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Aberrations of Orthogonal Arrays with removed runs

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1 Introduction

In this work, we consider multilevel Orthogonal arrays (OAs) under the Generalized Minimum Aberration (GMA) criterion, and we focus on the following problem. In several situations, it is hard to define *a priori* a fixed sample size. For example, budget constraints or time limitations may occur after the definition of the design, or even when the experiments are running, thus leading to an incomplete design. In such a situation, it is relevant not only to choose an OA with good properties, but also to define an order of the design points, so that the experimenter can stop the sequence of runs and loose as less information as possible. While OAs with added runs are well studied, see for instance [1] and the references therein, less has been done in the case of OAs with removed runs. Some results in this direction can be found in [3]. For a general reference on OAs, see [2].

2 Removing points from an OA

Let $\mathcal{D}$ be a full factorial design and $\mathcal{F}$ be a fraction. We use for each factor the complex coding of the levels. Moreover, denote with $\{X^\alpha, \alpha \in L\}$ the monomial basis of all complex functions defined on $\mathcal{D}$.

The counting function R of $\mathcal{F}$ is a polynomial defined over $\mathcal{D}$ so that for each $x \in \mathcal{D}$, $R(x)$ equals the number of appearances of x in the fraction. A $0-1$ valued counting function is called an indicator function of a single-replicate fraction $\mathcal{F}$. We denote by c_α the coefficients of the representation of R on $\mathcal{D}$ using the monomial basis $\{X^\alpha, \ \alpha \in L\}$:

$$R(x) = \sum_{\alpha \in L} c_\alpha X^\alpha(x), \ x \in \mathcal{D}, \ c_\alpha \in \mathbf{C} \, .$$

The Generalized Word-Length Pattern (GWLP) of a fraction $\mathcal{F}$ of the full factorial design $\mathcal{D}$ is a the vector $A_\mathcal{F} = (A_0(\mathcal{F}), A_1(\mathcal{F}), \ldots, A_m(\mathcal{F}))$, where

$$A_j(\mathcal{F}) = \sum_{|\alpha|_0 = j} a_\alpha \quad j = 0, \ldots, m \, ,$$

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$$a_\alpha = \left(\frac{\|c_\alpha\|}{c_0} \right)^2 , \qquad (1)$$

$|\alpha|_0$ is the number of non-null elements of α, and $c_0 := c_{(0,\ldots,0)} = \#\mathcal{F}/\#\mathcal{D}$.

The number a_α in Eq. (1) is the aberration of the term X^α. The GMA criterion consists in the lexicographic minimization of the GWLP $A_\mathcal{F}$.

In order to see how the GWLP changes when one or more design points are removed from an OA, we prove a general formula for the GWLP of the union of two or more fractions.

Given k fractions $\mathcal{F}_1, \ldots, \mathcal{F}_k$ with $n_1, \ldots, n_k$ runs respectively, consider their union $\mathcal{F} = \mathcal{F}_1 \cup \cdots \cup \mathcal{F}_k$. The j-th element of the GWLP of $\mathcal{F}$ is

$$A_j(\mathcal{F}) = \sum_{i=1}^{k} \frac{n_i^2}{n^2} A_i(\mathcal{F}_i) + 2\frac{(\#\mathcal{D})^2}{n^2} \sum_{i_1 < i_2} \sum_{|\alpha|_0 = j} \mathrm{Re}(c_\alpha^{(i_1)} \overline{c}_\alpha^{(i_2)}), \quad j = 0, \ldots, m \,. \qquad (2)$$

The formula in Eq. (2) can be applied with two different strategies to study the fractions obtained by deletion of runs:

- first, Eq. (2) ca be used to compute the aberrations of a fraction obtained by an OA by removing a few points;
- second, we can consider the decomposition of a large OA into smaller fractions.

Several examples concerning both approaches will be discussed.

References

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