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Symbolic method of cumulants for subordinated Brownian motions: the variance gamma case

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With symbolic calculus we mean a set of manipulation techniques aiming at performing algebraic calculation [4]. Symbolic calculus applies to Lévy processes since we can represent a Lévy process through its time one moment generating function, which has the following expression:

$$M(z) = \exp\{(K(z) - 1)\}. \tag{1}$$

In the ring of formal power series $\mathbb{R}[[z]]$, equation (1) is well defined independently of convergence radius [9]. Moreover if $M(z)$ and $K(z)$ have the following formal power series expansions:

$$M(z) = 1 + \sum_{i=1}^{\infty} \frac{a_i}{i!} z^i \quad K(z) = 1 + \sum_{i=1}^{\infty} \frac{c_i}{i!} z^i. \tag{2}$$

then $\{c_i\}$ are said the formal cumulants of $\{a_i\}$. Comparing (2) with (1), $\{c_i\}$ result to be cumulants of the time one distribution of the Lévy process and the moments of the Lévy process are sequence of binomial type whose coefficients are $\{c_i\}$.

Using this approach subordination of Lévy processes (see [1] and [8] as references on Lévy processes and subordination) becomes a formal series composition of cumulant generating functions. Therefore, using Faà di Bruno formula [4] we can calculate the cumulants of a subordinated Lévy process starting from the cumulants of the subordinand and of the subordinator.

Our aim is to develop an estimation procedure of the parameters of subordinated Brownian motions based on polykays. Polykays are corrected estimators of cumulant products with minimum variance [7]. We propose to apply this methodology in financial applications. In fact, subordinated Brownian motions are widely used to model asset returns, having the subordinator the appealing interpretation of economic time. In this framework, a famous subordinated Brownian motion is the variance gamma process [5], which is constructed using a gamma subordinator. Its time one moment generating function is therefore the composition of the moment generating function of a gamma process and a Brownian motion:

$$\log M(z) = \frac{1}{\nu} \log \frac{1}{1 - \nu\left(\mu z + \sigma^2 \frac{z^2}{2}\right)} \quad \nu > 0 \quad \mu \in \mathbb{R}, \quad \sigma > 0. \tag{3}$$

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Since the estimate of the variance gamma parameters is still an open issue ([2], [3] and [6]), we apply our procedure to this model.

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