Stochastic Mesh Method for Non-Linear Functionals on Paths

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Let x_s be a diffusion process $dx_s = a(x_s)dw_s + b(x_s)ds$, $y_t = \int_0^t h_s(x_s)ds$ be integrals on trajectories and $f_t = f_t(x_t, y_t)$ be some "reward" functions. The optimal stopping problem connected with f_t is formulated as calculation of

$$C_T = \sup_{\tau<T} \mathbf{E} f_\tau, \tag{1}$$

where τ are Markov moments.

Examples. It is shown in [1] that a problem of optimal stopping of resource extraction is described with "reward" functions $f_t = \int_0^t \alpha_u S_u du + \beta_t S_t$, where $S_t = \exp(x_t)$ is the resource price. If f_t has the form

$$f_t = \left(e^{\frac{1}{t}\int_0^t x_u du} - K \right)^+$$

then C_T is a price of geometrical average Asian option [2]. The latter gives an example of non-linear functional.

After discretization we receive the chain $z_n = (x_n, y_n)$ which approximates the process (x_t, y_t). The problem is reduced to the solution in the form of backward induction:

$$Y_N(z) = f_N(z), \quad Y_n(z) = \max\left(f_n(z), \mathbf{E}_{n,z} Y_{n+1}(z_{n+1})\right). \tag{2}$$

Mark Broadie and Paul Glasserman in [3] suggested a stochastic mesh method for the sequence (2) which does not depend on the dimension. But conditions for consistency of the method for the above problem are not fulfilled directly because transitional densities for z_n have singularities. It was shown in [4] that in the linear case the component y_n may be randomized and consistency of the method under some conditions was proved.

Here we consider the case when functions f_n are not linear, then this randomization does not work but we can get rid of singularity in a different way.

Let ξ_n be a sequence of independent standard normal variables, define

$$\begin{aligned} x_{2n+1} &= x_{2n} + a(x_{2n})\xi_{2n+1}\sqrt{\Delta} + b(x_{2n})\Delta, \\ x_{2n+2} &= x_{2n+1} + a(x_{2n})\xi_{2n+2}\sqrt{\Delta} + b(x_{2n})\Delta, \\ y_n &= \sum_{i=1}^{n} x_i \Delta. \end{aligned}$$

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We can consider the process $z_n = (x_{2n}, y_{2n})$. Transitional probabilities $p(z, z'')$ for the chain z_n have the form

$$p(z, z'') \quad = \quad \frac{1}{\Delta} \varphi \left(x + b(x)\Delta - x',\, a^2(x)\Delta \right) \varphi \left(x' + b(x)\Delta - x'',\, a^2(x)\Delta \right).$$

where $z = (x, y)$, $z'' = (x'', y'')$, $x' = (y'' - y)/\Delta - x''$, $\varphi(\cdot, \sigma^2)$ being the normal density with variance σ^2. These transitional probabilities are not singular but now conditions from [4] are not fulfilled because the "drift" component in $p(z, z'')$ is unbounded.

It is shown in the present investigation that inequalities from [4] needed for consistency are fulfilled, and thus stochastic mesh method works.

References

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