New algorithms of Monte Carlo method for investigation of criticality of particle scattering process with multiplication in random media

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A Monte Carlo algorithm admitting parallelization is constructed for estimation of probability moments of the spectral radius of the operator of the integral equation describing transfer of particles with multiplication in a random medium. A randomized homogenization method is developed with the same aim on the base of the theory of small perturbations and diffusive approximation. Test calculations performed for a one-group spherically symmetric model system have shown a satisfactory concordance of results obtained from two models.

1 Introduction

This work is focused on development of numerical statistical modelling algorithms for studying fluctuations of the criticality of particles transfer with multiplication in a random medium. For the sake of simplicity, we consider a one-velocity transfer process with isotropic indicatrices of scattering and splitting (replication), however, the structure of proposed algorithms does not practically change in the passage to more complex many-velocity and anisotropic models of the process. For the variant of the process presented here, it is expedient to consider the following homogeneous integro-differential transfer equation:

$$(\omega, grad\Phi) + \sigma\Phi = \frac{1}{k}\Big[\sigma_s\Phi_0 + \nu\sigma_f\Phi_0\Big].$$

Here $\Phi \equiv \Phi(r,\omega)$ is the density of the flow of particles (radiation intensity), $\sigma \equiv \sigma(r)$ is the total cross-section (attenuation coefficient), and $\sigma = \sigma_s + \sigma_c + \sigma_f$, where σ_s is the cross-section of scattering, σ_f is the cross-section of fission, σ_c is the cross-section of capture, ν is the mean number of particles emitted from the points of fission, $r \in R^3$ is a spatial point, ω is a unit direction vector, $\Phi_0(r) \equiv \int \Phi_0(r,\omega)d\omega$.

2 Formulation of the problem

If the parameters of the problem are random, then k is random as well and, in particular, it is practically important to estimate the probability $P(k > 1)$, i.e., the probability of over-criticality of the transfer problem of particles with

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multiplication in a random medium. This can be performed by estimating the probability moments Ek and Dk, i.e., the mean value and the variance of k. Using the approximate Gaussian property of the distribution of the random variable k, one can estimate the probability $P(k > 1)$ sufficiently well. Test numerical statistical modelling allows us to check the adequacy of this technique.

The value k is an effective coefficient of particle multiplication in generations of scattering acts. It can be estimated (see, e.g., review in [1]) by direct simulation of the transfer process. It is clear that this estimator is not very efficient for solving stochastic problems because it does not admit the use of double randomization. Therefore, we formulate Monte Carlo algorithms related to the limit relation [2]

$$k = \lim_{n\to\infty} k_n, \quad k_n = \sqrt[n]{\|K^n\|},$$

and

$$\|K^n\| = \sup_{x\in X} \int \dots \int k(x,x_1)k(x_1,x_2)\dots k(x_{n-1},x_n)\,dx_1\dots dx_n. \tag{1}$$

According to the definition of the substochastic density $k(x',x)$, the value $\|K^n\|$ equals to the mean number of collisions of the n-th number under the condition that the source normalized by one collision is positioned at the point x_0. Thus, we actually formulate an iterative algorithm of direct statistical modelling for estimation of the value $\|K^n\|$ with the special initial density function in the expression (1). Performing calculations simultaneously for different values of n according to (1), we can construct estimators for Ek and Dk based on approximation of the function $x^{1/n}$ and the corresponding double randomization [3].

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References

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