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On the evaluation of spatial-angular distributions of polarization characteristics of scattered radiation

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In this talk, we present an algorithm of the Monte Carlo method for evaluation of spatial-angular characteristics of scattered polarized radiation. Suggested algorithm is constructed on the idea of projective expansion of the vector function of the angular distribution of the radiation flux in terms of hemispherical harmonics, which are based on associated Jacobi polynomials.

In previous works ([1], [2]) we proposed a randomized projective method for estimating the corresponding one-dimensional spatial-angular distribution for the particular axisymmetric case of the problem. This method is based on the expansion of the vector integral intensity of radiation in terms of modified Jacobi polynomials, orthonormalized with "Lambertian" weight $|\cos\theta|$. Here θ is the latitude reckoned from the normal to the surface. Now the challenge is to construct the similar algorithm of numerical statistical modeling for evaluation of two-dimensional angular characteristics of partially polarized radiation, transmitted and reflected by layers of the scattering and absorbing medium.

To do this, we consider the system of orthonormal with weight $\mu = |\cos\theta|$ on the set $(0,1) \times (0, 2\pi)$ functions $\{H_{i,j}(\mu,\varphi),\ i = 0, ..., \infty, j = -i, ..., i\}$ in the Hilbert space $\mathcal{L}_{2,\mu}(\Omega_+)$ on the hemisphere of unit directions Ω_+. Here θ is the zenith angle, φ is the azimuth angle. The explicit form of the functions $H_{i,j}(\mu,\varphi)$ was obtained from the orthogonal system of hemispherical harmonics, first proposed in the paper [3]. In [4], the completeness of this system of hemispherical harmonics was proved.

After expansion of the vector function of the angular distribution of the radiation flux in terms of the orthonormal basis $\{H_{i,j}(\mu,\varphi),\ i = 0, ..., \infty, j = -i, ..., i\}$ and truncating it, we can get an estimator as a partial sum with coefficients that can be obtained with weighted vector Monte Carlo method [2].

We conclude by presenting some numerical results we have got for two-dimensional angular distributions of the intensity and the degree of polarization of radiation, transmitted and reflected by optically thick layers of a scattering and absorbing substance.

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