_T^{(2)}$:

$$\rho_{12}^{(w)} = \left(Y_T^{(1)}, Y_T^{(2)}\right)_w / \parallel Y_T^{(1)} \parallel_w \parallel Y_T^{(2)} \parallel_w$$

where $\left(Y_T^{(i)}, Y_T^{(j)}\right)_w = \sum_{k=1}^{T} w_k y_k^{(i)} y_k^{(j)}$, $w_k$=min$\{k, L, T-k\}$, $\parallel Y_T^{(i)} \parallel_w = \sqrt{\left(Y_T^{(i)}, Y_T^{(i)}\right)_w}$ $(i, j = 1, 2)$.

If two reconstructed components have zero $w$-correlation it means that these two components are well separated. Large values of $w$-correlations between reconstructed components indicate that the components should be considered as one group and possibly correspond to the same component in the SSA decomposition.

The first elementary matrix $\mathbf{E}_1$ with the norm $\sqrt{\lambda_1}$ has the highest contribution to the norm of $\mathbf{X}$ in (1) and the last elementary matrix $\mathbf{E}_d$ with the norm $\sqrt{\lambda_d}$ has the lowest contribution to the norm of $\mathbf{X}$. The plot of the eigenvalues $\lambda_1, \cdots, \lambda_d$ gives an overall view concerning the values of the eigenvalues and is essential in deciding where to truncate the summation of (1) in order to build a good approximation of the original matrix. A slowly decreasing sequence of eigenvalues typically indicate the presence of noise in the series. Similar values of the eigenvalues allow the identification of the eigentriples that correspond to the same harmonic component of the series. The periodogram analysis of the original time series also helps us in selecting the groups. Sharp sparks in the periodogram are associated with the harmonic components in the series.

We return to the discussion on parameter selection in Appendix B where we provide details of analysis for one of the series.

### SSA Forecasting

SSA forecasting method can be applied to the time series that approximately satisfy linear recurrent formulae [1]. The class of time series governed by linear recurrent formulae is rather wide; it includes harmonics, polynomial and exponential time series.

Let us briefly describe the so-called SSA recurrent forecasting algorithm (for more information see Golyandina et al., 2001). Define the original series $Y_T = (y_1, \ldots, y_T)$ and the reconstructed series $\widetilde{Y}_T = (\widetilde{y}_1, \ldots, \widetilde{y}_T)$. For an eigenvector $U \in \mathbb{R}^L$ we denote the vector of the first $L-1$ components of the vector $U$ as $U^{\nabla} \in \mathbb{R}^{L-1}$. Set $v^2 = \pi_1^2 + \ldots + \pi_r^2 < 1$, where $\pi_i$ is the last component of the eigenvector $U_i$ $(i = 1, \ldots, r)$. It can be proved that the last component $y_L$ of any vector $Y = (y_1, \ldots, y_L)^T$ is a linear combination of the first components $(y_1, \ldots, y_{L-1})$; that is, $y_L = a_1 y_{L-1} + \ldots + a_{L-1} y_1$ where the vector of coefficients $A = (a_1, \ldots, a_{L-1})$ can be expressed as $A = \sum_{i=1}^{r} \pi_i U_i^{\nabla} / (1 - v^2)$. The forecasts $\hat{y}_{T+1}, \ldots, \hat{y}_{T+h}$ are then obtained as

$$\hat{y}_i = \begin{cases} \widetilde{y}_i & \text{for } i = 1, \ldots, T \\ \sum_{j=1}^{L-1} a_j \hat{y}_{i-j} & \text{for } i = T+1, \ldots, T+h. \end{cases}$$

[1] We say that the time series $Y_T$ satisfies an linear recurrent formulae of order $L-1$ if there are numbers $a_1, \ldots, a_{L-1}$ such that

$$y_{T-i} = \sum_{k=1}^{L-1} a_k y_{T-i-k}, \qquad 0 \leq i \leq T - L.$$

### Bootstrapping

Assume that we have a time series $Y_T = \{y_t\}_{t=1}^T = Y_T^{(1)} + Y_T^{(2)}$ where $Y_T^{(1)}$ is the signal and $Y_T^{(2)}$ represents the noise. Let us consider a method of constructing average series for the signal $y_{T+M}^{(1)}$ at time $T+M$. In the unrealistic situation, when we know both the signal $Y_T^{(1)}$ and the true model of the noise $Y_T^{(2)}$, the Monte Carlo simulation can be applied to check the statistical properties of the forecast values $\widetilde{y}_{T+M}^{(1)}$ relative to the actual term $y_{T+M}^{(1)}$.

Indeed, assuming that the rules for the eigentriple selection are fixed, we can simulate $N$ independent copies $Y_{T,i}^{(2)}$ $(i = 1, \ldots, N)$ of the process $Y_T^{(2)}$ and apply the forecasting procedure to $N$ independent time series $Y_{T,i} = Y_T^{(1)} + Y_{T,i}^{(2)}$. Then the forecasting result will form a sample $\widetilde{y}_{T+M,i}^{(1)}$, which should be compared against $y_{T+M}^{(1)}$. In this way the Monte Carlo average series for the forecast can be built up.

Since in practice we do not know the signal $Y_T^{(1)}$, we can not apply this procedure. Under a suitable choice of the window length $L$ and the corresponding eigentriples, we have the representation $Y_T = \widetilde{Y}_T^{(1)} + \widetilde{Y}_T^{(2)}$, where $\widetilde{Y}_T^{(1)}$ (the reconstructed series) approximates $Y_T^{(1)}$, and $\widetilde{Y}_T^{(2)}$ is the residual series. Suppose now that we have a (stochastic) model for the residual $\widetilde{Y}_T^{(2)}$ (for instance, we can postulate some model for $Y_T^{(2)}$ and, since $\widetilde{Y}_T^{(1)} \approx Y_T^{(1)}$, we apply the same model for $\widetilde{Y}_T^{(2)}$ with the estimated parameters). Then, simulating $N$ independent copies $Y_{T,i}^{(2)}$ of the series $\widetilde{Y}_T^{(2)}$, we obtain $N$ series $Y_{T,i} = \widetilde{Y}_T^{(1)} + \widetilde{Y}_{T,i}^{(2)}$ and produce $M$ forecasting results $\widetilde{y}_{T+M,i}^{(1)}$ in the same manner as in the Monte Carlo simulation variant.

From the sample $\widetilde{y}_{T+M,i}^{(1)}$ $(1 \leq i \leq N)$ of the forecasts we can compute the average bootstrap forecast. This average bootstrap can then be compared with the value $\widetilde{y}_{T+M}^{(1)}$ obtained by Basic SSA forecast. Large discrepancy between these two forecast would typically indicate that the original SSA forecast is not reliable. Furthermore, using the sample of the bootstrap forecast results we can estimate the distribution of the forecast and compute, for example, confidence intervals for the true values. To do that, we need a stochastic model for $Y_T^{(2)}$; a standard assumption would be the assumption that $Y_T^{(2)}$ is the Gaussian white noise model. This assumption can be easily verified using the classical test for randomness and normality.

# 3 The data

The data in this study are taken from Eurostat, the official statistical agency of the European Community and represents eight major components of industrial production in Germany, France and the UK. The series used are seasonally unadjusted monthly indices for real output in Food Products, Chemicals, Basic Metals, Fabricated Metals, Machinery, Electrical Machinery, Vehicles and Electricity/Gas industries. Appendix A provides some information about the series. It should be noted that the series for Germany are the aggregated data following the reunification of the former East Germany and West Germany.

The same 24 series, ending in December 1995, have been previously examined in studies by Osborn et al. (1999) and Heravi et al. (2004). As explained in these papers,

Table 1: Descriptive statistics of the series.

| | Mean | | | S.D | | | Weight | | |
|---|---|---|---|---|---|---|---|---|---|
| Series | UK | GR | FR | UK | GR | FR | UK | GR | FR |
| Food products | 4.64 | 4.42 | 4.58 | 0.067 | 0.195 | 0.129 | 10.2 | 7.6 | 9.0 |
| Chemicals | 4.65 | 4.41 | 4.52 | 0.087 | 0.192 | 0.176 | 8.5 | 8.6 | 8.9 |
| Basic metals | 4.54 | 4.58 | 4.51 | 0.107 | 0.098 | 0.175 | 3.8 | 4.5 | 4.3 |
| Fabricated metal | 4.61 | 4.39 | 4.50 | 0.064 | 0.201 | 0.194 | 5.8 | 7.2 | 9.8 |
| Machinery | 4.63 | 4.51 | 4.55 | 0.078 | 0.152 | 0.163 | 7.5 | 13.6 | 8.6 |
| Electrical machinery | 4.47 | 4.37 | 4.57 | 0.105 | 0.256 | 0.138 | 3.0 | 5.6 | 3.9 |
| Vehicles | 4.64 | 4.29 | 4.39 | 0.133 | 0.315 | 0.405 | 4.7 | 10.4 | 7.1 |
| Electricity and gas | 4.62 | 4.48 | 4.54 | 0.176 | 0.172 | 0.204 | 6.7 | 6.5 | 9.6 |

these time series have been chosen primarily because of their importance to industrial production across the three countries. These eight time series account for at least half of total industrial production in each country. Plots of these time series are included in Osborn et al. (1999) and broadly represent a period of growth in the 1980s and stagnation or recession during the early 1990s. Here we have updated the data and in all cases the sample period ends in July 2007. However, the starting dates are different which reflects the availability of consistent data from Eurostat. The data for Germany starts from January 1978, for France starts from January 1990 and for the UK starts from 1998.

In all cases, the final two and a half years (30 observations) of data are retained for out-of-sample forecast accuracy tests. For comparability and in line with the usual convention for economic time series, all time series are analysed in logarithmic form and all subsequent results refer to the time series after this transformation. The descriptive statistics for these series are given in Table 1. For Germany, the vehicles series has the highest volatility, which is more than twice than the volatility of the other series. Similarly, the vehicles series has the highest volatility for France. The UK data, generally, are less volatile with gas and electricity series having highest volatilities.

Almost all of the industrial production series have complex structure with nonlinear trends and complex seasonality. SSA is well suited for non-stationary series with complex trend and periodicities and can be a powerful technique in modeling these industrial production series.

# 4 Forecasting Results

### Comparison of the accuracy of the forecasts

We consider forecasting performance of the SSA, ARIMA and Holt-Winter techniques at different horizons $h$, of up to a year. We provide results for $h = 1, 3, 6$ and 12 (months). We use the data up to the end of 2004 as training sample (to perform SSA decomposition and to estimate parameters of ARIMA and Holt-Winter models). Thus, with two and a half years of out-of-sample data, we have $N = 30, 28, 25$ and 19 out-of-sample forecast errors at the horizons $h = 1, 3, 6$ and 12, respectively.

We use the root mean squared error (RMSE) and the percentage of forecasts that correctly predict the direction of change to measure the forecast accuracy.[2] RMSE is the

[2]We have also computed other measures based on the magnitude of forecast errors, such as relative

most frequently quoted measure in forecasting literature (e.g., Zhang et al 1998). In order to save space, we only provide RMSE ratios of the SSA to that of the Holt-Winter and ARIMA models. [3] The ratios of RMSE are

$$RRMSE = \left(\sum_{i=1}^{N}(\hat{y}_{T+h,i} - y_{T+h,i})^2\right)^{\frac{1}{2}} / \left(\sum_{i=1}^{N}(\tilde{y}_{T+h,i} - y_{T+h,i})^2\right)^{\frac{1}{2}}$$

where $\hat{y}_{T+h}$ is the $h$-step ahead forecast obtained by SSA forecasting and $\tilde{y}_{T+h}$ is the $h$-step ahead forecast from either ARIMA or Holt-Winter model and $N$ is the number of the forecasts. If RRMSE $< 1$, then the SSA outperforms the other methods (either ARIMA or Holt-Winter).

In computing Box-Jenkins ARIMA forecasts, we need to choose the lags, the degree of differencing and the degree of seasonality $(p,d,q)$, $(P,D,Q)_s$ where $s = 12$. To do that we use the maximum order of lags, set by the software, and apply the Bayesian Information Criterion (BIC). Holt-Winter forecasts are also obtained by minimizing the BIC. The SSA parameters, the window length $L$ and the number of eigentriples $r$, are chosen based on the eigenvalue spectra and separability (see Appendix B). The parameters $(L, r)$ of the SSA and the orders $(p,d,q), (P,D,Q)_s$ of the ARIMA models are given when the models are estimated using data up to the end of 2004. Appendix B gives details of the analysis for fabricated metal series for Germany. Details for the other series are available from authors upon request.

Tables 2, 3 and 4 show the in-sample RMSE and RMSE ratios and out-of-sample RMSE ratios for the UK, France and Germany. Some summary statistics (average RMSE, RRMSE of SSA models to the Holt-Winter and ARIMA models for each country and horizon) are also given at the bottom of each table. The summary statistics are the RMSE and the RRMSE averages and the scores. The score is the number of times when SSA model yields lower RMSE. SSA has produced lower RMSE for all the series for the in-sample results [4].

The averages and the scores for 1-step ahead show that SSA forecasts are comparable with the forecasts obtained by ARIMA and Holt-Winter models. However, the performance of the SSA does improve relative to ARIMA and Holt-Winter models for forecasting at the horizons greater than one. The scores also confirm that the SSA forecasts outperform the forecasts produced by the ARIMA and Holt-Winter models, particularly at longer horizons. For all the series and three countries (24 cases), SSA outperforms the ARIMA $16, 18, 22$ and $23$ times at $h = 1, 3, 6$ and $12$ horizons respectively. It also outperforms the Holt-Winter models $16, 19, 23$ and $23$ times at $h = 1, 3, 6$ and $12$ horizons.

Table 5 summarizes the results of forecasts by ARIMA, Holt-Winter and SSA for all series. This table shows that the quality of 1-step ahead forecasts are similar for ARIMA and SSA; Holt-Winter forecasts being slightly worse. The quality of SSA forecasts at horizons $h = 3, 6$ and $12$ is much better than the quality of ARIMA and Holt-Winter forecasts. As $h$ increases, the quality of ARIMA and Holt-Winter forecasts becomes worse; the standard deviation of the ARIMA and Holt-Winter forecasts increases almost

---

root mean absolute errors. These measures yield qualitatively similar results to RMSE; we thus do not report them.

[3] Results of analysis (including values of RMSE for each series, method and horizon) are available from the authors upon request.

[4] SSA gives the highest $R^2$, although all three methods fit the data well in-sample, with $R^2 > 81\%$.

linearly with $h$. The situation is totally different for the SSA forecasts: the quality of SSA forecasts is almost independent of the value of $h$ (at least, in the range of values of $h$ considered in the paper). This observation serves as a confirmation of the following facts:

(i) most of the series considered here have a structure which can described via a deterministic trend and seasonality (for an example, see Appendix II);

(ii) this structure is well recovered by the SSA;

(iii) in most cases, the structure of the series is very stable as it is kept by the series for at least 12 months starting at any point.

Note that in the ideal situation, when we have a series which is a sum of a deterministic component (fully recovered by SSA) and a random noise, the error of SSA forecast will be exactly the same at any horizon. For more information, see Chapter 2 in Golyandina et al. (2001).

### Modified Diebold-Marino statistics

Using the modified Diebold-Marino statistics, given in Harvey et al. (1997), we test for the statistical significance of the results. The symbol $*$ indicates the results at the 10% level of significance or less. Comparing the SSA forecasts with the ARIMA, SSA outperforms the ARIMA significantly $2, 12, 9$ and 19 times at $h = 1, 3, 6$ and 12 horizons respectively at 10% significance level or less. SSA also outperforms the Holt-Winter significantly $6, 13, 16$ and 19 times at $h = 1, 3, 6$ and 12 horizons respectively at 10% significance level or less. Similar results have also been found when comparing the bootstrap forecasts, called in the table BSSA (to obtain bootstrap average series we have replicated the series 1000 times). In fact, the scores for all the horizons in tables 2, 3 and 4 show that both the SSA and bootstrap SSA methods have outperformed the ARIMA and Holt-Winter models exactly the same number of times (160 times out of 192 different cases).

We have also used the forecast encompassing test (Harvey et al. (1998)). The symbol $+$ indicates the results at the 10% level of significance or less. The results also confirms the superiority of the SSA, with 54% of cases significantly better at the 10% level of significance or less.

Cumulative distribution functions (c.d.f.) of the absolute values of the out-of-sample errors (for all eight series and 3 countries) obtained by SSA, ARIMA and Holt-Winter forecasts are presented in Fig. 1. If the c.d.f. graph produced by one method is strictly above the graph of another c.d.f., then we can say that the errors obtained by the first method are stochastically smaller than the errors represented by the second method. We can see from Fig. 1 that for $h = 3, 6$ and 12, SSA forecasting errors are stochastically much smaller than the errors of the other two methods. In addition, it can be seen that the ARIMA forecast errors are slightly smaller than the Holt-Winter forecast errors. In the case of $h = 1$ there is no evident prevalence of any method.

### Direction of change predictions

As another measure of forecast accuracy, in addition to RMSE, we also compute the percentage of forecasts that correctly predict the direction of change. Ash et al. (1997) argue that for some purposes, it may be more harmful to make a smaller prediction error yet misforecast the direction of change, than to make a larger directionally correct error. Clements and Smith (1999) discuss that the value of a model forecasts may be better measured by the direction of change. Heravi et al. (2004) argue that the direction of change forecasts are particularly important in economics for capturing the business cycle movement relating to expansion versus recession. Here the direction of change is interpreted only in terms of whether industrial production in a particular sector increases or decreases.

Table 6 provides the percentage of forecasts that correctly predict the direction of change at $h = 1, 3, 6$ and 12 horizons. It also shows whether they are significantly greater than the pure chance ($p = 0.50$). The symbols $*$ and $**$ in the table indicate the 5% and 1% levels of significance. A set of summary results is also given at the bottom of the table. The summary statistics are the average of correct signs for all eight series at $h = 1, 3, 6$ and 12 horizons and overall average for the three countries. The percentage of correct signs are generally better than those reported in Heravi et al. (2004). This is due the fact that the results for directional change are particulary sensitive to structural change in the out-of-sample period. The percentage of correct signs can be extremely high or low for all the three methods depending on whether there is a structural change in the series in the out-of-sample period. The overall percentage of correct signs for SSA are $90\%, 91\%, 92\%$ and $85\%$ at $h = 1, 3, 6$ and 12 respectively. For the Holt-Winter, these figures are $89\%, 91\%, 90\%$ and 82%, which are slightly lower than the SSA. ARIMA models have produced slightly better results (91% and 92%) at horizons $h = 1$ and $h = 3$ but they are lower (90% and 81%) at $h = 6$ and 12 horizons. For all 96 cases (3 countries, 8 series, $h = 1, 3, 6$ and 12 horizons) SSA has produced 93 significant cases at the 1% and 5% level. Similar results were obtained with the Holt-Winter and ARIMA models, giving 93 and 90 significant cases respectively.

# 5 Conclusion

In this paper, we compared Singular Spectrum Analysis (SSA), ARIMA and Holt-Winter methods for forecasting seasonally unadjusted monthly data on industrial production indicators in Germany, France and the UK. We demonstrated that SSA is a very powerful tool for analyzing and predicting economic data. SSA outperformed the ARIMA and Holt-Winter methods in predicting the values of the production series according to the RMSE criterion, particularly at horizons of $h = 3, 6$ and 12 months. We also found that SSA works well for small sample sizes, as for the UK with the sample size of 84 observations. The forecasts obtained by bootstrapping also confirm the findings. We also found that the three methods perform similarly well in predicting the direction of change. However, SSA outperforms the Holt-Winter and ARIMA models at longer horizons and hence can be considered as a reliable method for predicting recessions and expansions.

# Acknowledgments

The authors would like to thank the editor and the referees for their constructive comments which have led to substantial improvements in this paper. The authors are grateful to their colleagues Nina Golyandina and Vladimir Nekrutkin (University of St.Petersburg) for constructive remarks for useful suggestions. This research was in part supported by a grant from SRTC (No. 8711).

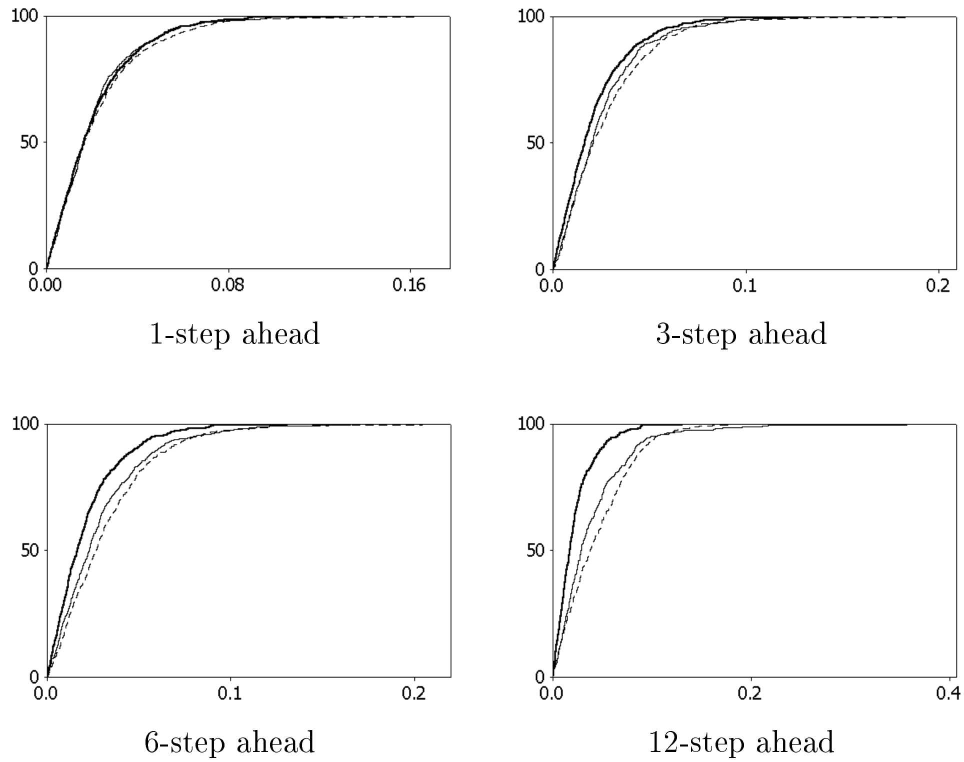


Figure 1: The cumulative distribution functions of the absolute values of the out-of-sample errors (for all eight series and 3 countries) obtained by SSA (thick-line), ARIMA (thin-line) and Holt-Winter (dashed-line)

Table 2: **Descriptive statistics of Out-of-sample and In-sample errors, UK**.

| | **Parameters** | | | **In-sample: RMSE** | | | **In-sample: RRMSE** | | | **Out-of-sample: RRMSE** | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Series | $L$ | r | $(p,d,q)(P,D,Q)_s$ | ARIMA | H-W | SSA | $\frac{SSA}{ARIMA}$ | $\frac{SSA}{H-W}$ | $h$ | $\frac{SSA}{ARIMA}$ | $\frac{SSA}{H-W}$ | $\frac{BSSA}{ARIMA}$ | $\frac{BSSA}{H-W}$ |
| da15 | 36 | 1-14 | (1,0,0)(0,1,1) | 0.012 | 0.010 | 0.007 | 0.58 | 0.70 | 1 | $0.90^{+}$ | $0.78^{*+}$ | 0.93 | $0.80^{*+}$ |
| | | | | | | | | | 3 | $0.83^{+}$ | $0.79^{+}$ | $0.92^{+}$ | $0.88^{+}$ |
| | | | | | | | | | 6 | $0.77^{+}$ | $0.63^{*+}$ | $0.84^{+}$ | $0.69^{*+}$ |
| | | | | | | | | | 12 | $0.21^{*+}$ | 0.95 | $0.23^{*+}$ | 1.04 |
| dg24 | 36 | 1-14 | (0,1,1)(0,1,1) | 0.019 | 0.015 | 0.009 | 0.47 | 0.60 | 1 | 0.87 | $0.77^{*+}$ | 0.93 | $0.83^{+}$ |
| | | | | | | | | | 3 | $0.65^{*+}$ | $0.67^{*+}$ | $0.70^{*+}$ | $0.71^{*+}$ |
| | | | | | | | | | 6 | $0.58^{*}$ | $0.57^{*+}$ | $0.61^{*}$ | $0.59^{*+}$ |
| | | | | | | | | | 12 | $0.74^{+}$ | $0.80^{+}$ | $0.77^{+}$ | $0.83^{+}$ |
| dj27 | 24 | 1-16 | (0,1,1)(0,1,1) | 0.034 | 0.028 | 0.005 | 0.15 | 0.18 | 1 | 0.96 | $0.90^{+}$ | 0.91 | $0.85^{*+}$ |
| | | | | | | | | | 3 | $0.81^{+}$ | $0.79^{+}$ | $0.90^{+}$ | $0.89^{+}$ |
| | | | | | | | | | 6 | 0.92 | 0.92 | 1.07 | 1.07 |
| | | | | | | | | | 12 | $0.30^{*+}$ | 0.80 | $0.34^{*+}$ | 0.92 |
| dj28 | 36 | 1-10 | (1,0,0)(1,1,0) | 0.026 | 0.020 | 0.019 | 0.73 | 0.95 | 1 | $0.86^{+}$ | 1.06 | 0.96 | 1.18 |
| | | | | | | | | | 3 | $0.84^{*+}$ | 0.99 | 1.02 | 1.21 |
| | | | | | | | | | 6 | $0.79^{+}$ | $0.81^{+}$ | 0.91 | 0.94 |
| | | | | | | | | | 12 | $0.42^{*+}$ | 0.83 | $0.46^{*+}$ | 0.93 |
| dk29 | 36 | 1-9 | (0,1,1)(0,1,1) | 0.026 | 0.023 | 0.021 | 0.81 | 0.91 | 1 | 1.21 | $0.83^{+}$ | 1.26 | $0.87^{+}$ |
| | | | | | | | | | 3 | 0.98 | $0.76^{*+}$ | 1.04 | 0.81 |
| | | | | | | | | | 6 | 0.98 | $0.59^{*+}$ | 0.93 | $0.56^{*+}$ |
| | | | | | | | | | 12 | $0.76^{+}$ | $0.48^{*+}$ | 0.82 | $0.52^{*+}$ |
| dl31 | 36 | 1-11 | (0,1,1)(0,1,0) | 0.037 | 0.025 | 0.020 | 0.54 | 0.80 | 1 | 1.30 | 1.48 | 1.20 | 1.37 |
| | | | | | | | | | 3 | 0.93 | 1.05 | 0.89 | 1.00 |
| | | | | | | | | | 6 | 0.81 | $0.76^{*}$ | 0.81 | 0.75 |
| | | | | | | | | | 12 | $0.42^{*+}$ | $0.47^{*+}$ | $0.56^{*+}$ | $0.63^{*+}$ |
| dm34 | 60 | 1-13 | (0,1,1)(1,1,0) | 0.059 | 0.046 | 0.027 | 0.46 | 0.59 | 1 | 1.00 | 0.96 | 1.07 | 1.02 |
| | | | | | | | | | 3 | $0.76^{*+}$ | $0.80^{*+}$ | $0.83^{+}$ | $0.87^{+}$ |
| | | | | | | | | | 6 | $0.67^{*+}$ | $0.73^{*+}$ | $0.81^{+}$ | $0.88^{+}$ |
| | | | | | | | | | 12 | $0.48^{*+}$ | $0.52^{*+}$ | $0.64^{*+}$ | $0.69^{*+}$ |
| e40 | 36 | 1-8 | (0,1,1)(0,1,0) | 0.035 | 0.024 | 0.020 | 0.57 | 0.83 | 1 | 0.93 | $0.81^{+}$ | 0.97 | $0.83^{+}$ |
| | | | | | | | | | 3 | 1.02 | $0.80^{*+}$ | 1.06 | $0.84^{+}$ |
| | | | | | | | | | 6 | $0.85^{+}$ | $0.67^{*+}$ | $0.92^{+}$ | $0.72^{*+}$ |
| | | | | | | | | | 12 | $0.65^{*+}$ | $0.42^{*+}$ | $0.67^{*+}$ | $0.43^{*+}$ |
| **Average** | | | | **0.031** | **0.024** | **0.016** | **0.54** | **0.70** | 1 | **1.00** | **0.95** | **1.03** | **0.97** |
| | | | | | | | | | 3 | **0.85** | **0.83** | **0.92** | **0.90** |
| | | | | | | | | | 6 | **0.80** | **0.71** | **0.86** | **0.78** |
| | | | | | | | | | 12 | **0.50** | **0.66** | **0.57** | **0.75** |
| **Score** | | | | | | | **8** | **8** | 1 | **5** | **6** | **5** | **5** |
| | | | | | | | | | 3 | **7** | **7** | **5** | **6** |
| | | | | | | | | | 6 | **8** | **8** | **7** | **7** |
| | | | | | | | | | 12 | **8** | **8** | **8** | **7** |

$*$ indicates significance for DM test at 10% or less, $+$ indicates significance for encompassing test at 10% or less.

Table 3: **Descriptive statistics of Out-of-sample and In-sample errors, Germany**.

| | **Parameters** | | | **In-sample: RMSE** | | | **In-sample: RRMSE** | | | **Out-of-sample: RRMSE** | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Series | $L$ | r | $(p,d,q)(P,D,Q)_s$ | ARIMA | H-W | SSA | $\frac{SSA}{ARIMA}$ | $\frac{SSA}{H-W}$ | $h$ | $\frac{SSA}{ARIMA}$ | $\frac{SSA}{H-W}$ | $\frac{BSSA}{ARIMA}$ | $\frac{BSSA}{H-W}$ |
| da15 | 60 | 1-12 | (0,1,1)(0,1,1) | 0.020 | 0.020 | 0.016 | 0.80 | 0.80 | 1 | 0.89 | 0.89 | 0.82 | 0.83 |
| | | | | | | | | | 3 | 0.69*+ | 0.62* | 0.69*+ | 0.63* |
| | | | | | | | | | 6 | 0.69*+ | 0.64*+ | 0.66*+ | 0.61*+ |
| | | | | | | | | | 12 | 0.49*+ | 0.61*+ | 0.56*+ | 0.70*+ |
| dg24 | 120 | 1-21 | (1,1,0)(0,1,1) | 0.024 | 0.023 | 0.017 | 0.71 | 0.74 | 1 | 0.89 | 0.84 | 0.98 | 0.97 |
| | | | | | | | | | 3 | 0.66*+ | 0.57* | 0.78+ | 0.67* |
| | | | | | | | | | 6 | 0.70*+ | 0.43*+ | 0.76+ | 0.47*+ |
| | | | | | | | | | 12 | 0.57*+ | 0.31*+ | 0.66*+ | 0.36*+ |
| dj27 | 60 | 1-19 | (0,1,1)(0,1,1) | 0.034 | 0.032 | 0.019 | 0.56 | 0.59 | 1 | 1.59 | 1.45 | 1.24 | 1.13 |
| | | | | | | | | | 3 | 1.25 | 1.18 | 1.01 | 0.95 |
| | | | | | | | | | 6 | 0.94 | 0.76*+ | 0.73*+ | 0.58*+ |
| | | | | | | | | | 12 | 0.56*+ | 0.47*+ | 0.44*+ | 0.37*+ |
| dj28 | 120 | 1-18 | (0,1,1)(0,1,1) | 0.028 | 0.027 | 0.021 | 0.75 | 0.78 | 1 | 0.97 | 0.89 | 0.87 | 0.79 |
| | | | | | | | | | 3 | 0.75* | 0.61* | 0.74* | 0.61* |
| | | | | | | | | | 6 | 0.49*+ | 0.40*+ | 0.50*+ | 0.41*+ |
| | | | | | | | | | 12 | 0.23*+ | 0.19*+ | 0.21*+ | 0.17*+ |
| dk29 | 48 | 1-18 | (2,1,0)(0,1,1) | 0.035 | 0.033 | 0.017 | 0.49 | 0.52 | 1 | 1.49 | 1.24 | 1.04 | 0.87 |
| | | | | | | | | | 3 | 1.37 | 1.03 | 1.00 | 0.75 |
| | | | | | | | | | 6 | 1.01 | 0.74*+ | 0.78*+ | 0.57*+ |
| | | | | | | | | | 12 | 0.65*+ | 0.47*+ | 0.52*+ | 0.38*+ |
| dl31 | 48 | 1-18 | (0,1,1)(0,1,1) | 0.029 | 0.028 | 0.015 | 0.52 | 0.54 | 1 | 1.48 | 1.41 | 1.31 | 1.25 |
| | | | | | | | | | 3 | 1.17 | 1.22 | 1.05 | 1.09 |
| | | | | | | | | | 6 | 0.82+ | 0.79+ | 0.75*+ | 0.72*+ |
| | | | | | | | | | 12 | 0.54* | 0.49*+ | 0.45* | 0.42*+ |
| dm34 | 60 | 1-18 | (0,1,2)(0,1,1) | 0.096 | 0.092 | 0.064 | 0.67 | 0.70 | 1 | 0.72*+ | 0.45*+ | 0.84+ | 0.52+ |
| | | | | | | | | | 3 | 0.73*+ | 0.41*+ | 0.79+ | 0.44*+ |
| | | | | | | | | | 6 | 0.74+ | 0.40*+ | 0.53*+ | 0.29*+ |
| | | | | | | | | | 12 | 0.85 | 0.44*+ | 0.83 | 0.43*+ |
| e40 | 60 | 1-15 | (0,1,1)(0,1,1) | 0.029 | 0.028 | 0.019 | 0.66 | 0.68 | 1 | 0.97 | 0.96 | 0.94 | 0.92 |
| | | | | | | | | | 3 | 0.75*+ | 0.76*+ | 0.71*+ | 0.71*+ |
| | | | | | | | | | 6 | 0.69*+ | 0.70*+ | 0.67*+ | 0.68*+ |
| | | | | | | | | | 12 | 0.62*+ | 0.62*+ | 0.61*+ | 0.61*+ |
| **Average** | | | | **0.037** | **0.035** | **0.023** | **0.65** | **0.67** | 1 | **1.12** | **1.02** | **1.01** | **0.91** |
| | | | | | | | | | 3 | **0.92** | **0.80** | **0.85** | **0.73** |
| | | | | | | | | | 6 | **0.76** | **0.60** | **0.67** | **0.54** |
| | | | | | | | | | 12 | **0.57** | **0.45** | **0.53** | **0.43** |
| **Score** | | | | | | | **8** | **8** | 1 | **5** | **5** | **5** | **6** |
| | | | | | | | | | 3 | **5** | **5** | **5** | **7** |
| | | | | | | | | | 6 | **7** | **8** | **8** | **8** |
| | | | | | | | | | 12 | **8** | **8** | **8** | **8** |

$*$ indicates significance for DM test at 10% or less, + indicates significance for encompassing test at 10% or less.

Table 4: **Descriptive statistics of Out-of-sample and In-sample errors, France.**

| | Parameters | | | In-sample: RMSE | | | In-sample: RRMSE | | | Out-of-sample: RRMSE | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Series | $L$ | r | $(p,d,q)(P,D,Q)_s$ | ARIMA | H-W | SSA | $\frac{SSA}{ARIMA}$ | $\frac{SSA}{H-W}$ | $h$ | $\frac{SSA}{ARIMA}$ | $\frac{SSA}{H-W}$ | $\frac{BSSA}{ARIMA}$ | $\frac{BSSA}{H-W}$ |
| da15 | 60 | 1-12 | (1,0,0)(0,1,1) | 0.024 | 0.023 | 0.014 | 0.58 | 0.61 | 1 | 0.91 | 0.78 | 0.78* | 0.67* |
| | | | | | | | | | 3 | 0.76*$^+$ | 0.68*$^+$ | 0.70*$^+$ | 0.64*$^+$ |
| | | | | | | | | | 6 | 0.75*$^+$ | 0.73$^+$ | 0.71*$^+$ | 0.69$^+$ |
| | | | | | | | | | 12 | 0.80*$^+$ | 0.67*$^+$ | 0.76*$^+$ | 0.63*$^+$ |
| dg24 | 120 | 1-21 | (0,1,1)(0,1,1) | 0.028 | 0.024 | 0.017 | 0.61 | 0.71 | 1 | 0.82 | 0.79* | 0.78 | 0.75* |
| | | | | | | | | | 3 | 0.92 | 0.90 | 0.90 | 0.89 |
| | | | | | | | | | 6 | 0.85 | 0.81 | 0.92 | 0.88 |
| | | | | | | | | | 12 | 1.01 | 1.00 | 1.23 | 1.15 |
| dj27 | 60 | 1-14 | (1,1,0)(0,1,1) | 0.031 | 0.029 | 0.019 | 0.61 | 0.66 | 1 | 0.93 | 0.99 | 0.83 | 0.89 |
| | | | | | | | | | 3 | 0.70* | 0.74*$^+$ | 0.67* | 0.71*$^+$ |
| | | | | | | | | | 6 | 0.50*$^+$ | 0.56*$^+$ | 0.51*$^+$ | 0.56*$^+$ |
| | | | | | | | | | 12 | 0.39*$^+$ | 0.53*$^+$ | 0.40*$^+$ | 0.56*$^+$ |
| dj28 | 120 | 1-18 | (0,1,3)(1,1,0) | 0.029 | 0.026 | 0.017 | 0.59 | 0.65 | 1 | 0.77* | 0.62*$^+$ | 0.80 | 0.65*$^+$ |
| | | | | | | | | | 3 | 0.74* | 0.57*$^+$ | 0.76* | 0.60*$^+$ |
| | | | | | | | | | 6 | 0.63 | 0.48*$^+$ | 0.66 | 0.50*$^+$ |
| | | | | | | | | | 12 | 0.56*$^+$ | 0.38*$^+$ | 0.57*$^+$ | 0.38*$^+$ |
| dk29 | 48 | 1-18 | (3,1,0)(0,1,1) | 0.028 | 0.029 | 0.019 | 0.68 | 0.66 | 1 | 1.06 | 1.08 | 0.98 | 1.01 |
| | | | | | | | | | 3 | 1.15 | 1.05 | 1.08 | 0.99 |
| | | | | | | | | | 6 | 1.15 | 1.03 | 1.12 | 1.00 |
| | | | | | | | | | 12 | 0.98 | 0.73*$^+$ | 0.90 | 0.67*$^+$ |
| dl31 | 48 | 1-18 | (0,1,1)(0,1,1) | 0.034 | 0.033 | 0.022 | 0.65 | 0.67 | 1 | 1.16 | 1.10 | 1.19 | 1.14 |
| | | | | | | | | | 3 | 1.06 | 0.99 | 1.11 | 1.03 |
| | | | | | | | | | 6 | 0.82 | 0.79$^+$ | 0.83 | 0.80$^+$ |
| | | | | | | | | | 12 | 0.61*$^+$ | 0.67*$^+$ | 0.70*$^+$ | 0.76*$^+$ |
| dm34 | 60 | 1-18 | (0,1,1)(0,1,0) | 0.081 | 0.077 | 0.074 | 0.91 | 0.96 | 1 | 0.94 | 1.01 | 0.84 | 0.90 |
| | | | | | | | | | 3 | 0.80 | 0.91 | 0.75 | 0.82 |
| | | | | | | | | | 6 | 0.65*$^+$ | 0.81$^+$ | 0.60*$^+$ | 0.75$^+$ |
| | | | | | | | | | 12 | 0.42*$^+$ | 0.57*$^+$ | 0.40*$^+$ | 0.55*$^+$ |
| e40 | 60 | 1-15 | (0,0,8)(1,1,0) | 0.048 | 0.037 | 0.018 | 0.38 | 0.49 | 1 | 0.93$^+$ | 0.86*$^+$ | 0.87$^+$ | 0.80*$^+$ |
| | | | | | | | | | 3 | 0.75*$^+$ | 0.78*$^+$ | 0.69*$^+$ | 0.71*$^+$ |
| | | | | | | | | | 6 | 0.65* | 0.75*$^+$ | 0.58*$^+$ | 0.68*$^+$ |
| | | | | | | | | | 12 | 0.68* | 0.71* | 0.63* | 0.66* |
| **Average** | | | | **0.038** | **0.035** | **0.025** | **0.63** | **0.68** | 1 | **0.94** | **0.90** | **0.88** | **0.85** |
| | | | | | | | | | 3 | **0.86** | **0.83** | **0.83** | **0.80** |
| | | | | | | | | | 6 | **0.75** | **0.75** | **0.74** | **0.74** |
| | | | | | | | | | 12 | **0.69** | **0.66** | **0.70** | **0.67** |
| **Score** | | | | | | | **8** | **8** | 1 | **6** | **5** | **7** | **6** |
| | | | | | | | | | 3 | **6** | **7** | **6** | **7** |
| | | | | | | | | | 6 | **7** | **7** | **7** | **7** |
| | | | | | | | | | 12 | **7** | **7** | **7** | **7** |

$*$ indicates significance for DM test at 10% or less, $+$ indicates significance for encompassing test at 10% or less.

Table 5: **Descriptive statistics of out-of-sample errors.**

| Method | N | Mean | S.D | Min | Median | Max |
|---|---|---|---|---|---|---|
| **1-step ahead** | | | | | | |
| Holt-Winter | 720 | 0.00297 | 0.03109 | -0.13771 | 0.00440 | 0.16733 |
| ARIMA | 720 | 0.00014 | 0.02808 | -0.13844 | 0.00165 | 0.10497 |
| SSA | 720 | 0.00010 | 0.02837 | -0.08982 | -0.00034 | 0.087198 |
| **3-step ahead** | | | | | | |
| Holt-Winter | 672 | 0.00521 | 0.03555 | -0.15961 | 0.00728 | 0.19733 |
| ARIMA | 672 | 0.00085 | 0.03281 | -0.14697 | 0.00284 | 0.10402 |
| SSA | 672 | -0.00025 | 0.02855 | -0.09839 | -0.00069 | 0.088908 |
| **6-step ahead** | | | | | | |
| Holt-Winter | 600 | 0.00920 | 0.04115 | -0.18965 | 0.01150 | 0.20733 |
| ARIMA | 600 | 0.00347 | 0.03853 | -0.20505 | 0.00695 | 0.11062 |
| SSA | 600 | 0.00003 | 0.02903 | -0.13882 | 0.00063 | 0.08908 |
| **12-step ahead** | | | | | | |
| Holt-Winter | 456 | 0.01767 | 0.05278 | -0.18090 | 0.02029 | 0.14733 |
| ARIMA | 456 | 0.00938 | 0.05452 | -0.35677 | 0.01424 | 0.19970 |
| SSA | 456 | 0.00146 | 0.02952 | -0.13039 | 0.00110 | 0.09062 |

Table 6: **Out-of-sample percentage of forecasts of correct sign**.

| | **Holt-Winter** | | | | **ARIMA** | | | | **SSA** | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Series | 1 | 3 | 6 | 12 | 1 | 3 | 6 | 12 | 1 | 3 | 6 | 12 |
| | **UK** | | | | | | | | | | | |
| Food product | 0.87** | 0.89** | 1.00** | 0.89** | 0.83** | 0.96** | 1.00** | 0.68 | 0.90** | 0.96** | 0.92** | 0.74* |
| Chemicals | 0.97** | 0.96** | 0.92** | 0.89** | 0.97** | 0.93** | 0.96** | 0.79** | 0.97** | 0.93** | 0.80** | 0.89** |
| Basic metals | 0.80** | 0.93** | 0.76** | 0.84** | 0.80** | 0.86** | 0.72* | 0.79** | 0.73** | 0.82** | 0.80** | 0.74* |
| Fabricated metal | 0.97** | 0.93** | 0.88** | 0.84** | 0.93** | 0.89** | 0.92** | 0.84** | 0.93** | 0.96** | 1.00** | 0.74* |
| Machinery | 0.90** | 0.93** | 0.80** | 0.74* | 1.00** | 1.00** | 0.96** | 0.84** | 0.90** | 0.93** | 1.00** | 0.95** |
| Electrical machinery | 0.87** | 0.86** | 0.84** | 0.58 | 0.93** | 0.82** | 0.92** | 0.53 | 0.77** | 0.89** | 0.92** | 0.74* |
| Vehicles | 0.90** | 0.93** | 0.96** | 0.84** | 0.90** | 0.93** | 0.96** | 0.84** | 0.97** | 0.79** | 0.92** | 0.84** |
| Electricity and gas | 0.93** | 0.93** | 1.00** | 0.84** | 0.97** | 0.96** | 0.44 | 0.89** | 1.00** | 1.00** | 1.00** | 0.68 |
| **Average** | **0.90** | **0.92** | **0.90** | **0.81** | **0.92** | **0.92** | **0.86** | **0.78** | **0.90** | **0.91** | **0.92** | **0.79** |
| | **Germany** | | | | | | | | | | | |
| Food product | 0.90** | 0.78** | 0.92** | 0.79** | 0.90** | 0.75** | 0.88** | 0.84** | 0.93** | 0.86** | 0.92** | 0.95** |
| Chemicals | 0.86** | 0.89** | 0.72* | 0.79** | 0.87** | 0.89** | 0.92** | 0.89** | 0.87** | 0.93** | 0.92** | 1.00** |
| Basic metals | 0.83** | 0.79** | 0.84** | 0.63 | 0.87** | 0.82** | 0.84** | 0.68 | 0.80** | 0.75** | 0.88** | 0.89** |
| Fabricated metal | 0.87** | 0.93** | 0.88** | 0.63 | 0.90** | 0.93** | 0.88** | 0.63 | 0.77** | 0.96** | 1.00** | 1.00** |
| Machinery | 0.97** | 0.96** | 0.92** | 0.79** | 0.97** | 0.96** | 0.96** | 0.84** | 0.90** | 0.89** | 0.88** | 1.00** |
| Electrical machinery | 0.90** | 0.93** | 0.96** | 0.89** | 0.90** | 0.96** | 0.96** | 0.89** | 0.83** | 0.86** | 0.96** | 1.00** |
| Vehicles | 0.80** | 0.75** | 0.88** | 0.58 | 0.87** | 0.89** | 0.92** | 0.79** | 0.90** | 0.86** | 0.96** | 0.95** |
| Electricity and gas | 0.93** | 0.93** | 1.00** | 0.84** | 0.97** | 0.89** | 1.00** | 0.84** | 0.90** | 0.93** | 0.92** | 0.68 |
| **Average** | **0.88** | **0.87** | **0.89** | **0.74** | **0.90** | **0.89** | **0.92** | **0.80** | **0.86** | **0.88** | **0.93** | **0.93** |
| | **France** | | | | | | | | | | | |
| Food product | 0.90** | 93** | 0.92** | 0.84** | 0.93** | 1.00** | 0.92** | 0.95** | 0.93** | 0.93** | 1.00** | 0.79** |
| Chemicals | 0.90** | 1.00** | 0.88** | 0.95** | 0.90** | 1.00** | 0.92** | 0.95** | 0.93** | 0.93** | 0.76** | 0.95** |
| Basic metals | 1.00** | 0.86** | 0.88** | 0.95** | 1.00** | 0.89** | 0.80** | 0.89** | 1.00** | 0.96** | 1.00** | 0.89** |
| Fabricated metal | 0.97** | 0.93** | 0.92** | 1.00** | 0.93** | 1.00** | 1.00** | 0.95** | 0.97** | 0.96** | 1.00** | 0.95** |
| Machinery | 0.93** | 1.00** | 0.96** | 0.95** | 0.90** | 1.00** | 0.96** | 1.00** | 0.97** | 0.86** | 0.80** | 0.89** |
| Electrical machinery | 0.83** | 0.86** | 0.84** | 0.89** | 0.87** | 0.89** | 0.84** | 0.84** | 0.97** | 0.93** | 0.88** | 0.89** |
| Vehicles | 0.93** | 0.96** | 0.84** | 0.84** | 0.87** | 0.89** | 0.84** | 0.63 | 0.87** | 0.93** | 0.80** | 0.84** |
| Electricity and gas | 0.77** | 0.96** | 1.00** | 0.89** | 0.87** | 0.96** | 1.00** | 0.89** | 0.87** | 0.96** | 1.00** | 0.53 |
| **Average** | **0.90** | **0.94** | **0.91** | **0.91** | **0.91** | **0.96** | **0.91** | **0.89** | **0.94** | **0.93** | **0.91** | **0.84** |
| **Overall Average** | **0.89** | **0.91** | **0.90** | **0.82** | **0.91** | **0.92** | **0.90** | **0.81** | **0.90** | **0.91** | **0.92** | **0.85** |

$*$ indicates significance at 5% and $**$ indicates significance at 1%.

## References


[1] Ash, J. C. K, Smyth, D. J., & Heravi, S.(1997). The accuracy of OECD forecasts for Japan. *Pacific Economic Review*, *2 (1)*, pp. 25–44.

[2] Clements, M. P., & Smith, J. (1999). A Monte Carlo investigation of forecasting performance of emprical SETAR model. *Journal of Applied Econometrics*, *14*, 123–141.

[3] Diebold, F. X. and R. S. Mariano. (1995). Comparing predictive accuracy, *Journal of Business, Economics and Statistics*, *13*, 253–263.

[4] Elsner, J. B., & Tsonis, A. A. (1996). *Singular Spectral Analysis. A New Tool in Time Series Analysis*. Plenum Press, New York and London.

[5] Ghil M., Allen, R. M., Dettinger, M. D., Ide, K., Kondrashov, D., Mann, M. E., Robertson, A., Saunders, A., Tian, Y., Varadi, F., & Yiou, P. (2002). Advanced spectral methods for climatic time series. *Rev. Geophys.*, *40(1)*, 3.1-3.41, 10.1029/2000GR000092.

[6] Golyandina, N., Nekrutkin, V., & Zhigljavsky, A. (2001). *Analysis of Time Series Structure: SSA and related techniques*. Chapman & Hall/CRC, New York - London.

[7] Hassani, H. (2007). Singular Spectrum Analysis: Methodology and Comparison, *Journal of Data Science*, *5(2)*, 239–257.

[8] Harvey, D. I., Leybourne, S. J. & Newbold, P. (1997). Testing the Equality of Prediction Mean Squared Errors, *International Journal of Forecasting*, *13*, 281–291.

[9] Harvey, D. I., Leybourne, S. J. & Newbold, P. (1998). Tests for Forecast Encompassing, *Journal of Business and Economic Statistics*, **16**, 254-59.

[10] Heravi, S., Osborn, D. R., & Birchenhall, C. R. (2004). Linear Versus Neural Network Forecasts for European Industrial Production Series. *International Journal of Forecasting*, *20*, 435–446.

[11] Miron, J. A. (1996). *The Economics of Seasonal Cycles*, MIT Press, Cambridge, MA.

[12] Moody, J., Levin, U., & Rehfuss, S. (1993). Predicting the U.S. index of industrial production.*Neural Network World*, *3*, 791–794.

[13] Moskvina, V. G., & Zhigljavsky, A. (2003) An algorithm based on singular spectrum analysis for change-point detection. *Communication in Statistics - Simulation and Computation*, *32*, No. 2, 319–352.

[14] Osborn, D. R., Heravi, S., & Birchenhall, C. R. (1999). Seasonal unit roots and forecasts of two-digit European industrial production. *International Journal of Forecasting*, *15*, 27–47.

[15] Stephens, M. A. (1974). EDF statistics for goodness of fit and some comparisons. *Journal of the American Statistical Association*, *69*, 730–737.

[16] Swanson, N. R., & White, H. (1997a). A model selection approach to real time 26 macroeconomic forecasting using linear models and artificial neural networks, *Review of Economics and Statistics*, it 39, 540–550.

[17] Swanson, N. R., & White, H. (1997b). Forecasting economic time series using adaptive versus nonadaptive and linear versus nonlinear econometric models.*International Journal of Forecasting*, *13*, 439–461.

[18] Vautard, R., P. Yiou, & Ghil, M. (1992), Singular-spectrum analysis: A toolkit for short, noisy chaotic signals. *Physica D*, *58*, 95–126.

[19] Zhang, G., Patuwo, B. E., & Hu, M. Y. (1998). Forecasting with artificial neural networks: the state of the art. *International Journal of Forecasting*, *14*, 35-62.

# Appendix A: European industrial production series

The two-digit categories examined in this paper are:

Table 7: **Industrial production series**.

| Short name | Detail |
|---|---|
| Food product (da15) | Manufacture of food products and beverages |
| Chemicals (dg24) | Manufacture of chemicals and chemical product |
| Basic metals (dj27) | Manufacture of basic metals |
| Fabricated metal (dj28) | Manufacture of fabricated metal products, except machinery and equipment |
| Machinery (dk29) | Manufacture of machinery and equipment N.E.C. |
| Electrical machinery (dl31) | Manufacture of electrical machinery and apparatus N.E.C. |
| Vehicles (dm34) | Manufacture of motor vehicles, trailers and semi-trailers |
| Electricity and gas (e40) | Electricity, gas and water supply |

For more information about these series and some graphs depicting them (up to 1995), see Osborn et al. (1999).

# Appendix B: Application of SSA for the Fabricated metal series in Germany

We shall now use the Fabricated metal series for Germany as an example to illustrate the selection of the SSA parameters and to show the reconstruction of the original series in details. To perform the analysis, we have used the SSA software[5]. Fig. 2 presents the series, indicating a complex trend and strong seasonality.

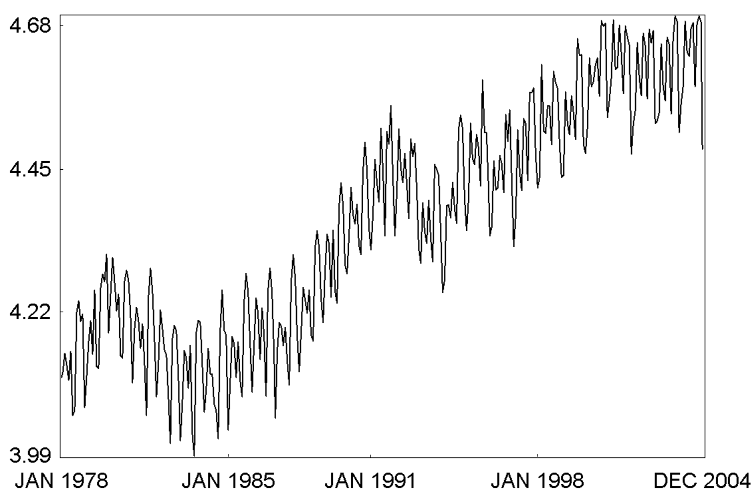


Figure 2: Fabricated metal series in Germany

[5]http://www.gistatgroup.com/cat/index.html

### Selection of the window length $L$

The window length $L$ is the only parameter in the decomposition stage. Knowing that the time series may have a periodic component with an integer period, to achieve a better separability of this periodic component it is advisable to take the window length proportional to that period. For example, the assumption that there is an annual periodicity in the series suggests that we must pay attention to the frequencies $k/12$ $(k = 1, ..., 12)$. As it is advisable to choose $L$ reasonably large (but smaller than $T/2$ which is 162 in this case), we choose $L = 120$.

### Selection of $r$

Information from auxiliary methods help us in choosing the parameters of the models. Here, we briefly explain some methods, which are useful in the separation of the signal component from noise. Usually a harmonic component produces two eigentriples with close singular values (except for the frequency 0.5 which provides one eigentriple with the saw-tooth singular vector). Another useful insight is provided by checking breaks in the eigenvalue spectra. Additionally, a pure noise series typically produces a slowly decreasing sequence of singular values.

Choosing $L = 120$ and performing SVD of the trajectory matrix $\mathbf{X}$, we obtain 120 eigentriples, ordered by their contribution (share) in the decomposition. Fig. 3 depicts the plot of the logarithms of the 120 singular values.

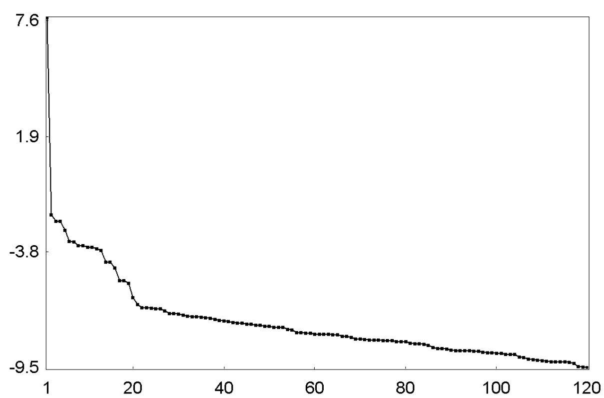


Figure 3: Logarithms of the 120 eigenvalues.

Here a significant drop in values occurs around component 19 which could be interpreted as the start of the noise floor. Six evident pairs, with almost equal leading singular values, correspond to six (almost) harmonic components of the series: eigentriple pairs 3-4, 6-7, 8-9, 10-11, 14-15 and 17-18 are related to the harmonics with specific periods (we show later that they correspond to the periods of 6, 4, 12, 3, 36 and 2.4 months).

Another way of grouping is to examine the matrix of the absolute values of the $w$-correlations. Fig. 4 shows the $w$-correlations for the 120 reconstructed components in a 20-grade grey scale from white to black corresponding to the absolute values of correlations from 0 to 1. Based on this information, we select the first 18 eigentriples for the reconstruction of the original series and consider the rest as noise.

The principal components (shown as time series) of the first 18 eigentriples are shown in Fig. 5. Consider a pure harmonic with a frequency $w$, certain phase, amplitude and

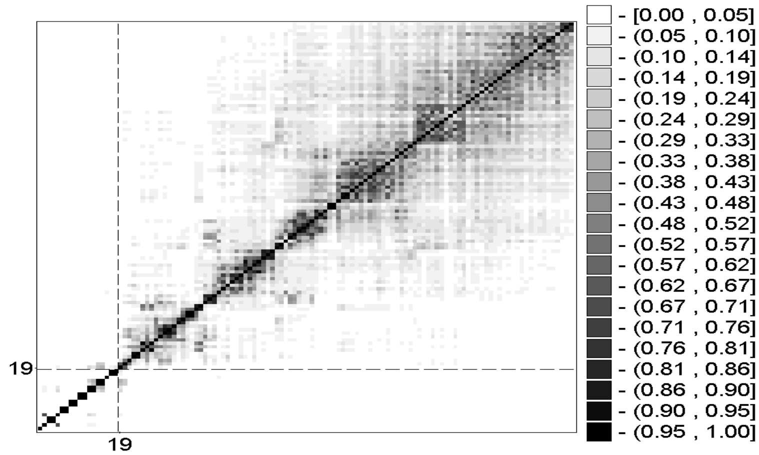


Figure 4: Matrix of $w$-correlations for the 120 reconstructed components.

the ideal situation where the period $P = 1/w$ is a divisor of both the window length $L$ and $K = T - L + 1$. In this ideal situation, the left eigenvectors and principal components have the form of sine and cosine sequences with the same period $P$ and the same phase. Thus, the identification of the components that are generated by a harmonic is reduced to the determination of these pairs.

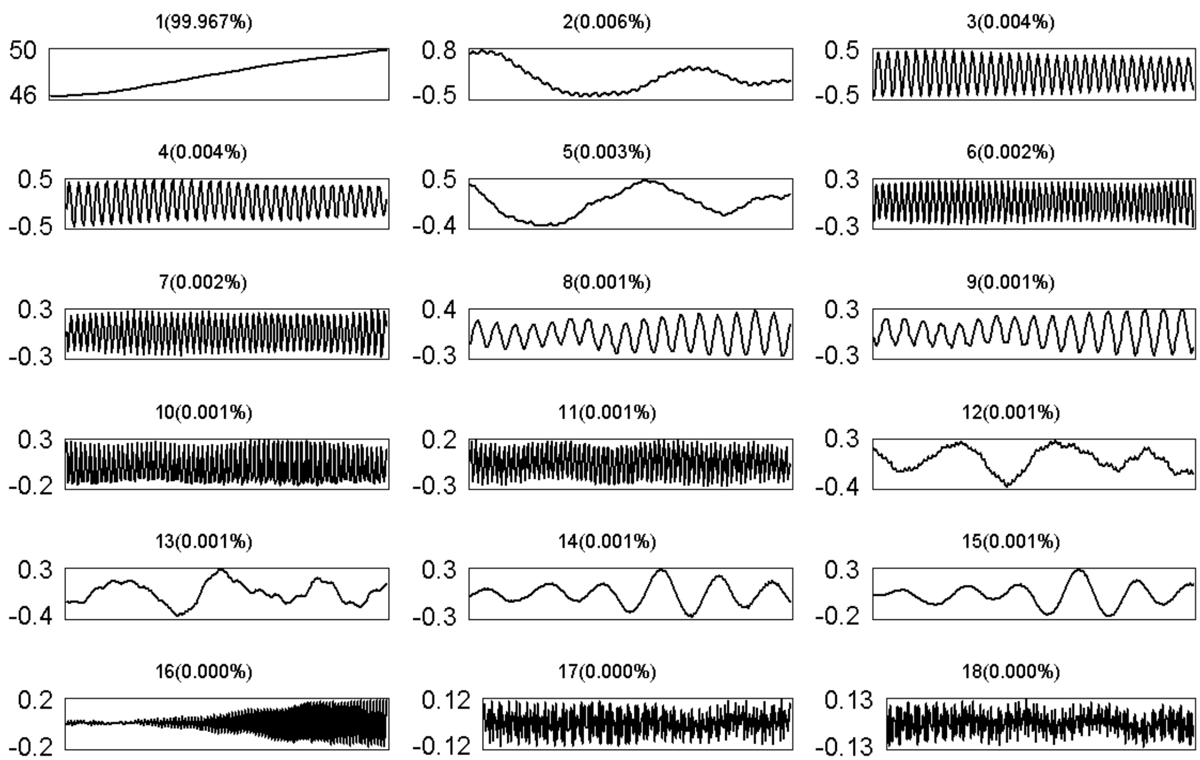


Figure 5: The first 18 principal components plotted as time series

Fig. 6 depicts the scatterplots of the paired principal components in the series, corresponding to the harmonics with periods 6, 4, 12, 3, 36 and 2.4 months. They are ordered by their contribution (share) in the SVD step (from left to right).

The periodograms of the paired eigentriples (3-4 , 6-7, 8-9, 10-11 and 17-18) also

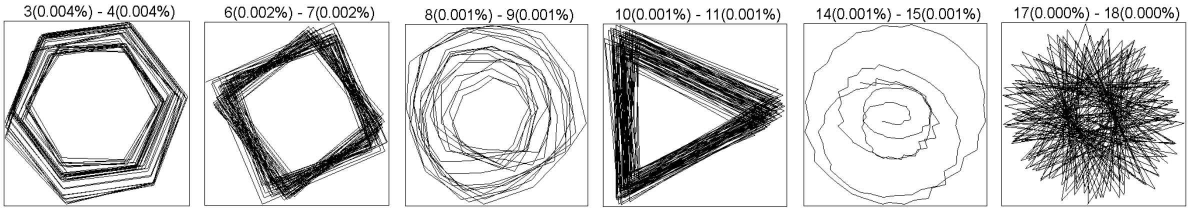


Figure 6: Scatterplots (with lines connecting consecutive points) corresponding to the paired harmonic principal components.

confirm that the eigentriples correspond to the periods of 6, 4, 12, 3, 36 and 2.4 months.

**Construction of Trend, Harmonics and Noise**

Trend is a slowly varying component of a time series which does not contain oscillatory components. Thus to capture the trend in the series, we should look for slowly varying eigenvectors. Fig. 7 (top) shows the extracted trend which is obtained from the eigentriples 1, 2, 5, and 12-13. It clearly follows the trend in the series.

Fig. 7 (middle) represents all the harmonic components and clearly shows the same pattern of seasonality as in the original series. Thus, we can classify the rest of the eigentriples components (19–120) as noise. Fig. 7 (bottom) shows the residuals which are obtained from these eigentriples. The $w$-correlation between the reconstructed series (the eigentriples 1-18) and the residuals (the eigentriples 19-120) is equal to 0.0006 confirms that this grouping is very reasonable. The $p$-value of Anderson-Darling test (Stephens, 1974) for testing normality is 0.6 implying that the residual series has a distribution very close to the normal distribution.

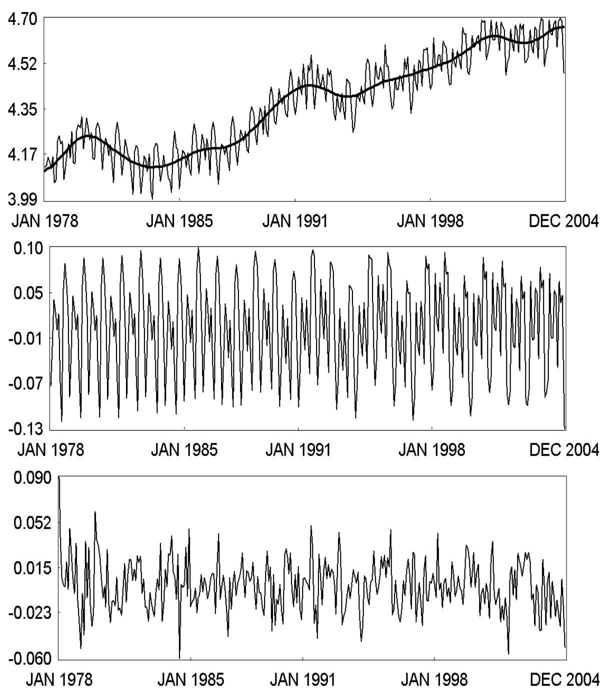


Figure 7: Reconstructed trend (top), harmonic (middle) and noise (bottom).