Some properties of quasi random numbers and their randomizations

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A sequence of vectors  in the unit *s*-dimensional hypercube is called quasi-random (low discrepancy) sequence, if its star-discrepancy has the best order . The well-known Koksma-Hlawka inequality [1] in this case gives the estimation of discrepancy decreasing of numeral integration error in the class of functions of bounded variation in the sense of Hardy and Krause also as . Since the logarithm is a function growing as at an arbitrarily small , this suggests that the quasi-Monte Carlo methods are significantly better than the Monte Carlo method. This is true at moderate values of  and large values of . However at large  (, for example), good asymptotic can occur at the excessively large values of , practically unrealized on a computer (compare  and  at  and ). For moderate , another asymptotic is valid. The paper gives a number of numeral examples that support these considerations.

Thus, only randomization of sequences can be recommended for the method quasi-Monte Carlo error estimation. These methods have close connection with the theory of quadrature formulas with one free node [2]. In the paper, these connections are discussed and are illustrated by numeral examples. We also propose the reception of the Holton sequence [3] transformation taking into account the behavior of the integrand. Advantage of such approach is confirmed by numeral examples.

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**References**

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