The probability of the capital staying above zero long enough in the Cramér-Lundberg model

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We consider the classical Cramér-Lundberg model, in which the capital X_t of an insurance company at time t is represented by the stochastic process

$$X_t = x + ct - \sum_{n=1}^{N_t} Y_n,$$

where $x \geq 0$ is the initial capital, $c > 0$ is the constant premium rate, N_t is a standard Poisson process with intensity λ, $\{Y_n\}$ are independent random variables distributed exponentially with parameter α, which are independent of N_t.

Let τ_d^X be the time of Parisian ruin, i. e., the first time when the capital has stayed below zero for at least d. For this relatively simple model, there is an explicit formula for the probability of ultimate Parisian ruin $\mathsf{P}\left(\tau_d^X < \infty\right)$, it can be found in [1].

Let η_l^X be the first time when the capital has stayed above zero for at least l. Then $P(\eta_l^X < \tau_d^X)$ is the probability of this happening before ultimate Parisian ruin. Using an argument similar to one in [1], we have derived an explicit formula for this probability:

$$\mathsf{P}\left(\eta_l^X < \tau_d^X\right) = \bar{G}_{12}(l) + \frac{G_{12}(l) P_{21}(d) \bar{P}_{12}(l)}{1 - P_{21}(d) P_{12}(l)},$$

where

$$\begin{aligned} g_{12}(t) &= \frac{\lambda e^{-\alpha x} e^{-(c\alpha+\lambda)t}}{ct + x} \left(x I_0 \left(2\sqrt{\alpha\lambda t(ct + x)} \right) \right. \\ &\quad \left. + \frac{ct}{\sqrt{\alpha\lambda t(ct + x)}} I_1 \left(2\sqrt{\alpha\lambda t(ct + x)} \right) \right), \quad t \geq 0, \\ G_{12}(l) &= \int_0^l g_{12}(t)\, dt, \quad \bar{G}_{12}(l) = 1 - G_{12}(l), \\ P_{12}(l) &= \sqrt{\frac{\lambda}{c\alpha}} \int_0^l e^{-(\lambda + c\alpha)t} t^{-1} I_1 \left(2t\sqrt{\lambda c\alpha} \right) dt, \quad \bar{P}_{12}(l) = 1 - P_{12}(l), \\ P_{21}(d) &= \sqrt{\frac{c\alpha}{\lambda}} \int_0^d e^{-(\lambda + c\alpha)t} t^{-1} I_1 \left(2t\sqrt{\lambda c\alpha} \right) dt, \end{aligned}$$

I_ν is the modified Bessel function.

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We have also performed an analysis of the sensitivity of the function $P(\eta_l^X < \tau_d^X)$ to the parameters α, λ, x, c, d and l. Namely, large samples of these parameters were generated and the influence of each parameter was analyzed using scatterplots.

Also, another way of analyzing sensitivity was used. Applying an algorithm from the book [2], the numerical value of the first-order sensitivity index of the main function on each parameter was found, as well as the total-effect index.

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References

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