Abstracts of the Ninth Workshop on Simulation (2018) 1-3

A Nonhomogeneous Risk Model

Alice Gross [1]

1 Introduction

This paper deals with the problem of ruin in the compound binomial model that describes the temporal evolution of the surplus of an insurance company at fixed dates. The premium income process is deterministic, and the successive claim amounts are independent and nonidentically distributed. Our purpose is to evaluate the probability of ruin over any finite time horizon under the assumption of the model where the claim sizes are distributed as a discrete phase-type distribution.

2 Formulation of the problem

We consider a discrete-time model in which the surplus process of an insurance company $\{U_n, n \in \mathbb{N}\}$ is defined as

$$U_n = u + n - S_n,$$

where

- $u \in \mathbb{N}$ is the initial surplus;
- $S_n = \sum_{i=1}^{n} Y_i$, $n = 1, 2, \ldots$;
- Y_i is the claim amount at time i, $\{Y_i\}_{i>0}$ are assumed to be independent and are allowed to have different distributions: $\mathbf{P}(Y_i = k) = p_k^{(i)}$ for $k, i = 0, 1, 2, \ldots$.

At the beginning of each period, the company receives a constant premium, assumed to be equal to 1, as a prepayment to cover the risk during that period. Thus, the evolution of the insurance portfolio in discrete time is given by the process mentioned above.

Premiums are assumed to be constants in the classical model, whereas claims are forming a sequence of uniformly distributed, independent random variables. In the present work claims are still independent, but we assume that they can have different distributions, which increases the accuracy of our model.

One of the main characteristics in risk theory is the finite-time ruin probability defined by $\mathbf{P}_u(\tau^0 < t) \equiv \mathbf{P}(\tau^0 < t \mid U_0 = u)$ for the ruin time

$$\tau^0 = \inf\{n \in \mathbb{N} \colon U_n < 0\}$$

[1]Faculty of Mechanics and Mathematics, Moscow State University, 119991, Moscow, Russia. E-mail: grossak@yandex.ru

and fixed time horizon t (see [1], [2]).

We define the Parisian ruin time τ^d as

$$\tau^d = \inf\{n \in \mathbb{N} \colon n - \sup\{s < n \colon U_s > 0\} > d, U_n < 0\},$$

and calculate the Parisian ruin probability over a finite horizon $\mathbf{P}_u(\tau^d < t)$. With $d = 0$ we arrive at the classical ruin probability. Thus, we extend the concept of ruin to the Parisian type of ruin (see [3]).

The paper aims to calculate the Parisian ruin probability of an insurance company under the assumption of a discrete-time risk model where the claims are distributed as a discrete phase-type distribution.

References

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