Abstracts of the Ninth Workshop on Simulation (2018) 1-2

Excess of D-optimal Designs for Cobb-Douglas model

Yu. D. Grigoriev[1], V. B. Melas[2], P. V. Shpilev[3](✉)

1 Introduction

The problem of searching nonsingular optimal designs with the minimal number of support points is quite important since the use of such designs allows to decrease experimental expenses. Many works were devoted to the study of this problem, see [7], [8] and [1]). In pioneer paper [2] was shown that D-optimal designs are always saturated for polynomial regression models, i. e. the number n of support points of these designs coincides with the number p of unknown parameters $\theta \in \Theta \subseteq R^p$ of the model $\eta(x, \theta)$. In [6], this result is called the de la Garza phenomenon. On the other hand, for nonlinear in parameters models, cases in which optimal designs arise with the number of support points $n > p$ are not rare. In our recent paper [4] we proposed to call such cases the *excess phenomenon*, and the corresponding designs *excess* designs. The series of papers [9], [10], [11] and [3] consider the question on transferring de la Garza result to nonlinear models. For example, in [9] and [11] it was provided an approach allowing to decrease the upper estimate for the number of support points which is equal to $\frac{p(p+1)}{2}$ for D-optimal designs following from the Caratheodori theorem. In some cases, the Yang method directly leads to the conclusion that the locally optimal design $\xi = \xi(\theta)$ for the model in question is saturated. One manages to decrease the upper estimate for the number of support points in the Yang-Stufken method due to analysis of the structure of the information matrix elements. It is evident that, if there are equal elements among them or if they possess certain specific properties, then the account for this information leads to reducing the upper estimate.

Most authors are concentrated their attention on models with one explaining variable whereas many regression models used in practice are multidimensional. These models are much more difficult to study and methods that work good enough for one dimensional models (such as Yang-Stufken method and its specifications for example) usually can't be generalized to a multidimensional case. To a large extent, this is related to the fact that Chebyshev systems of functions do not exist in this case, see [3].

Excess phenomenon also takes place for the locally optimal design for multidimensional models. The analytical solution of the problem of finding the dependence between the number of the locally optimal design support points and the

[1]St.-Petersburg State Electrotechnical University, Russia, E-mail: yuri_grigoriev@mail.ru

[2]St.-Petersburg State University, Russia, E-mail: vbmelas@post.ru

[3]St.-Petersburg State University, Russia, E-mail: p.shpilev@spbu.ru

lengths of the design intervals could be a very useful instrument which allows the investigator to choose the most suitable design space to reduce experimental expenses. In our recent paper [4] we got some results for Ayen-Peters model but to our best knowledge no other analytical solutions of this problem for multidimensional models are available in the literature. The purpose of the present paper is to study excess phenomenon on the example of the two-dimensional Cobb-Douglas model which is used in microeconomics [5]. In our work, we show that, for some homotheties $T\colon \mathcal{X} \to \mathcal{X}'$ of the design space $\mathcal{X}$, saturated locally D-optimal designs for two-dimensional model can become excess and vice versa. We find saturated designs in the explicit form. In our paper we provide an analytical solution of the problem we've discussed.

References

[1] Atkinson, A. C., Donev, A. N., Khuri, A. I., Mukerjee, B., Sinha, B. K., and Tobias, R. D. Optimum experimental design. with SAS *Oxford University Press*, Oxford, 2007.

[2] de la Garza, A.. *Spacing of information in polynomial regression.* // Math. Statist., 1954, 25:123–130.

[3] Dette, H. and Melas, B. *A note on the de la Garza phenomenon for locally optimal designs.* // Ann. Statist., 2011, 39(2):1266–1281.

[4] Grigoriev, Yu. D., Melas, V. B., Shpilev, P. V. *Excess of locally D-optimal designs and homothetic transformations.* // Vestnik St. Petersburg University: Mathematics, 2017, 50(4): 329-336.

[5] Heady, E. O. and Dillon, J. L. *Agricultural Production Functions* Ames: Iowa State University Press, 1961.

[6] Khuri, A. I., Mukerjee, B., Sinha, B. K., and Ghosh, M. *Design issues for generalized linear models: A review.* // Statistical Science, 2006, 21(3):376–399.

[7] Melas, V. B. *Functional Approach to Optimal Experimental Design.* Springer Science + Business Media, Inc., New York, 2006.

[8] Pukelsheim, F. *Optimal Design of Experiments.* SIAM, Philadelphia, 2006.

[9] Yang, M. and Stufken, J. *Support points of locally optimal designs for nonlinear models with two parameters.* // Ann. Statist., 2009, 37:518–541.

[10] Yang, M. and Stufken, J. *Identifying locally optimal designs for nonlinear models: a simple extension with profound consequences.* // Ann. Statist., 2012, 40(3):1665–1681.

[11] Yang, M. *On the de la Garza phenomenon.* // Ann. Statist., 2010, 38:2499–2524.