Abstracts of the Ninth Workshop on Simulation (2018) 1-1

The ϵ-complexity of finite dimensional continuous maps

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The concept of complexity for different mathematical objects is constantly in sight of many outstanding scientists. In our talk, we recall some basic ideas in approach to the problem of complexity, and then present our concept of the *ϵ-complexity* of continuous maps.

One of the first efforts to provide a quantitative approach to the concept of the "complexity of a physical system" was made in the 1870s by Boltzmann, who introduced the notion of entropy in equilibrium statistical physics. The greater the entropy, the more "complicated" the system is.

In the 1940s Shannon developed the concept of entropy to measure the uncertainty of a discrete random variable. He interpreted the entropy as a measure of the "degree of uncertainty" peculiar to a particular probability distribution. It can be shown that Shannon's entropy of stationary and ergodic random sequence is the coefficient in the asymptotic of the logarithm of the number of typical trajectories when time goes to infinity. Therefore, it is possible to use Shannon's entropy as the measure of "complexity" for stationary and ergodic random sequence.

In the 1950s Kolmogorov and Sinai introduced the entropy concept to the theory of dynamical systems. The entropy of a dynamical system is the coefficient of the asymptotic behavior of the logarithm of the number of different types of trajectories of a dynamical system when the time goes to infinity. Again, the entropy of a dynamical system may serve as a measure of "complexity": the more "complex" the system, the richer variety of its trajectories.

In the mid-sixties Kolmogorov has offered the general idea about "complexity of an object." At the semantic level, this idea can be described as follows: *a "complex" object requires a lot of information for its reconstruction and, for a "simple" object, little information is needed.* The "complexity" of an object should be measured by the length of its *"shortest description"*.

We show that neither Shannon's entropy, nor entropy of dynamic systems, nor Kolmogorov complexity are good tools for assessment of complexity of continuous maps. In this talk, we propose the concept of the *ϵ-complexity* for finite-dimensional continuous maps. This concept is in line with Kolmogorov's idea, but it is not confined to it. Our approach is based on a recovery of a continuous map by its values at some uniform grid. This approach allows receiving effective characterization of a complexity for Hölder maps. Employing the ϵ-complexity it was possible to offer *model-free* approach to the problems of classification and segmentation for data of arbitrary nature.

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