Multivariate small sample tests for two-way designs with applications to industrial statistics

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# Introduction

In this paper, we present a novel nonparametric approach for multivariate analysis of two-way crossed factorial design based on NonParametric Combination applied to Synchronized Permutation tests. This nonparametric hypothesis testing procedure not only allows to overcome the shortcomings of MANOVA test like violation of assumptions such as multivariate normality or covariance homogeneity, but, in an extensive simulation study, reveals to be a powerful instrument both in case of small sample size and many response variables. We contextualize its application in the field of industrial experiments and we assume a linear additive model for the data set analysis. Indeed, the linear additive model interpretation well adapts to the industrial production environment because of the way control of production machineries is implemented. The case of small sample size reflects the frequent needs of practitioners in the industrial environment where there are constraints or limited resources for the experimental design. Furthermore, an increase in rejection rate can be observed under alternative hypothesis when the number of response variables increases with fixed number of observed units. This could lead to a strategical benefit considering that in many real problems it could be easier to collect more information on a single experimental unit than adding a new unit to the experimental design. An application to industrial thermoforming processes is useful to illustrate and highlight the benefits of the adoption of the herein presented nonparametric approach.

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