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# Bounds on the Rate of Convergence for Constant Retrial Rate Queueing Model with Two Servers

Yacov Satin [1], Evsey Morozov, Ruslana Nekrasova [2], Alexander Zeifman [3], Ksenia Kiseleva, Anna Sinitcina, Alexander Sipin, Galina Shilova [4], Irina Gudkova [5]

The paper deals with a $M/M/2/2$-type retrial queueing system with a constant retrial rate denoted as $\Sigma$. The exogenous (primary) customers arrive to the system according to a Poisson process with rate $\lambda$. The system has two stochastically identical servers with i. i. d. exponential service times, general service time $S$ and rate $\mu := 1/ES$.

The system has the following special feature: if a primary customer finds both servers busy it goes to some kind of infinite-capacity repository (the so-called orbit that can be considered as a FIFO queue). Let us also assume that the orbit works as a single FIFS server, in which a head line (the oldest) secondary customer tries to enter the arbitrary server after an exponentially distributed time with (orbit) rate $\mu_0$. (It returns to the orbit if both servers are busy). Since each secondary costumer has (potentially) an infinite number of attempts to enter servers, the system has no losses.

First, a single-server retrial queue with constant retrial rate was suggested in [5] and used to simulate a telephone exchange system. The authors of ([1, 2] extended the model with multiple servers and waiting places. The paper [2] deals with the performance and stability analysis of the model by means of matrix-analytical method for $c \geq 2$ servers. The authors found the stability criteria of the model in an explicit form depending on system parameters $\lambda$, $\mu$, $\mu_0$. In recent paper [3, 6]), the retrial model with constant retrial rate was considered in a general way and includes a multi-server case with general service times and multi-class customers. Nevertheless, the above mentioned researches assume the positive recurrence of the process (regenerative, not necessary Markovian) and as a result the existence of the stationary distribution. However, none of these papers analyze the rate of convergence to stationarity.

We extend the analysis of this type of systems by finding the rate of convergence to stationarity of the corresponding Markov process. We describe a two-server retrial model and thereby extend the similar analysis developed for the simplest

[1]Vologda State University, Vologda, Russia

[2]IAMR, KarRC RAS and Petrozavodsk State University, Petrozavodsk, Russia

[3]Vologda State University, Vologda, Russia, IPI FRC CSC RAS; VolSC RAS, E-mail: a_zeifman@mail.ru

[4]Vologda State University, Vologda, Russia

[5]Peoples Friendship University of Russia (RUDN University), IPI FRC CSC RAS, Moscow, Russia

$M/M/1$-type retrial model in [9]. The retrial queueing systems with constant retrial rate could be applied for a wide range of applications, e.g. for unslotted Carrier Sense Multiple Access with Collision Detection (CSMA/CD) protocol ([4]) or for the optical-electrical hybrid contention resolution scheme for Optical Packet Switching (OPS) networks ([7, 8]). In the paper, we derive the sufficient conditions for the null ergodicity and ergodicity of the corresponding process and obtain the upper bounds on the rate of convergence for the both situations. Numerical examples of bounding the rate of convergence for the corresponding models are also given.

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