# On steady state probabilities of renewable systems with Marshal-Olkin failure model.

V. Rykov (Moscow, Russia)

e-mail vladimir_rykov@mail.ru

March 12, 2018

**Abstract**

**1.** Stability of systems is a key problems in all natural sciences. For stochastic systems stability often means insensitivity or low sensitivity of their output characteristics to the shapes of some input distributions. There are some papers devoted to these problems (for the bibliography see, for example, [1]). Most of these investigations deal with system which components fail independently. In 1967 Marshall and Olkin [2] proposed bivariate distribution with depend components that can be used as a failure model for two-component reliability system. In the paper [1] the reliability function of the same system with Marshal-Olkin (MO) failure model for the its elements has been studied. In the present paper this investigations has been developed for the steady state probabilities (s.s.p) of the same system.

**2.** Consider a heterogeneous hot double redundant repairable system, the failure of which components satisfies to the MO model which can be represented by stochastic equation

$$(T_1, T_2) = (\min(A_1, A_3), \min(A_2, A_3)). \tag{1}$$

This means that there exists three reasons (shocks), which leads to the system failure. The first shock, time to which is a r.v. $A_1$, act only to the first component, the second one, time to which is a r.v. $A_2$, act only to the second one, while the third one, time to which is a r.v. $A_1$, act to both components and leads to the system failure. The r.v.'s $A_1, A_2, A_3$ are supposed to be independent exponentially distributed r.v.'s with parameters $\alpha_i$ $(i = 1, 2, 3)$.

In this paper it is supposed that after partial failure (when only one component fails) the partial repair begins, which means that the system prolong to work and the failed component begins to repair. But after the system failure the renewal of whole system begins that demand some random time, say $B_3$ with some c.d.f. $B_3(t)$, and after this time the system goes to the state 0. In any case the repair times $B_i$ $(i = 1, 2, 3)$ of components and the whole system has absolute continuous distributions with cumulative distribution functions (c.d.f.) $B_i(x)$ $(i = 1, 2, 3)$ and probability density functions (p.d.f.) $b_i(x)$ $(i = 1, 2, 3)$ correspondingly. All repair times are independent. In further we will use the following notations.

- $\alpha = \alpha_1 + \alpha_2 + \alpha_3$ the summary intensity of the system failure;
- $\bar{\alpha}_k = \alpha_k + \alpha_3$ the intensity of the $k$-th and third shock;
- $b_k = \int_0^\infty (1 - B_k(x))dx$ $(k = 1, 2, 3)$ $k$-th element repair time expectations;
- $\rho_k = b_k\alpha_i$;
- $\beta_k(x) = (1 - B_k(x))^{-1} b_k(x)$ $(k = 1, 2, 3)$ $k$-th element conditional repair intensity given elapsed repair time is $x$;
- $\tilde{b}_k(s) = \int_0^\infty e^{-sx} b_k(x)dx$ $(k = 1, 2, 3)$ Laplace transform (LT) of the $k$-th element repair time distribution.

Under considered assumptions the system state space can be represented as $E = \{0,\ 1,\ 2,\ (3,1),\ (3,2)\}$, which means: 0 — both components are working, 1 — the first component is repaired, and the second one is working, 2 — the second component is repaired, and the first one is working, $(3,1)$ — both components are in down states, and the first one is repaired, $(3,2)$ — both components are in down states, and the second is repaired.

For s.s.p. calculation we will use the method of additional variables introduction or so called markovization method that consists in introduction of some special additional variables in order to describe the system behavior with Markov processes. For our case as additional variables we use elapsed time of the failed component. Thus let us consider two-dimensional Markov process $Z = \{Z(t),\ t \geq 0)\}$, with $Z(t) = (J(t), X(t))$ where $J(t)$ is the system state in the time $t$, and $X(t))$ represents elapsed time of the failed component. The process phase space is $\mathcal{E} = \{0, (1, x), (2, x), (3, 1, x), (3, 2, x)\}$.

**3.** The main results are represented in the following theorems

**Theorem 1.** The s.s.p.'s of the system under consideration has the form

$$\begin{aligned}
\pi_1 &= \frac{\alpha_1}{\bar{\alpha}_2}(1-\tilde{b}_1(\bar{\alpha}_2))\pi_0, \\
\pi_2 &= \frac{\alpha_2}{\bar{\alpha}_1}(1-\tilde{b}_2(\bar{\alpha}_1))\pi_0, \\
\pi_3 &= [\alpha_1(1-\tilde{b}_1(\bar{\alpha}_2))+\alpha_2(1-\tilde{b}_1(\bar{\alpha}_2))+\alpha_3]b_3\pi_0 \qquad (2)
\end{aligned}$$

where $\pi_0$ is given by

$$\pi_0 = \left[1+\alpha_1(1-\tilde{b}_1(\bar{\alpha}_2))\left(b_3+\frac{1}{\bar{\alpha}_2}\right)+\alpha_2(1-\tilde{b}_2(\bar{\alpha}_1))\left(b_3+\frac{1}{\bar{\alpha}_1}\right)+\alpha_3 b_3\right]^{-1}. \qquad (3)$$

This theorem show an evident dependence of the system s.s.p. from the shapes of elements repair time distributions. However under rare failures this dependence became negligible

**Theorem 2.** Under the rare components' failures, when $q = \max[\alpha_1, \alpha_2, \alpha_3] \to 0$ the s.s.p. of the considered system take the form

$$\begin{aligned}
\pi_0 &\approx [1+\rho_1+\rho_2+\rho_3+b_3(\bar{\alpha}_1\rho_1+\bar{\alpha}_2\rho_2)]^{-1}, \\
\pi_i &\approx \rho_i\pi_0 \quad (i=1,2), \\
\pi_3 &\approx [\rho_3+b_3(\bar{\alpha}_1\rho_1+\bar{\alpha}_2\rho_2)]\pi_0. \qquad (4)
\end{aligned}$$

**Remark.** One can see that the part $b_3(\bar{\alpha}_1\rho_1+\bar{\alpha}_2\rho_2)$ in probabilities $\pi_0$ and $\pi_3$ has the second order with respect to $q$, and therefore using only the first order of this value the above formulas can be rewrite as follows

$$\begin{aligned}
\pi_0 &\approx [1+\rho_1+\rho_2+\rho_3]^{-1}, \\
\pi_i &\approx \rho_i\pi_0 \quad (i=1,2,3).
\end{aligned}$$

**Acknowlegements**

The publication has been prepared with the support of the "RUDN University Program 5-100" and funded by RFBR according to the research projects No. 17-01-00633 and No. 17-07-00142, and has been supported by FAPESP Grant No. 2013/07375-0.

# References

[1] D Kozyrev, V Rykov, N Kolev (2018). Reliability Function of Renewable System under Marshall-Olkin Failure Model.// RTA. No. 2, 2018. Accepted to the publication

[2] A. Marshall, and I. Olkin (1967). A multivariate exponential distribution.// Journal of American Statistical Association, **6**2, 30-44.