Approximate spectral model of periodically correlated air temperature time-series

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The article is devoted to the issues related to the use of periodically correlated processes for the simulation of real time series. Two types of models of the periodically correlated processes are considered. The first model is based on the representation of the periodically correlated process of a discrete argument in the form of a vector stationary process with a given matrix correlation function. The second model is based on the one special spectral representation of a periodically correlated process. Each of these models has its advantages and disadvantages. For example, the first model is convenient for modeling long Gaussian sequences with a constant time step. And the second model is, in general case, approximately Gaussian, but this model allows constructing processes both in equidistant and not equidistant points in time. We used six-hour measurement observations data of surface air temperature for the month of May with the period 1938-1984 at the meteorological "Sverdlovsk" station in Russia to construct models for real processes.

The Gaussian periodically correlated process being interpreted as vector stationary processes with a given matrix correlation function is considered as the first model. For a process with discrete time on a limited interval matrix correlation function corresponds to the block-Toeplitz correlation matrix and the process is constructed on the basis of the conditional distributions method. The vector models of autoregression of a fixed order are used to model long-length vector sequences. In this model, a sample matrix correlation function of time series of air temperature evaluated by real data is used as the matrix correlation function.

As the second model of a periodically correlated process, a nonrandomized spectral model of a random process in the form:



is considered , where  and  - are independent standard normal values,

,

where  is partition of the spectral space, , and spectral densities  are periodic functions of time .

In this paper, we proposed an approach to estimate the input characteristics of this model on the basis of real temperature data. The approach is based on the Fourier transformation of the correlation functions obtained by means of a special approximation of the sample correlation functions of stationary processes for each observation period. The correlation functions for each observation period are used as the input characteristics when constructing the spectral model, but cross-correlation is determined by the model under consideration. However, a periodically correlated process with all the correlations taken into account can be constructed with the help of the first model. We constructed the time dependencies on the process correlations based on the sample models for various time shifts based on the first and second models. 1 000 000 simulated trajectories were used for modeling. Comparison of the model correlations with the actual ones has shown that the first model reproduces the real character of the correlation dependence on time more accurately than the second one. This is because cross-correlation is not the input parameters of the second model. So, it is expedient to use the first model for a realistic description of the time series of the surface air temperature. The second model can be useful for modeling such time series for which the correlation structure is well described by the class of correlation functions corresponding to the given model.

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