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Statistical modelling algorithm for solving the nonlinear Boltzmann equation on base of the projection method

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A statistical modelling algorithm is constructed for solution of the nonlinear kinetic Boltzmann equation based on the projection method. Hermite polynomials are used as an orthonormalized basis. The algorithm was tested on calculations for the problem of one-dimensional relaxation of gas with a known solution.

1 Introduction

In this report we consider solution of the Cauchy problem for a nonlinear kinetic Boltzmann equation in a spatially homogeneous case. It is well known that the approximate solution to this problem can be estimated by the Monte Carlo method simulating a Markov process of evolution of the corresponding N-particle ensemble whose phase states change due to pairwise interactions of particles (see, e.g., [1], [3], [4]).

The spatially homogeneous relaxation of a simple gas is described by the following Cauchy problem for the nonlinear Boltzmann equation (in dimensionless form) [2]:

$$\frac{\partial f(\mathbf{v},t)}{\partial t} = \int [f(\mathbf{v}',t)f(\mathbf{v}_1',t) - f(\mathbf{v},t)f(\mathbf{v}_1,t)]w(\mathbf{v}',\mathbf{v}_1' \to \mathbf{v},\mathbf{v}_1)d\mathbf{v}'d\mathbf{v}_1'd\mathbf{v}_1,$$

$$f(\mathbf{v},t)\,\Big|_{t=0} = f_0(\mathbf{v}),$$

where

$$w(\mathbf{v}',\mathbf{v}_1' \to \mathbf{v},\mathbf{v}_1) = 4\sigma(g,\cos\vartheta)\delta^{(3)}(\mathbf{v}'+\mathbf{v}_1'-\mathbf{v}-\mathbf{v}_1)\delta\Big(\frac{\mathbf{v}'^2+\mathbf{v}_1'^2}{2} - \frac{\mathbf{v}^2+\mathbf{v}_1^2}{2}\Big),$$

and $g = |\mathbf{v}'-\mathbf{v}_1'|$, $\sigma(g,\cos\vartheta)-$ is the differential cross-section of scattering of two particles, $\vartheta-$ is the angle of rotation of relative velocity in the system of the center of inertia of scattering particles.

Here$f(\mathbf{v},t)$ is the distribution density of gas particles over velocities $\mathbf{v} \in \mathbf{R}^3$ at the time moment $t \geq 0$, and $\int f(\mathbf{v},t)d\mathbf{v} = 1$.

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Let us pose the problem of calculation of the distribution density of particles over the absolute value of the velocity, i.e.,

$$\varphi(|\mathbf{v}|, t) = |\mathbf{v}|^2 \int f(\mathbf{v}, t) d\omega, \quad \mathbf{v} = |\mathbf{v}| \cdot \omega.$$

For the sake of convenience, introduce the notation $x = |\mathbf{v}|$.

This report is focused on calculation of the solution to the problem formulated above by the Monte Carlo method based on the projection method.

References

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