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Asymptotics for High-Dimensional Covariance Matrices of Factor Models

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In this talk we consider high dimensional vector time series $Y_1, ..., Y_n$ with a factor model structure

$$Y_i \;=\; BF_i + E_i, \quad i = 1, ..., n,$$

where the vector F_i contains all factors at time i, for $i = 1, ..., n$. It is assumed that the number of factors is allowed to go to infinity. The matrix B contains the corresponding factor loadings and E_i is the error component.

Factor models are widely used in various areas including psychometrics, marketing, finance as well as natural sciences and technology. For example, factor models are a key tool for financial risk analysis and macroeconomic forecasting of indicators such as the GDP (Gross Domestic Product) and inflation. Since large datasets are becoming increasingly available in many disciplines the analysis of high dimensional time series has become an highly active area. The estimation of high-dimensional variance-covariance matrices is of particular interest, but often only an intermediate step, since interest focuses on the behavior of functions of the sample variance-covariance matrix, especially bilinear forms which naturally arise when studying projection type statistics.

We establish new results on distributional approximations for bilinear functions of sample variance-covariance matrices in terms of Brownian motions. In the high dimensional setting, where also the dimension is allowed to go to infinity, these approximations by Gaussian processes hold true without any constraints on the dimension, the sample size or their ratio. Our results are valid for uniformly ℓ_1-bounded projection vectors which arise either naturally or by construction in many statistical problems like change-point analysis, sparse financial portfolio selection and shrinkage estimation.

A simulation study illustrates the performance of this theory. For example we will use our results to detect changes in the variance of a projection.

References

[1] Kouritzin M. *Strong approximation for cross-covariances of linear variables with long-range dependence* // Stochastic Process. Appl., 1995, v. 60(2), p. 343–353.

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[2] Philipp W. *A note on the almost sure approximation of weakly dependent random variables* // Mh. Math., 1986, v. 102(3), p. 227–236.

[3] Steland A., von Sachs R. *Large sample approximations for variance-covariance matrices of high-dimensional time series* // Bernoulli, 2017, v. 23(4A), p. 2299–2329.

[4] Steland A., von Sachs R. *Asymptotics for high-dimensional covariance matrices and quadratic forms with applications to the trace functional and shrinkage* // Stochastic Process. Appl., to appear.